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Multi-Sample Scale Tests for Comparing Scale Parameters with Equal and Unequal Location Differences

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Abstract: In this paper we have discussed some tests which is used for equality of scale parameters under equal and unequal location parameters. Test considered here are Levene's test, Bartlett's test, Box – Anderson Test, Janckknife test, test based on bootstrap and Lepage test. We have found out some results to know the performance (in terms of level and power) of these tests using simulation technique. Results are display in various table and graphs. Discussions and conclusions are made on the basis of results obtained.

Keywords: Scale parameters, Multi-sample test, Simulation, Power.

1. INTRODUCTION

For testing the equality of scale parameters (Variance) in various conditions, many research works had already been done. Whenever we give a look in the literature, it is observed that many tests has been developed among them some of reported at the best. But this is controversial and some of the statistician (viz. Brown and Forsythe, 1974; Conover et al., 1981; Geng et al., 1979; Hall, 1972; Keselman et al., 1979; Sharma, 1991) suggest that there is no test which is uniformly best for all distribution and sample size configurations. One test which usually stands out in terms of power and robustness against non-normality is Levene's test using the sample median as an estimate of the location parameter.

Levene's test is the one –way analysis of variance F – test on $|x_{ij} - \overline{x_i}|$, the absolute deviation of the x_{ij} from their group mean $\overline{x_i}$. (Draper and Hunter, 1969). Various modification of Levene's test have been proposed and investigated. Brown and Forsythe (1974) consider the median and 10 % trimmed mean, which are more robust estimates of location. Loh (1987) examines the effectiveness of applying Satterthwaite's method of correcting degrees of freedom and data-based power transformation on Levene's test with group medians in place of group means. He finds that Satterthwaite's method may improve the robustness of Levene's test for small samples. Yitnosumarto and O'Neill (1986) give another method for modifying the degrees of freedom of the F-test. Boos and Brownie (1989) study the bootstrap versions of Bartlett's test and Layard's (1973) k-sample generalization of Miller's (1968) two – sample jackknife test. They prove that for location-scale families the bootstrap version of Bartlett's test is consistent under H_0 as Min $\{n_1, n_2 \ldots\ldots n_i\} \rightarrow \infty$. their simulation results show that the bootstrap versions of Bartlett's test and the jackknife test perform better than the original versions. However, when the data come from an exponential distribution, the size of the bootstrap tests tend to be rather large.

Our aim of study is to compare the eight existing tests viz. Levene's test, Bartlett test, Modified Bartlett test, Box-Anderson Test, the three Jackknife's test and Lepage test; for detecting scale parameters. We here use Monte Carlo simulation technique for find the results by generating observations for normal and logistic distribution. We here developed the computer program ourselves for the same. Necessary discussion and conclusion are given on the basis of computed results. Results are tabulated for convenience.

2. TEST STATISTICS

Given a variable X with sample of size **N** divided into k subgroups, where n_i is the sample size of the i^{th} subgroup from the i^{th} population with mean μ_i ,variance σ_i^2 and distribution function $F = \{(x - \mu)/\sigma_i\}$.
The null hypothesis is $H_0 : \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \cdots = \sigma_k^2$ against the alternative hypothesis

$H_1 : \sigma_i^2 \neq \sigma_j^2$ for at least one pair (i, j)

Define the group mean $\overline{x_i} = \sum_{j=1}^{n_i} \frac{x_{ij}}{n_i}$, group variance $s_i^2 = \sum_{j=1}^{n_i} (x_{ij} - \overline{x_{i.}})^2 / (n_i - 1)$,
and the total sample size $N = \sum_{i=1}^{k} n_i$.

I. Levene's test (Levene 1960) is used to test if k samples have equal variance, some statistical test, for example the ANOVA, assume that variances are equal across groups or samples. The Levene's test can be used to verify that assumption.

