

# $\Pi g^{\wedge} B^*$ -Continuity in Topological Space

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**Abstract:** In this Paper using  $\pi g^{\wedge} b^*$ -closed set in topological spaces we introduce a new class of sets called  $\pi$  generalized  $\wedge b^*$ -continuous functions (briefly  $\pi g^{\wedge} b^*$ -continuous functions). Further the concept of almost  $\pi g^{\wedge} b^*$ -continuous function and  $\pi g^{\wedge} b^*$ -irresolute function are discussed.

**Key words:**  $\pi g^{\wedge} b^*$ -continuous function,  $\pi g^{\wedge} b^*$ -irresolute function, almost  $\pi g^{\wedge} b^*$ -continuous function.

## I. INTRODUCTION

Levine[10] and Andrijevic[2] introduced the concept of generalized open sets and b-open sets respectively in topological spaces. The class of b-open sets is contained in the class of semipre-open sets and contains the class of semi-open and the class of pre-open sets. Since then several researches were done and the notion of generalized semi-closed, generalized pre-closed and generalized semipre-open sets were investigated. In 1968 Zaitsev[18] defined  $\pi$ -closed sets.

Later Dontchev and Noiri[6] introduced the notion of  $\pi g$ -closed sets. Park defined  $\pi gp$ -closed sets. Then Aslim, Caksu and Noir[3] introduced the notion of  $\pi gs$ -closed sets. D. Sreeja and S. Janaki[17] studied The idea of  $\pi gb$ -closed sets and introduced the concept of  $\pi gb$ -continuity. Later the properties and characteristics of  $\pi gb$ -closed and  $\pi gb$ -continuity were introduced by Sinem Caglar and Gulhan Ashim[16]. Dhanya. R and A. Parvathi[4] introduced the concept of  $\pi gb^*$ -closed sets and  $\pi gb^*$ -continuity in topological spaces. Hussain[7] introduced the concept of almost continuity in topological spaces.

## II. PRELIMINARIES

Throughout this paper  $(X, \tau)$  represents non empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. A subset A of a topological space  $(X, \tau)$ ,  $cl(A)$  and  $int(A)$  denote the closure of A and interior of A respectively.  $(X, \tau)$  will be replaced by X if there is no chance of confusion.

### Definiton 2.1

Let  $(X, \tau)$  be a topological space. A subset A of  $(X, \tau)$  is called

- (1) a **semi-closed set** if  $int(cl(A)) \subseteq A$ .
- (2) a  **$\alpha$ -closed set** if  $cl(int(cl(A))) \subseteq A$ .
- (3) a **pre-closed set** if  $cl(int(A)) \subseteq A$ .
- (4) a **semipre-closed set** if  $int(cl(int(A))) \subseteq A$ .
- (5) a **regular-closed set** if  $A = cl(int(A))$ .
- (6) a **b-closed set** if  $cl(int(A)) \cap int(cl(A)) \subseteq A$ .
- (7) a  **$b^*$ -closed set** if  $int(cl(A)) \subset U$ , whenever  $A \subset U$  and U is b-open.

the complements of the above mentioned sets are called semi-open,  $\alpha$ -open, pre-open, semi-open, regular open, b-open,  $b^*$ -open sets respectively. The intersection of all semi-closed (resp.  $\alpha$ -closed, pre-closed, semipre-closed, regular-closed and b-closed) subsets of  $(X, \tau)$  containing A is called the semi-closure (resp.  $\alpha$ -closure, pre-closure, semipre-closure, regular-closure and b-closure) of A and is denoted by  $scl(A)$  (resp.  $\alpha cl(A)$ ,  $pcl(A)$ ,  $spcl(A)$ ,  $rcl(A)$  and  $bcl(A)$ ). A subset A of  $(X, \tau)$  is called clopen if it is both open and closed in  $(X, \tau)$ .

### Definition 2.2

A subset A of a space  $(X, \tau)$  is called  $\pi$ -closed if A is finite intersection of regular closed sets.

