IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

ISO 3297:2007 Certified

Vol. 3, Issue 8, August 2016

Čech MP-closed sets in closure spaces

T. Muthu Priya[1], A. Francina Shalini[2]

Research Scholar, Department of Mathematics, Nirmala College for women, Coimbatore, India[1]

Assistant Professor, Department of Mathematics, Nirmala College for women, Coimbatore, India [2]

Abstract: The purpose of this paper is to define and study the notion of Čech MP-closed and Čech MP-open sets in Čech closure spaces and investigate their characterizations.

Keywords: Čech MP-closed sets , Čech MP-open sets.

I. INTRODUCTION

Čech spaces were introduced by Eduard Čech [5] (i.e., sets endowed with a grounded , Extensive and additive closure operators) and studied by many others[6][9].N. Levine [9] introduced g-closed sets. The concept of generalized closed sets and generalized continuous maps of topological spaces extended to closure space in [4].D. Andrijevic [1] initiated the study of β-open sets and β-closed sets. In this paper we introduce the concept of Čech MP-closed sets and Čech MP-open sets and discuss some of its properties.

II. PRELIMINARIES

A map k:P(X)→P(X) defined on the power set P(X) of a set X is called a closure operator on X and the pair (X,k) is called a closure space if the following axioms are satisfied.

1. k(ϕ)=ϕ
2. A⊆k(A) for every A⊆X
3. k(A∪B)=k(A)∪k(B) for all A,B⊆X

A closure operator k on X is called idempotent if k(A)=k[k(A)] for all A⊆X.

Definition 2.1: A subset of a Čech - closure space (X,k) will be called Čech closed if k(A)=A and Čech -open if its complement is closed. i.e., if k(X-A)=X-A.

Definition 2.2: A subset A of a Čech closure space (X,K) is said to be

1. Čech regular open if A=int(k(A)) and Čech regular closed if A=k(int(A))
2. Čech pre open if A⊆int(k(A)) and Čech pre closed if k(int(A))⊆A
3. Čech semi open if A⊆k(int(A))
4. Čech α-open if A⊆int (k(int(A))) and Čech α-closed if K(int(k(A)))⊆A
5. Čech β-open if A⊆k(int(k(A))) and Čech β-closed if int(k(int(A)))⊆A

Definition 2.3: Let (X, k) be a Čech closure space. A subset A⊆X is called a Čech w-closed set if k(A)⊆G whenever A⊆G and G is semi-open subset of (X,k). A subset A of X is called a w-open set if its complement is a w-closed subset of (X,k).

Definition 2.4: Let (X,k) be a Čech closure space. A subset A⊆X is called a Čech g-closed set if k(A)⊆G whenever A⊆G and G is Čech-open subset of (X,k). A subset A⊆X is called a generalized open set, briefly a g-open set, if its complement is g-closed.

Definition 2.5: Let (X,k) be a Čech closure space. A subset A⊆X is called a Čech αψ-closed set if k(A)⊆G whenever A⊆G and G is α-open subset of (X,k).

Definition 2.6: Let (X,k) be a Čech closure space. A subset A⊆X is called a J-Čech closed set if $k_\alpha(A)$⊆G, whenever A⊆G and G is semi-open subset of (X,k),where $k_\alpha(A)$ is the smallest α-closed set containing A.

Definition 2.7: Let (X,k) be a Čech closure space. A subset A⊆X is called a Čech πgβ-closed set if k(A)⊆G whenever A⊆G and G is π-open subset of (X,k).

III. ČECH MP-CLOSED SETS

Definition 3.1: Let (X,k) be a Čech closure space.A subset A⊆X is called a Čech MP-closed set closed set containing A if $k_\beta(A)$⊆G whenever A⊆G and G is π-open subset of (X,k),where $k_\beta(A)$ is the smallest β-closed set containing A. A subset A of X is called a MP-open set if its complement is a MP-closed subset of (X, k).

Definition 3.2: Let (X,k) be a Čech closure space. A subset A⊆X is called a Čech M-closed set if $k_\beta(A)$⊆G whenever A⊆G and G is Čech-open subset of (X,k),where $k_\beta(A)$ is the smallest β-closed set containing A.

Definition 3.3: Let (X,k) be a Čech closure space. A subset A⊆X is called a Čech N-closed set if $k_\beta(A)$⊆G whenever A⊆G and G is α-open subset of (X,k),where $k_\beta(A)$ is the smallest β-closed set containing A.

Definition 3.4: Let (X,k) be a Čech closure space. A subset A⊆X is called a Čech T-closed set if k_β(A)⊆G whenever A⊆G and G is semi-open subset of (X,k),where k_β(A) is the smallest β-closed set containing A.