Levene's test is an alternative to the Bartlett test. The levene test is less sensitive than the Bartlett test to departure from normality. But if the data come from a normal or nearly normal distribution, then Bartlett's test has better performance.
The Levene test statistics is

$$w = \frac{(N-k)}{(k-1)} \frac{\sum_{i=1}^{k} n_i (\bar{z}_i - \bar{z}_{..})^2}{\sum_{i=1}^{k} \sum_{j=1}^{n_i} (z_{ij} - \bar{z}_{i.})^2} \quad \ldots (1)$$

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Where, $z_{ij} = |x_{ij} - \bar{x}_{i.}|$, here $\bar{x}_{i.}$ is the mean or median or 10% trimmed mean of the ith subgroup and $\bar{z}_{i.}$ are the group means of the z_{ij} and $\bar{z}_{..}$ is the overall mean of the z_{ij}.
The three choices for defining z_{ij} determine the robustness and power of Levene's test. By robustness we mean the ability of the test to not falsely detect unequal variance when the underlying data are not normally distributed and the variables are in fact equal.
Trimmed mean performed best when the underlying data followed a Cauchy distribution (i.e. heavy tailed) and the median performed best when the underlying data followed a χ_4^2 (i.e. skewed) distribution.
Using the mean provided the best power for symmetric, moderate tailed distributions the Levene test reject the hypothesis that the variance are equal if $w > F_{\alpha,k-1,n-k}$, where $F_{\alpha,k-1,n-k}$ is the upper critical value of the F – distribution with k -1 and n - k d.f. at level of α.

II. Bartlett's test (Snedecar and Cochron, 1983) is used to test if k sample have equal variances. Equal variances across samples is called homogeneity of variance. It is sensitive to departures from normality. i.e. if sample comes from normal distribution, then Bartlett's test may simply be testing for non-normality. The Levene's Test is an alternative to the Bartlett's test i.e. less sensitive to departures from normality. Some statistical test, e.g. ANOVA assume that variance are equal across group or variances are equal across group or samples. The Bartlett's test can be used to verify that assumption. The test statistics for Bartlett's test for equality of variance across group against the alternative that variances are unequal for at least two group.

$$B = \frac{(N-k)\ln S_p^2 - \sum_{i=1}^{k}(n_i-1)\ln S_i^2}{1+\left(1/(3(k-1))\right)\left(\left(\sum_{i=1}^{k} 1/(n_i-1)\right)\right) - 1/(N-k)} \quad \ldots \quad (2)$$

Where s_i^2 is the variance of the ith group, N is the total sample size, n_i is the sample size of the i^{th} group, k is the number of groups and s_p^2 is the pooled variance. The pooled variance is a weighted average of the group variances and is defined as

$$S_p^2 = \sum_{i=1}^{k}(n_i - 1)\, S_i^2 / (N-k)$$

Critical Region: the variances are judged to be unequal if $T > \chi^2_{1-\alpha,k-1}$, Where $\chi^2_{1-\alpha,k-1}$ is the critical value of the chi-square distribution with k -1 d.f. and a significance level of α.

III. We also consider a modification of Bartlett's test investigated by Boos and Brownie (1989). The modified test statistics is

$$B_1 = dB \quad \ldots \quad (3)$$

$$\text{where } d = \frac{2}{(\hat{\beta}_2 - 1)} \quad \ldots \quad (4)$$

$$\text{and } \hat{\beta}_2 = \frac{N\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^4}{\left\{\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2\right\}^2} \quad \ldots \quad (5)$$

The modification is motivated by the fact that under weak regularity conditions, $B \rightarrow \frac{1}{2}(\beta_2 - 1)\chi^2_{k-1}$ in distribution (Box, 1953). The critical point for B_1 is the same as that for B.

IV. Box- Andersen test is another variation of Bartlett's test, which is recommended by Miller (1968). The test statistics is

$$B_2 = \frac{2}{(\hat{\beta}_2 - 1)}\{(N-k)\ln S_p^2 - \sum_{i=1}^{k}(n_i - 1)\ln S_i^2\}. .. \quad (6)$$

Where $\hat{\beta}_2$ is given by (5)
The null hypothesis is rejected when $B_2 > 100(1-\alpha)^{th}$ percentile of the chi-squared distribution with (k - 1) degrees of freedom.