### Definition 2.3

A subset A of a space  $(X, \tau)$  is called

- (1) a **g-closed set** if  $cl(A) \subset U$  whenever  $A \subset U$  and U is open in  $(X, \tau)$ .
- (2) a **gp-closed set** if  $pcl(A) \subset U$  whenever  $A \subset U$  and U is open in  $(X, \tau)$ .
- (3) a **gs-closed set** if  $scl(A) \subset U$  whenever  $A \subset U$  and U is open in  $(X, \tau)$ .
- (4) a **gb-closed set** if  $bcl(A) \subset U$  whenever  $A \subset U$  and U is open in  $(X, \tau)$ .
- (5) a **ga-closed set** if  $\alpha cl(A) \subset U$  whenever  $A \subset U$  and U is open in  $(X, \tau)$ .
- (6) a  **$\pi g$ -closed set** if  $cl(A) \subset U$  whenever  $A \subset U$  and U is  $\pi$ -open in  $(X, \tau)$ .
- (7) a  **$\pi ga$ -closed set** if  $\alpha cl(A) \subset U$  whenever  $A \subset U$  and U is  $\pi$ -open in  $(X, \tau)$ .
- (8) a  **$\pi gp$ -closed set** if  $pcl(A) \subset U$  whenever  $A \subset U$  and U is  $\pi$ -open in  $(X, \tau)$ .
- (9) a  **$\pi gs$ -closed set** if  $scl(A) \subset U$  whenever  $A \subset U$  and U is  $\pi$ -open in  $(X, \tau)$ .
- (10) a  **$\pi gb$ -closed set** if  $bcl(A) \subset U$  whenever  $A \subset U$  and U is  $\pi$ -open in  $(X, \tau)$ .

Complement of  $\pi$ -closed set is called  $\pi$ -open set.

Complement of g-closed, gp-closed, gs-closed, gb-closed, ga-closed,  $\pi ga$ -closed,  $\pi gp$ -closed,  $\pi gs$ -closed and  $\pi gb$ -closed sets are called g-open, gp-open, gs-open, gb-open,

$g\alpha$ -open,  $\pi g\alpha$ -open,  $\pi gp$ -open,  $\pi gs$ -open and  $\pi gb$ -open sets respectively.

**Definition 2.4**

A function  $f: (X,\tau) \rightarrow (Y,\sigma)$  is called continuous (resp.  $\alpha$ -continuous, pre-continuous,  $g$ -continuous, regular continuous,  $gb$ -continuous,  $b^*$ -continuous) if  $f^{-1}(V)$  is closed (resp.  $\alpha$ -closed, pre-closed,  $g$ -closed, regular closed,  $gb$ -closed,  $b^*$ -closed) in  $(X,\tau)$  for every closed set  $V$  in  $(Y,\sigma)$ .

**Definition 2.5**

A function  $f: (X,\tau) \rightarrow (Y,\sigma)$  is called  $\pi$ -continuous (resp.  $\pi\alpha$ -continuous,  $\pi gp$ -continuous,  $\pi g$ -continuous,  $\pi gb$ -continuous,  $\pi gb^*$ -continuous) if  $f^{-1}(V)$  is closed (resp.  $\pi\alpha$ -closed,  $\pi gp$ -closed,  $\pi g$ -closed,  $\pi gb$ -closed,  $\pi gb^*$ -closed) in  $(X,\tau)$  for every closed set  $V$  in  $(Y,\sigma)$ .

**III.  $\pi g^*b^*$ -CONTINUITY**

**Definition 3.1**

A function  $f: (X,\tau) \rightarrow (Y,\sigma)$  is called  $\pi g^*b^*$ -continuous if  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed in  $(X,\tau)$  for every closed set  $V$  of  $(Y,\sigma)$ .

**Definition 3.2**

A function  $f: (X,\tau) \rightarrow (Y,\sigma)$  is called  $\pi g^*b^*$ -irresolute if  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed in  $(X,\tau)$  for every  $\pi g^*b^*$ -closed set  $V$  in  $(Y,\sigma)$ .

**Definition 3.3**

A function  $f: (X,\tau) \rightarrow (Y,\sigma)$  is called  $\pi g^*b^*$ -closed if  $f(V)$  is  $\pi g^*b^*$ -closed in  $(Y,\sigma)$  for every  $\pi g^*b^*$ -closed set  $V$  in  $(X,\tau)$ .