Definition 3.5: Let (X,k) be a Čech closure space. A subset A⊆X is called a Čech D-closed set if k_β(A)⊆G whenever A⊆G and G is pre-open subset of (X,k),where k_β(A) is the smallest β-closed set containing A.

Theorem 3.6: Every Čech closed set is Čech MP-closed set

Proof: Let G be a π-open set of (X,k) such that A⊆G. Since A is closed k(A)=A. Therefore k_β(A)⊆k(A)=A⊆G. i.e., k_β(A)⊆G, where G is π-open. Therefore A is MP-closed set. Hence Every Čech closed set is Čech MP-closed set.

Remark 3.7: Converse of the above theorem need not be true which can be seen from the following example

Example 3.8: Let X={a, b ,c} and define the closure operator k on X by k{ϕ}=ϕ, k{a}={a},k{b}={b ,c},k{c}=k{a, c}={a ,c}, k{a ,b}=k{b, c}=k X=X
Čech closed sets of X ={ϕ, X,{a},{a ,c}}
Čech MP-closed of X={ϕ, X,{a},{b},{c},{a, b},{b, c},{a, c}}
Then A={a,b} is Čech MP-closed set but not Čech closed set.

Theorem 3.9:
(a) Every Čech w-closed set is Čech MP-closed set.
(b) Every Čech g-closed set is Čech MP-closed set.
(c) Every Čech αψ-closed set is Čech MP-closed set.
(d) Every J-Čech closed set is Čech MP-closed set.
(e) Every Čech πgβ-closed set is Čech MP-closed set.

Proof: (a) Let A be a Čech w-closed closed set. Then k_β(A)⊆G whenever A⊆G and G is π-open in X. But k(A)⊆ k_β(A) whenever A⊆G,G is π-open in G. Now we have k_β(A)⊆G,G is π-open. Therefore A is Čech MP-closed set.
Proof is obvious for others.

Remark 3.10: Converse of the above theorem need not be true which can be seen from the following example

Example 3.11: Le t X={a, b, c, d} and define the closure operator on X by k{ϕ}={ϕ},k{a}=k{a, b}={a, b},k{b}= k{b, c}=k{a, b, c}= {a, b, c},k{c}=k{a, c}={a, c},k{d}= k{a, d}=k{a, b, d}={a, b, d},k{b, d}= k{c, d}=k{b, c, d}= k{a, c, d}=k X=X
Čech w- closed set of X={ϕ ,X,{a ,b},{a, c},{a, b ,c},{a, b, d}}
Čech MP-closed set of X={ϕ, X,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c}{b, d}, {c, d},{a, b, c},{a, c, d},{a, b, d},{b, c, d}}
Then A={c,d} is Čech MP-closed set but not in Čech w-closed set.

Example 3.12: Let X={a, b, c, d} and define the closure operator on X by k{ϕ}={ϕ},k{a}=k{a ,b}={a ,b},k{b}= k{b ,c}=k{a ,b, c}= {a ,b ,c},k{c}=k{a ,c}={a ,c}, k{d}= k{a ,d}=k{a, b, d}={a ,b, d},k{b, d}=k{c, d}= k{b, c, d}=k{a, c, d }=k X=X
Čech g-closed set of X={ϕ, X,{a},{a, b},{a, c},{a, d},{b, c},{a, b, c},{a, c, d}, {a, b, d}{b, c, d}}
Čech MP-closed set of X={ϕ, X,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c}{b, d}, {c, d},{a, b, c},{a, c, d},{a, b, d,{b, c, d}}}
Then A={b} is Čech MP-closed set but not in Čech g-closed set.

Example 3.13: Let X={a, b, c, d} and define the closure operator on X by k{ϕ}={ϕ},k{a}=k{a, b}={a, b},k{b}= k{b, c}=k{a, b, c}= {a, b, c},k{c}=k{a, c}={a, c}, k{d}=k{a, d}=k{a, b, d}={a, b, d},k{b, d}=k{c, d}= k{b, c, d} =k{a, c, d}=k X=X
Čech αψ –closed set of X={ϕ ,X,{a},{a ,b},{a, b, c},{a, b, d}}
Čech MP-closed set of X={ϕ, X,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c}{b, d},{c, d},{a, b, c},{a, c, d},{a, b, d},{b, c, d}}
Then A={b, c, d} is Čech MP-closed set but not in Čech αψ-closed set.

Example 3.14: Let X={a, b, c} and define the closure operator on X by k{ϕ}=ϕ, k{a}={a},k{b}={b, c},k{c}=k{a, c}={a, c},k{a, b}=k{b, c}=k X=X
J-Čech closed set of X ={ϕ, X,{a},{c},{a, c}}
Čech MP-closed set of X={ϕ, X,{a},{b},{c},{a, b},{b ,c},{a ,c}}
Then A={a, b} is Čech MP-closed set but not in J-Čech closed set.