V. Jackknife test
The test statistics is

$$J = \frac{\sum_{i=1}^{k} n_i(\bar{u}_{i.} - \bar{u}_{..})^2/(k-1)}{\sum_{i=1}^{k}\sum_{j=1}^{n_i}(u_{ij} - \bar{u}_{i.})^2/(N-k)} \quad \ldots \quad (7)$$

where $u_{ij} = n_i \log S_i^2 - (n_i - 1)\log S_{ij}^2$, $\bar{u}_{i.} = \sum_{j=1}^{n_i}\frac{u_{ij}}{n_i}$, $\bar{u}_{..} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{u_{ij}}{N}$, $s_i^2 = \sum_j \frac{(x_{ij}-\bar{x}_{i.})^2}{(n_i-1)}$, $s_{ij}^2 = \frac{\left[(n_i-1)s_i^2 - n_i(x_{ij}-\bar{x}_{i.})^2/(n_i-1)\right]}{(n_i-2)}$

For j = 1,2, ..., n_i , i = 1, 2, ..., k the null hypothesis is rejected when J exceeds the $100(1-\alpha)^{th}$ percentile of the F – distribution with (k – 1) and (N – k) degrees of freedom.
It is Layard's (1973) k – sample generalization of Miller's (1968) two – sample Jackknife procedure. O' Brien (1978) note that the null behaviour of this statistics is adversely affected, for unequal sample size by the dependence of n_i of both the mean and variance of the.... value u_{ij} . empirical result in O' Brien (1978) showing positive within group correlations between u_{ij} for the exponential distribution explain the liberal nature of this test at the exponential.

VI. O' Brien (1978) recommends using the alternative Jackknife pseudovalues

$$q_{ij} = n_i s_i^2 - (n_i - 1)s_{i(j)}^2 \quad \ldots \quad (8)$$

which can be computed by the formula

$$q_{ij} = \frac{\{n_i(x_{ij}-\bar{x}_{i.})^2 - s_i^2\}}{(n_i-2)} \quad \ldots \quad (9)$$

The resulting test statistic, which is the one way analysis of variance F-statistic based on the q_{ij}, will be called $\mathbf{J_1}$. The critical point is the same as that for J.

VII. Sharma (1991) proposes the modification of **Layard's** (1973) Jackknife procedure. It is based on Jacknifing one group of observations at a time instead of one observation in each group. The pseudovalues are defined by

$$\xi_i = k\log s^2 - (k-1)\log s_{-i}^2,\ i = 1, 2, ..., k \quad \ldots \quad (10)$$

where $s^2 = \sum_{i=1}^{k}(n_i - 1)s_i^2 / \sum_{i=1}^{k}(n_i - 1)$ and $s_{-i}^2 = \sum_{k\neq i}(n_k - 1)s_k^2 / \sum_{k\neq i}(n_k - 1)$

The test statistic is

$$J_2 = \frac{k^{1/2}\bar{\xi}}{\left\{\sum_{i=1}^{k}(\xi_i - \bar{\xi}_.)^2/(k-1)\right\}^{1/2}} \quad \ldots \quad (11)$$

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Where $\bar{\xi}_{.} = \sum_{i=1}^{k} \xi_i / k$. The null hypothesis is rejected when the absolute value of J_2 is greater than the 100(1-α/2)th percentile of the t distribution with (k-1) degrees of freedom.

VIII. Lepage Test:

This test is basically based on ranks. It is a combination of the Wilcoxon – Mann – Whitney and Ansari Bradley test statistics. The multisample version of the Ansari – Bradley statistics is defined by the formula

$$T_B = \frac{1}{v_N^2} \sum_{j=1}^{k} n_j \left(\frac{S_j^{(b)}}{n_j} - \tilde{\mu}_N \right)^2 = \frac{1}{v_N^2} \sum_{j=1}^{k} \frac{\left(S_j^{(b)} - n_j \tilde{\mu}_N\right)^2}{n_j} \quad .. \quad (12)$$

where $S_j^{(b)} = \sum_{i=1}^{n_j} b_N\left(R_{ji}\right)$, j = 1, 2, ..., k

R_{ji} are the ranks of the sample from the j^{th} population,

$$v_N^2 = \begin{cases} \frac{N(N^2-4)}{48(N-1)} & N \text{ even} \\ \frac{(N+1)(N^2+3)}{48N} & N \text{ odd} \end{cases}$$

$$\tilde{\mu}_N = \begin{cases} \frac{(N+2)}{4} & N \text{ even} \\ \frac{(N+1)^2}{4N} & N \text{ odd} \end{cases}$$