**Example 3.1(a)**

Consider  $X = \{a,b,c,d\}$ ,  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$  and  $Y = \{a,b,c,d\}$  with topology  $\sigma = \{Y, \Phi, \{a\}, \{a,b\}\}$ . Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be defined by  $f(a)=a; f(b)=b; f(c)=c$ , then  $f$  is  $\pi g^*b^*$ -continuous.

**Example 3.2(a)**

Consider  $X = \{a,b,c\}$ ,  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$  and  $Y = \{a,b,c\}$  with topology  $\sigma = \{Y, \Phi, \{a\}\}$ . Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be defined by  $f(a)=a; f(b)=b; f(c)=c$ , then  $f$  is  $\pi g^*b^*$ -irresolute.

**Theorem 3.1**

Every continuous function is  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be a continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is continuous  $f^{-1}(V)$  is closed in  $X$ . As every closed set is  $\pi g^*b^*$ -closed.  $f^{-1}$  is  $\pi g^*b^*$ -closed. Hence  $f$  is  $\pi g^*b^*$ -continuous.

**Remark 3.1**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.1**

Consider  $X = \{a,b,c\}$ ,  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$  and  $Y = \{a,b,c\}$  with topology  $\sigma = \{Y, \Phi, \{b,c\}\}$ . Let

$f: (X,\tau) \rightarrow (Y,\sigma)$  be defined by  $f(a)=a; f(b)=b; f(c)=c$ , then  $f$  is  $\pi g^*b^*$ -continuous but it is not continuous.

**Theorem 3.2**

Every  $\pi$ -continuous function is  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be a  $\pi$ -continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is  $\pi$ -continuous  $f^{-1}(V)$  is  $\pi$ -closed in  $X$ . As every  $\alpha$ -closed set is  $\pi g^*b^*$ -closed.  $f^{-1}$  is  $\pi g^*b^*$ -closed. Hence  $f$  is  $\pi g^*b^*$ -continuous.

**Remark 3.2**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.2**

Consider  $X = \{a,b,c\}$ ,  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$  and  $Y = \{a,b,c,d\}$  with topology  $\sigma = \{Y, \Phi, \{b\}, \{b,c\}\}$ . Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be defined by  $f(a)=a; f(b)=b; f(c)=c, f(d)=d$  then  $f$  is  $\pi g^*b^*$ -continuous but it is not  $\pi$ -continuous.

**Theorem 3.3**

Every  $\alpha$ -continuous function is  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be a  $\alpha$ -continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is  $\alpha$ -continuous  $f^{-1}(V)$  is  $\alpha$ -closed in  $X$ . As every  $\alpha$ -closed set is  $\pi g^*b^*$ -closed.  $f^{-1}$  is  $\pi g^*b^*$ -closed. Hence  $f$  is  $\pi g^*b^*$ -continuous.

**Remark 3.3**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.3**

Consider  $X = \{a,b,c\}$ ,  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$  and  $Y = \{a,b,c\}$  with topology  $\sigma = \{Y, \Phi, \{a\}, \{c\}, \{a,c\}\}$ . Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be defined by  $f(a)=a; f(b)=b; f(c)=c$ , then  $f$  is  $\pi g^*b^*$ -continuous but it is not  $\alpha$ -continuous.

**Theorem 3.4**

Every  $g$ -continuous function is  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be a  $g$ -continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is  $g$ -continuous  $f^{-1}(V)$  is  $g$ -closed in  $X$ . As every  $g$ -closed set is  $\pi g^*b^*$ -closed.  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed. Hence  $f$  is  $\pi g^*b^*$ -continuous.

**Remark 3.4**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.4**

Consider  $X = \{a,b,c\}$ ,  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$  and  $Y = \{a,b,c\}$  with topology  $\sigma = \{Y, \Phi, \{a\}, \{b,c\}\}$ . Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be defined by  $f(a)=a; f(b)=b; f(c)=c$ . Then  $f$  is  $\pi g^*b^*$ -continuous but it is not  $g$ -continuous.