And the Wilcoxon test statistic is

$$T_K = \frac{1}{w_N^2} \sum_{j=1}^{k} n_j \left(\frac{S_j}{n_j} - \frac{N+1}{2} \right)^2 = \frac{1}{w_N^2} \sum_{j=1}^{k} \frac{\left(S_j - n_j \frac{N+1}{2}\right)^2}{n_j} \quad ... \quad (13)$$

where $w_N^2 = \frac{N(N+1)}{12}$ and $S_j = \sum_{i=1}^{n_j} R_{ji}$, partial Sum

Putting,

$$\mathbf{L = T_B + T_K} \quad ... \quad (14)$$

This statistics is asymptotically χ^2 - distributed with 2(k-1) degrees of freedom.

3. MONTE CARLO STUDY

To Study the significance level and power of the tests, we generate a random sample from the distributions viz. (i). Normal distribution and (ii) Logistic Distribution.

The null hypothesis of equal variances is studied along with three alternatives: $(\sigma_1, \sigma_2, \sigma_3, \sigma_4)$= (1, 2, 3, 4), (4, 3, 2, 1) and (1, 3, 5, 7)

For each sample size value of the tests are calculated and compare with the theoretical value of accept or reject the hypothesis. If it is rejected it is counted and repeat the process. We have repeated 10, 000 times for each sample size and calculate the proportion of rejection i.e. number of rejected the null hypothesis divided by the total number of repetition is calculated and tabulated.

Here Table 1 and Table 2 gives the estimates of significance level (5%) and power of the eight tests for normal distribution. Whereas Table 3 and Table 4 gives the estimates of significance level (5%) as well as power of the seven tests respectively for logistic distribution.

Table 1: Monte Carlo estimates of significance level under the null hypothesis based on10, 000 replications.

Sample Sizes	W	B	B_1	B_2	J	J_1	J_2	L
(5,5,5,5)	.041	.046	. 053	.069	.033	.011	.071	.030
(10,10,10,10)	.048	.039	.042	.055	.042	.025	.068	.041
(15,15,15,15)	.050	.040	.045	.048	.043	.027	.080	.047
(20,20,20,)	.040	.046	.050	.053	.057	.036	.058	.037
(5,5,5,5,5)	.037	.041	.050	.066	.025	.010	.046	.038
(10,10,10,10,10)	.033	.039	.042	.055	.037	.024	.068	.034
(15,15,15,15,15)	.043	.044	.045	.053	.050	.031	.063	.042
(20,20,20,20,20)	.041	.047	.050	.054	.046	.037	.063	.043
(5,5,5,5,5,5)	.053	.046	.057	.072	.035	.013	.049	.030
(10,10,10,10,10,10)	.041	.035	.037	.048	.038	.022	.056	.030
(15,15,15,15,15,15)	.041	.035	.038	.044	.043	.028	.052	.040
(20,20,20,20,20,20)	.052	.046	.048	.054	.049	.038	.049	.041
(5,5,10,10)	.046	.047	.055	.072	.049	.026	.071	.032
(5,10,15,20)	.039	.052	.054	.064	.069	.038	.084	.036
(5,5,20,20)	.032	.051	.053	.065	.078	.039	.085	.041

Table 2 Monte Carlo Estimates of Power Based on 10, 000 replications.

Sample Sizes	σ^2	W	B	B_1	B_2	J	J_1	J_2	L
5,5,5,5)	(1,2,3,4)	.402	.508	.329	.399	.286	078	.959.	.295
	(4,3,2,1)	.117	.518	.318	.400	.282	.068	.957	.270
	(1,3,5,7)	.569	.840	.577	.665	.557	.103	.998	.473
(5,5,10,10)	(1,2,3,4)	.470	.660	.495	.547	.512	.116	.938	.435
	(4,3,2,1)	.484	.823	.599	.665	.688	.295	.996	.666
	(1,3,5,7)	.684	.943	.771	.823	.791	.158	1.00	.416

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(10,10,10,10)	(1,2,3,4)	.696	.949	.819	.843	.863	.442	.997	.325
	(4,3,2,1)	.666	.953	.821	.847	.873	.449	.994	.328
	(1,3,5,7)	.893	1.00	.963	.974	.994	.582	1.00	.927
(5,10,15,20)	(1,2,3,4)	.671	.841	.702	.739	.747	.267	1.00	.601
	(4,3,2,1)	.918	.978	.702	.949	.970	.850	.959	.828
	(1,3,5,7)	.856	.864	.926	.757	.767	.469	1.00	.833
(15,15,15,15)	(1,2,3,4)	.975	.933	.980	.983	.988	.820	1.00	.922
	(4,3,2,1)	.951	1.00	.973	.974	.988	.814	1.00	.908
	(1,3,5,7)	.999	1.00	.998	.999	1.00	.904	1.00	.999
(20,20,20,20)	(1,2,3,4)	.995	1.00	.994	.994	.999	.961	1.00	.978
	(4,3,2,1)	.994	1.00	.997	.998	1.00	.958	1.10	.973
	(1,3,5,7)	1.00	1.00	1.00	.999	1.00	.987	1.00	.999

Figure 1.1

Empirical Power of Test Under $\sigma_1 = 1$, $\sigma_2 = 3$, $\sigma_3 = 5$, $\sigma_4 = 7$

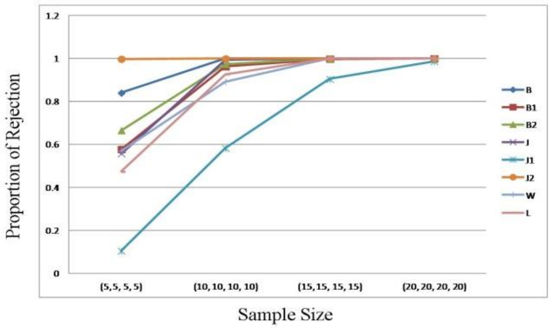

Figure 1.2

Empirical Power of Test Under Sample Size (10, 10, 10, 10)

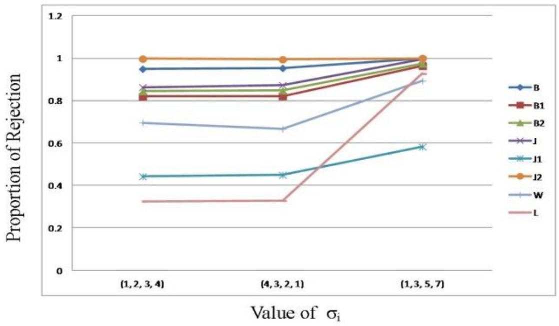

Figure 1.3

Empirical Power of Test Under Sample Size (05, 10, 15, 20)

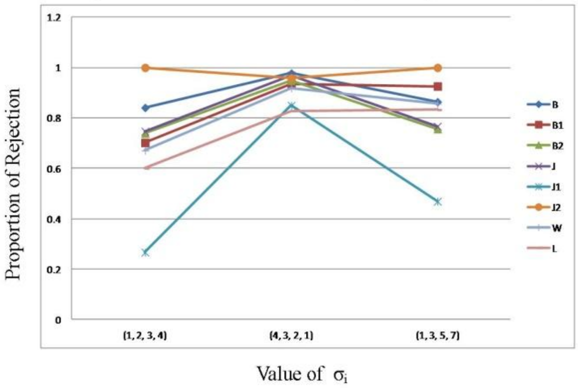

Figure 2.1

Empirical Power of Test Under $\sigma_1 = 1$, $\sigma_2 = 3$, $\sigma_3 = 5$, $\sigma_4 = 7$