**Theorem 3.5**

Every pre continuous function is  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be a pre continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is pre continuous  $f^{-1}(V)$  is pre-closed in  $X$ . As every pre-closed set is  $\pi g^*b^*$ -closed.  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed. Hence  $f$  is  $\pi g^*b^*$ -continuous.

**Remark 3.5**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.5**

Consider  $X=\{a,b,c,d\}$ ,  $\tau=\{X,\Phi,\{a\},\{d\},\{a,d\},\{c,d\},\{a,c,d\}\}$  and  $Y=\{a,b,c,d\}$  with topology  $\sigma=\{Y,\Phi,\{a\},\{b\},\{a,b\},\{b,c\}\}$ . Let  $f:(X,\tau)\rightarrow(Y,\sigma)$  be defined by  $f(a)=b; f(b)=c; f(c)=a; f(d)=d$ . Then  $f$  is  $\pi g^*b^*$ -continuous but it is not pre-continuous.

**Theorem 3.6**

Every  $g_b$ -continuous function is  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow(Y,\sigma)$  be a  $g_b$ -continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is  $g_b$ -continuous  $f^{-1}(V)$  is  $g_b$ -closed in  $X$ . As every  $g_b$ -closed set is  $\pi g^*b^*$ -closed.  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed. Hence  $f$  is  $\pi g^*b^*$ -continuous.

**Remark 3.6**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.6**

Consider  $X=\{a,b,c,d\}$ ,  $\tau=\{X,\Phi,\{b\},\{c,d\},\{b,c,d\}\}$  and  $Y=\{a,b,c,d\}$  with topology  $\sigma=\{Y,\Phi,\{a,c,d\}\}$ . Let  $f:(X,\tau)\rightarrow(Y,\sigma)$  be defined by  $f(a)=a; f(b)=b; f(c)=c; f(d)=d$ . Then  $f$  is  $\pi g^*b^*$ -continuous but it is not  $g_b$ -continuous.

**Theorem 3.7**

Every  $\pi g_\alpha$ -continuous function is  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow(Y,\sigma)$  be a  $\pi g_\alpha$ -continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is  $\pi g_\alpha$ -continuous  $f^{-1}(V)$  is  $\pi g_\alpha$ -closed in  $X$ . As every  $\pi g_\alpha$ -closed set is  $\pi g^*b^*$ -closed.  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed. Hence  $f$  is  $\pi g^*b^*$ -continuous.

**Remark 3.7**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.7**

Consider  $X=\{a,b,c\}$ ,  $\tau=\{X,\Phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$  and  $Y=\{a,b,c\}$  with topology  $\sigma=\{Y,\Phi,\{a\}\}$ . Let  $f:(X,\tau)\rightarrow(Y,\sigma)$  be defined by  $f(a)=b; f(b)=c; f(c)=c$ . Then  $f$  is  $\pi g^*b^*$ -continuous but it is not  $\pi g_\alpha$ -continuous.

**Theorem 3.8**

Every  $\pi g^*b^*$ -continuous function is  $\pi g_b$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow(Y,\sigma)$  be a  $\pi g^*b^*$ -continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is  $\pi g^*b^*$ -continuous  $f^{-1}(V)$  is  $\pi g_b$ -closed in  $X$ . As every  $\pi g^*b^*$ -closed set is  $\pi g_b$ -closed.  $f^{-1}(V)$  is  $\pi g_b$ -closed. Hence  $f$  is  $\pi g_b$ -continuous.

**Remark 3.8**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.8**

Consider  $X=\{a,b,c\}$ ,  $\tau=\{X,\Phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$  and  $Y=\{a,b,c\}$  with topology  $\sigma=\{Y,\Phi,\{a\}\}$ . Let  $f:(X,\tau)\rightarrow(Y,\sigma)$  be defined by  $f(a)=b; f(b)=b; f(c)=c$ . Then  $f$  is  $\pi g_b$ -continuous but it is not  $\pi g^*b^*$ -continuous.

**Theorem 3.9**

Every  $\pi g^*b^*$ -continuous function is  $\pi g_s$ -continuous.

**Proof**

Let  $f: (X,\tau) \rightarrow(Y,\sigma)$  be a  $\pi g^*b^*$ -continuous function. Let  $V$  be a closed set in  $Y$ . Since  $f$  is  $\pi g^*b^*$ -continuous  $f^{-1}(V)$

is  $\pi g_s$ -closed in  $X$ . As every  $\pi g^*b^*$ -closed set is  $\pi g_b$ -closed.  $f^{-1}(V)$  is  $\pi g_s$ -closed. Hence  $f$  is  $\pi g_s$ -continuous.

**Remark 3.9**

The converse of the above theorem need not be true as seen from the following example.

**Example 3.9**

Consider  $X=\{a,b,c\}$ ,  $\tau=\{X,\Phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$  and  $Y=\{a,b,c\}$  with topology  $\sigma=\{Y,\Phi,\{a\}\}$ . Let  $f:(X,\tau)\rightarrow(Y,\sigma)$  be defined by  $f(a)=b; f(b)=c; f(c)=a$ . Then  $f$  is  $\pi g_s$ -continuous but it is not  $\pi g^*b^*$ -continuous.

**Remark 3.10**

$\pi g_p$ -continuous and  $\pi g^*b^*$ -continuous are independent of each other. It is shown in the following example.

**Example 3.10**

Let  $X=\{a,b,c\}$ ,  $\tau=\{X,\Phi,\{a\},\{b\},\{a,b\},\{a,c\}\}$  and  $Y=\{a,b,c\}$  with topology  $\sigma=\{Y,\Phi,\{b\},\{b,c\}\}$ . Let  $f:(X,\tau)\rightarrow(Y,\sigma)$  be defined by  $f(a)=a; f(b)=c; f(c)=b$ . Then  $f^{-1}\{a\}=\{a\}$  is  $\pi g^*b^*$ -continuous but it is not  $\pi g_p$ -continuous and  $f^{-1}\{a,c\}=\{a,b\}$  is  $\pi g_p$ -continuous but it is not  $\pi g^*b^*$ -continuous.

**Remark 3.11**

$\pi g$ -continuous and  $\pi g^*b^*$ -continuous are independent of each other. It is shown in the following example.

**Example 3.11**

Let  $X=\{a,b,c,d\}$ ,  $\tau=\{X,\Phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},\{a,b,d\}\}$  and  $Y=\{a,b,c,d\}$  with topology  $\sigma=\{Y,\Phi,\{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\}\}$ . Let  $f:(X,\tau)\rightarrow(Y,\sigma)$  be an identity function. Then  $f^{-1}\{a,b,d\}=\{a,b,d\}$  is  $\pi g$ -continuous but it is not  $\pi g^*b^*$ -continuous and  $f^{-1}\{a\}=\{a,b\}$  is  $\pi g^*b^*$ -continuous but it is not  $\pi g$ -continuous.

**IV.  $\pi g^*b^*$ -CONTINUITY AND ITS CHARACTERISTICS**

**Theorem 4.1**

Let  $f: X \rightarrow Y$  be a function. Then the following statements are equivalent:

- (1)  $f$  is  $\pi g^*b^*$ -continuous;
- (2) The inverse image of every open set in  $Y$  is  $\pi g^*b^*$ -open in  $X$ .

**Proof**

(1)  $\Rightarrow$ (2)

Let  $U$  be open subset of  $X$ . Then  $(Y-U)$  is closed in  $Y$ . Since  $f$  is  $\pi g^*b^*$ -continuous,  $f^{-1}(Y-U)=X-f^{-1}(U)$  is  $\pi g^*b^*$ -closed in  $X$ . Hence  $f^{-1}(U)$  is  $\pi g^*b^*$ -open in  $X$ .

(2)  $\Rightarrow$ (1)

Let  $V$  be a closed subset of  $Y$ . Then  $(Y-V)$  is open in  $Y$ , hence by hypothesis (2)  $f^{-1}(Y-V)=X-f^{-1}(V)$  is  $\pi g^*b^*$ -open in  $X$ . Hence  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed in  $X$ . Therefore,  $f$  is  $\pi g^*b^*$ -continuous.