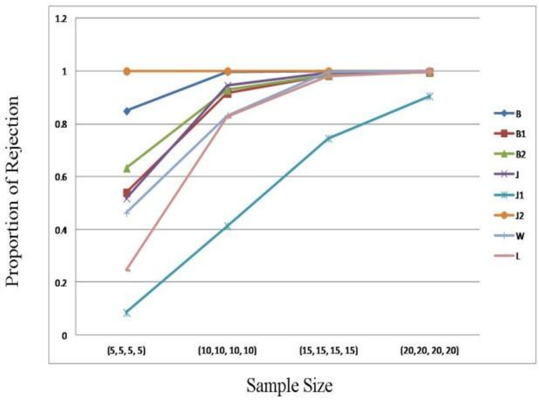

Figure 2.2

Empirical Power of Test Under Sample Size (10, 10, 10, 10)

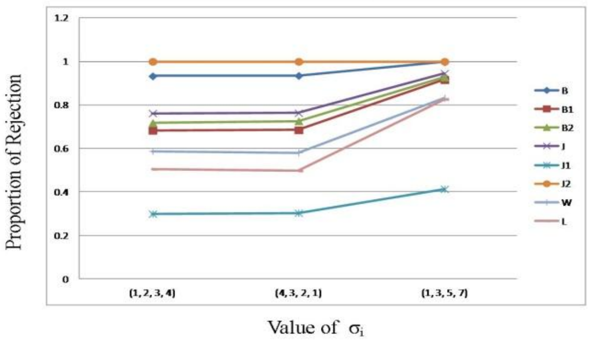

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Table 3 Monte Carlo estimates of significance level under the null hypothesis based on 10,000 replications

Sample Size	W	B	B_1	B_2	J	J_1	L
(5,5,5,5)	.037	.088	.049	.074	.045	.013	.033
(10,10,10,10)	.035	.118	.048	.054	.067	.023	.039
(15,15,15,15)	.045	.145	.043	.047	.066	.030	.048
(20,20,20,20)	.045	.156	.046	.049	.070	.038	.049
(5,5,5,5,5)	.040	.100	.047	.072	.046	.015	.029
(10,10,10,10,10)	.032	.140	.040	.049	.061	.022	.039
(15,15,15,15,15)	.050	.156	.046	.048	.064	.032	.046
(20,20,20,20,20)	.040	.173	.043	.047	.067	.033	.050
(5,5,5,5,5,5)	.042	.107	.045	.066	.042	.014	.033
(10,10,10,10,10,10)	.045	.150	.039	.047	.065	.022	.038
(15,15,15,15,15,15)	.049	.176	.037	.041	.069	.028	.041
(20,20,20,20,20,20)	.054	.189	.043	.046	.070	.0757	.047
(5,5,10,10)	.036	.107	.048	.058	.067	.018	.035
(5,10,15,20)	.039	.123	.043	.053	.075	.030	.043
(5,5,20,20)	.042	.112	.041	.053	.089	.038	.041

Table 4 Monte Carlo Estimates of Power Based on 10,000 replications

Sample size	σ^2	W	B	B_1	B_2	J	J_1	J_2	L
(5,5,5,5)	(1,2,3,4)	.313	.551	.301	.381	.284	.064	.999	.139
	(4,3,2,1)	.108	.561	.297	.379	.286	.061	.998	.145
	(1,3,5,7)	.462	.849	.540	.633	.518	.085	1.00	.246
(5,5,10,10)	(1,2,3,4)	.339	.695	.389	.456	.465	.065	1.00	.187
	(4,3,2,1)	.436	.813	.527	.597	.584	.255	.999	.393
	(1,3,5,7)	.503	.937	.674	.726	.740	.089	1.00	.350
(10,10,10,10)	(1,2,3,4)	.588	.933	.683	.718	.762	.299	1.00	.504
	(4,3,2,1)	.581	.935	.685	.726	.763	.303	1.00	.497
	(1,3,5,7)	.832	.998	.916	.928	.946	.413	1.00	.826
(5,10,15,20)	(1,2,3,4)	.510	.846	.545	.585	.661	.138	1.00	.326
	(4,3,2,1)	.843	.966	.829	.860	.865	.701	1.00	.704
	(1,3,5,7)	.728	.983	.801	.833	.887	.205	1.00	.593
(15,15,15,15)	(1,2,3,4)	.943	.990	.891	.904	.924	.606	1.00	.783
	(4,3,2,1)	.890	.991	.888	.901	.925	.601	1.00	.776
	(1,3,5,7)	.995	1.00	.983	.985	.992	.745	1.00	.980
(20,20,20,20)	(1,2,3,4)	.982	.997	.963	.969	.975	.813	1.00	.920
	(4,3,2,1)	.981	.998	.965	.969	.974	.816	1.00	.925
	(1,3,5,7)	.999	1.00	.995	.996	.998	.905	1.00	.998