**Theorem 4.2**

Every  $\pi g^*b^*$ -irresolute function is  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: X \rightarrow Y$  be  $\pi g^*b^*$ -irresolute function. Let  $V$  be closed set in  $Y$ , then  $V$  is  $\pi g^*b^*$ -closed in  $Y$ . since  $f$  is  $\pi g^*b^*$ -

irresolute  $f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -closed in  $X$ . Hence  $f$  is  $\pi g^{\wedge} b^*$ -continuous.

**Remark 4.2**

The converse of the above theorem need not be true it can be seen from the following example.

**Example 4.2**

Consider  $X=Y=\{a,b,c\}$ ,  $\tau=\{X,\Phi,\{a\},\{b\},\{a,b\}\}$ ,  $\sigma=\{X,\Phi,\{a\}\}$ . Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be the identity map. Then  $f$  is  $\pi g^{\wedge} b^*$ -continuous but it is not  $\pi g^{\wedge} b^*$ -irresolute.

**Remark 4.3**

Composition of two  $\pi g^{\wedge} b^*$ -continuous is need not be  $\pi g^{\wedge} b^*$ -continuous.

**Example 4.3**

Let  $X=\{a,b,c,d\}$ ,  $\tau=\{X,\Phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$ ,  $\sigma=\{Y,\Phi,\{a\},\{c\},\{a,c\}\}$ ,  $\eta=\{Z,\Phi,\{a\},\{b\},\{a,b\},\{a,b,d\}\}$ . Define  $f: (X,\tau) \rightarrow (X,\sigma)$  by  $f(a)=a$ ;  $f(b)=d$ ;  $f(c)=b$ ;  $f(d)=c$ . Define  $g: (X,\sigma) \rightarrow (X,\eta)$  by  $g(a)=a$ ;  $g(b)=c$ ;  $g(c)=b$ ;  $g(d)=d$ . Then  $f$  and  $g$  are  $\pi g^{\wedge} b^*$ -continuous but  $g \circ f$  is not  $\pi g^{\wedge} b^*$ -continuous.

**Theorem 4.4**

Let  $f: X \rightarrow Y$  be a function. Then the following statements are equivalent:

- (1) For each  $x \in X$  and each open set  $V$  containing  $f(x)$  there exists a  $\pi g^{\wedge} b^*$ -open set  $U$  containing  $x$  such that  $f(U) \subset V$ .
- (2)  $f(\pi g^{\wedge} b^* \text{-cl}(A)) \subset \text{cl}(f(A))$  for every subset  $A$  of  $X$ .

**Proof**

(1)  $\Rightarrow$  (2)

Let  $y \in f(\pi g^{\wedge} b^* \text{-cl}(A))$  then, there exists an  $x \in \pi g^{\wedge} b^* \text{-cl}(A)$  such that  $y=f(x)$ . we claim that  $y \in \text{cl}(f(A))$  and let  $V$  be any open neighborhood of  $y$ . Since  $x \in \pi g^{\wedge} b^* \text{-cl}(A)$  there exists an  $\pi g^{\wedge} b^*$ -open set  $U$  such that  $x \in U$  and  $U \cap A \neq \Phi$ ,  $f(U) \subset V$ . Since  $U \cap A \neq \Phi$ ,  $f(A) \cap V \neq \Phi$ . Therefore,  $y=f(x) \in \text{cl}(f(A))$ . Hence  $f(\pi g^{\wedge} b^* \text{-cl}(A)) \subset \text{cl}(f(A))$ . Hence  $f(\pi g^{\wedge} b^* \text{-cl}(A)) \subset \text{cl}(f(A))$ .

(2)  $\Rightarrow$  (1)

Let  $x \in X$  and  $V$  be any open set containing  $f(x)$ . Let  $A=f^{-1}(Y-U)$ , since  $f(\pi g^{\wedge} b^* \text{-cl}(A)) \subset \text{cl}(f(A)) \subset (Y-V) \Rightarrow \pi g^{\wedge} b^* \text{-cl}(A) \subset f^{-1}(Y-V)=A$ . Hence  $\pi g^{\wedge} b^* \text{-cl}(A)=A$ . Since  $f(x) \in V \Rightarrow x \in f^{-1}(V) \Rightarrow x \notin \pi g^{\wedge} b^* \text{-cl}(A)$ . Thus there exists an open set  $U$  containing  $x$  such that  $U \cap A = \Phi$ . Therefore  $f(U) \subset V$ .

**Definition 4.1**

A topological space  $(X,\tau)$  is  $\pi g^{\wedge} b^*$ -space if every  $\pi g^{\wedge} b^*$ -closed set is closed.

**Theorem 4.5**

Every  $\pi g^{\wedge} b^*$ -space is  $\pi g^{\wedge} b^* \text{-} T_{1/2}$  space.

**Proof**

Let  $(X,\tau)$  be a  $\pi g^{\wedge} b^*$ -space and let  $A \subset X$  be  $\pi g^{\wedge} b^*$ -closed set in  $X$ . Then  $A$  is closed  $\Rightarrow A$  is  $b^*$ -closed  $\Rightarrow (X,\tau)$  is a  $\pi g^{\wedge} b^* \text{-} T_{1/2}$  space.

**Theorem 4.6**

Let  $f: (X,\tau) \rightarrow (Y,\sigma)$  be a function then,

- (1) If  $f$  is  $\pi g^{\wedge} b^*$ -irresolute and  $X$  is  $\pi g^{\wedge} b^* \text{-} T_{1/2}$  space, then  $f$  is  $b^*$ -irresolute.

- (2) If  $f$  is  $\pi g^{\wedge} b^*$ -continuous and  $X$  is  $\pi g^{\wedge} b^* \text{-} T_{1/2}$  space, then  $f$  is  $b^*$ -continuous.

**Proof**

(1) Let  $V$  be  $b^*$ -closed in  $Y$ , then  $V$  is  $\pi g^{\wedge} b^*$ -closed in  $Y$ . Since  $f$  is  $\pi g^{\wedge} b^*$ -irresolute,  $f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -closed in  $X$ . Since  $X$  is  $\pi g^{\wedge} b^* \text{-} T_{1/2}$  space,  $f^{-1}(V)$  is  $b^*$ -closed. Therefore  $f$  is  $b^*$ -continuous.

(2) Let  $V$  be closed in  $Y$ . Since  $f$  is  $\pi g^{\wedge} b^*$ -continuous,  $f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -closed in  $X$ . Since  $X$  is  $\pi g^{\wedge} b^* \text{-} T_{1/2}$  space,  $f^{-1}(V)$  is  $b^*$ -closed. Therefore  $f$  is  $b^*$ -continuous.

**Definition 4.2**

A function  $f: X \rightarrow Y$  is said to be almost  $\pi g^{\wedge} b^*$ -continuous if  $f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -closed in  $X$  for every regular closed set  $V$  of  $Y$ .

**Theorem 4.7**

For a function  $f: X \rightarrow Y$ , the following statements are equivalent:

- (1)  $f$  is almost  $\pi g^{\wedge} b^*$ -continuous.
- (2)  $f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -open in  $X$  for every regular open set  $V$  of  $Y$ .
- (3)  $f^{-1}(\text{int}(\text{cl}(V)))$  is  $\pi g^{\wedge} b^*$ -open in  $X$  for every open set  $V$  of  $Y$ .
- (4)  $f^{-1}(\text{cl}(\text{int}(V)))$  is  $\pi g^{\wedge} b^*$ -closed in  $X$  for every closed set  $V$  of  $Y$ .

**Proof**

(1)  $\Rightarrow$  (2)

Suppose  $f$  is almost  $\pi g^{\wedge} b^*$ -continuous. Let  $V$  be a regular open subset of  $Y$ . Since  $(Y-V)$  is regular closed and  $f$  is almost  $\pi g^{\wedge} b^*$ -continuous,  $f^{-1}(Y-V) = X - f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -closed in  $X$ . Hence  $f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -open in  $X$ .

(2)  $\Rightarrow$  (1)

Let  $V$  be a regular closed subset of  $Y$ . Then  $(Y-V)$  is regular open. By the hypothesis,  $f^{-1}(Y-V) = X - f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -open in  $X$ . Hence  $f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -closed. Thus  $f$  is  $\pi g^{\wedge} b^*$ -continuous.