4. DISCUSSION

Under Normal Distribution, Figure (1.1) shows the empirical power of the tests for the different sample sizes under the alternative (1, 3, 5, 7). Similarly, Figure (1.2) and Figure (1.3) shows the empirical power of the tests for the alternatives under the sample sizes (10, 10, 10, 10) and (5, 10, 15, 20) respectively.

If we consider the alternative (1, 3, 5, 7) ,the following results are found for the mentioned sample sizes:

(i). For the sample size (5, 5, 5, 5) the power of $\mathbf{J_2}$ is highest than the remaining test and $\mathbf{J_1}$ shows the lowest power.

(ii). For the sample size (5, 5, 10, 10) i.e. unequal samples, here also $\mathbf{J_2}$ is the most power full and its power is 1.0 and $\mathbf{J_1}$ has the lowest power i.e. 0.158

(iii). For the sample size (15, 15, 15, 15) i.e. equal and moderate, Bartlett and $\mathbf{J_2}$ shows the highest power and $\mathbf{J_1}$ shows the lowest power i.e. 0.582

(iv). For the sample size (5, 10, 15, 20) i.e. unequal and moderate, the power of $\mathbf{J_2}$ is highest and $\mathbf{J_1}$ shows the lowest power.

(v). For the sample size (20, 20, 20, 20) i.e. large and equal, all the tests shows the same power.

That means, $\mathbf{J_2}$ shows the highest power and $\mathbf{J_1}$ shows the lowest power for the considered sample sizes. **Bartlett's test** also shows the highest power as $\mathbf{J_2}$ for equal and moderate sample size. One important point is that for large sample size, all the tests show the highest power.

Similarly, at the alternative (1, 2, 3, 4), the power of $\mathbf{J_2}$ is high and $\mathbf{J_1}$ shows lowest power for the mentioned sample

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sizes, but for sample sizes (10, 10, 10, 10), Lepage shows the lowest power.

Again at the alternative (4, 3, 2, 1), $\mathbf{J_2}$ shows the highest and $\mathbf{J_1}$ shows the lowest power. But for sample size (10, 10, 10, 10) and (5, 10, 15, 20) Lepage shows the lowest power.

Under the logistic distribution, Figure (2.1) shows the empirical power of the tests for the different sample sizes under the alternative (1, 3, 5, 7). Similarly, Figure (2.2) and Figure (2.3) shows the empirical power of the tests for the alternatives under the sample sizes (10, 10, 10, 10) and (5, 10, 15, 20) respectively. For these figures, it is observed that Jackknife (J_2) shows the highest power against all the considered alternatives, for all the sample sizes. Where on the other hand Bartlett (B) shows highest power only against the alternative (1, 3, 5, 7). Simultaneously Leveny (W) also shows the highest power for the large sample size. Here it is observed that power increases with increase of sample of sample size, for all the tests.

Modified Bartlett (B_1), Box Anderson (B_2), Levene (W) and Lepage (L) satisfied that the three mentioned significance level (10%, 05%, 01%). But on the other hand Bartlett (B) and Jackknife tests less satisfies the significance levels.

5. CONCLUSION

From the above discussion we concluded that Jackknife test $\mathbf{J_2}$ being the more powerful for both normal and logistic distribution and Jackknife test $\mathbf{J_1}$ shows the lowest power for small sample sizes. But for moderate equal and unequal sample size Lepage test being the least powerful for normal distribution. So it can be concluded that both Jackknife test $\mathbf{J_2}$ and Bartlett (**B**) test best fit under normal distribution as well as logistic distribution. Also it is assumed that all the tests are best for large sample size for the both distribution.

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