(2)  $\Rightarrow$  (3)

Let  $V$  be an open subset of  $Y$ . Then  $\text{int}(\text{cl}(V))$  is regular open in  $Y$ . By the hypothesis,  $f^{-1}(\text{int}(\text{cl}(V)))$  is  $\pi g^{\wedge} b^*$ -open in  $X$ .

(3)  $\Rightarrow$  (2)

Let  $V$  be a regular open subset of  $Y$ . Since  $V = \text{int}(\text{cl}(V))$  and every regular open set is open then  $f^{-1}(V)$  is  $\pi g^{\wedge} b^*$ -open in  $X$ .

(3)  $\Rightarrow$  (4)

Let  $V$  be a closed subset of  $Y$ . Then  $(Y-V)$  is open in  $Y$ . By the hypothesis,  $f^{-1}(\text{int}(\text{cl}(Y-V))) = f^{-1}(Y - \text{cl}(\text{int}(V))) = X - f^{-1}(\text{cl}(\text{int}(V)))$  is  $\pi g^{\wedge} b^*$ -open in  $X$ . Therefore  $f^{-1}(\text{cl}(\text{int}(V)))$  is  $\pi g^{\wedge} b^*$ -closed in  $X$ .

(4)  $\Rightarrow$  (3)

Let  $V$  be an open subset of  $Y$ . Then  $(Y-V)$  is closed. By the hypothesis  $f^{-1}(Y - \text{cl}(\text{int}(Y-V))) = X - f^{-1}(\text{int}(\text{cl}(V)))$  is  $\pi g^{\wedge} b^*$ -closed in  $X$ . Therefore,  $f^{-1}(\text{int}(\text{cl}(V)))$  is  $\pi g^{\wedge} b^*$ -open in  $X$ .

**Theorem 4.8**

Every  $\pi g^{\wedge} b^*$ -continuous function is almost  $\pi g^{\wedge} b^*$ -continuous.

**Proof**

Let  $f: X \rightarrow Y$  be  $\pi g^*b^*$ -continuous function. Let  $V$  be regular closed set in  $Y$ , then  $V$  is closed in  $Y$ . Since  $f$  is  $\pi g^*b^*$ -continuous function  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed in  $X$ . Therefore  $f$  is almost  $\pi g^*b^*$ -continuous.

**Theorem 4.9**

Every almost  $b^*$ -continuous function is almost  $\pi g^*b^*$ -continuous.

**Proof**

Let  $f: X \rightarrow Y$  be almost  $b^*$ -continuous function and let  $V$  be regular closed set in  $Y$ . Then  $f^{-1}(V)$  is  $b^*$ -closed in  $X$ , hence  $f^{-1}(V)$  is  $\pi g^*b^*$ -closed in  $X$ . Therefore  $f$  is almost  $\pi g^*b^*$ -continuous.

**Theorem 4.10**

Let  $X$  be a  $\pi g^*b^*T_{1/2}$  space. Then  $f: X \rightarrow Y$  is almost  $\pi g^*b^*$ -continuous if and only if  $f$  is almost  $b^*$ -continuous.

**Proof**

Suppose  $f: X \rightarrow Y$  is almost  $\pi g^*b^*$ -continuous. Let  $A$  be a regular closed subset of  $Y$ . Then  $f^{-1}(A)$  is  $\pi g^*b^*$ -closed in  $X$ . Since  $X$  is  $\pi g^*b^*T_{1/2}$  space,  $f^{-1}(A)$  is  $b^*$ -closed in  $X$ . Hence  $f$  is almost  $\pi g^*b^*$ -continuous.

Conversely, suppose that  $f: X \rightarrow Y$  is almost  $b^*$ -continuous and  $A$  be a regular closed subset of  $Y$ . Then  $f^{-1}(A)$  is  $b^*$ -closed in  $X$ . Since every  $b^*$ -closed set is  $\pi g^*b^*$ -closed,  $f^{-1}(A)$  is  $\pi g^*b^*$ -closed. Therefore,  $f$  is almost  $\pi g^*b^*$ -continuous.

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