Example 3.15 : Let X={a, b, c, d} and define the closure operator on X by k{ϕ}=ϕ, k{a}=k{d}=k{a, b}= k{a, d}=k{a, b, d}={a, b, d},k{b}={b}, k{c}= k{b, d}=k{c, d}=k{b, c, d}= {b, c, d},k{a, c}=k{a, c, d}= {a, c, d},k{b, c}=k{a, b, c}=k X=X
Čech πgβ-closed set of X={ϕ, X,{b},{a, d},{b, d},{c, d},{a, b, c},{a ,c, d},{a, b, d},{b ,c, d}}
Čech MP-closed set of X={ϕ, X,{b},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c},{a, c, d},{a, b, d},{b, c, d}}
Then A={a ,b} is Čech MP-closed set but not in Čech πgβ-closed set.

Theorem 3.16:
a. Every Čech M-closed set is Čech MP-closed set.
b. Every Čech N-closed set is Čech MP-closed set.
c. Every Čech T-closed set is Čech MP-closed set.
d. Every Čech D-closed set is Čech MP-closed set.

Proof: (a) Let G be a Čech π-open subset of (X,k) such that A⊆G. Since A is closed k(A)=A⊆G. Let A be Čech M-closed set that implies k_β(A)⊆G whenever A⊆G,G is open. But we have k_β(A)⊆k(A)=A⊆G that implies

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
ISO 3297:2007 Certified
Vol. 3, Issue 8, August 2016

k_{β}(A)⊆G, where G is π-open. Hence Ever Čech M-closed set is Čech MP-closed set.

Proof is obvious for others.

Remark 3.17: Converse of the above theorem need not be true which can be seen from the following example.

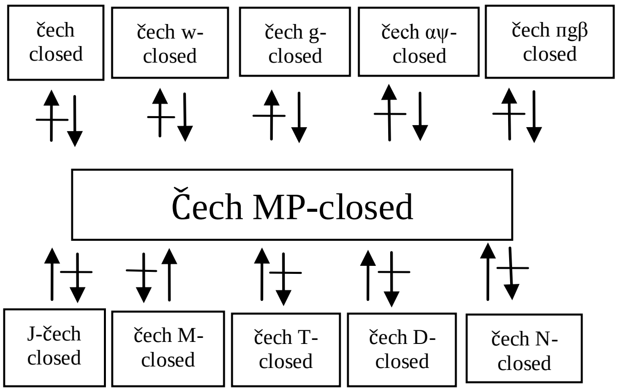
Example 3.18: Let X={a, b, c, d} and define the closure on X by k{ϕ}={ϕ},k{a}=k{a, b}={a, b},k{b}= k{b, c}=k{a ,b, c}= {a, b, c},k{c}=k{a, c}= {a, c},k{d}=k{a, d}=k{a, b, d}={a, b, d},k{b, d}= k{c, d}=k{b, c, d}=k{a, c, d}=k X=X
Čech M- closed set of X={ϕ, X, {a},{b},{c},{a, b},{a, c},{a, d}, {b, c},{b, d},{a, b, c},{a, c, d},{a, b, d},{b, c, d}}
Čech MP-closed set of X={ϕ, X,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c}{b, d},{c, d},{a, b, c},{a ,c, d},{a, b, d},{b, c, d}}
Then A={c, d} is Čech MP-closed set but not in Čech M-closed set.

Example 3.19: Let X={a, b, c, d} and define the closure on X by k{ϕ}={ϕ},k{a}=k{a,b}={a,b},k{b}=k{b,c}=k{a,b,c}={a,b,c},k{c}=k{a,c}={a,c},k{d}= k{a, d}=k{a, b, d}={a, b, d},k{b, d}=k{c, d}=k{b, c, d}=k{a, c, d}=k X=X
Čech N –closed set of X={ϕ, X,{a},{b},{c},{d},{a, b},{a, c},{b, c},{b, d},{a, b, c}, {a ,b, d}{b, c, d}}
Čech MP-closed set of X={ϕ, X,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c}{b, d}, {c, d},{a, b, c},{a, c, d},{a, b, d},{b, c, d}}}
Then A={a, c, d} is Čech MP-closed set but not in Čech N-closed set.

Example 3.20: Let X={a, b, c, d} and define the closure on X by k{ϕ}={ϕ},k{a}=k{a, b}={a, b},k{b}= k{b, c}=k{a, b, c}= {a, b, c},k{c}=k{a, c}={a, c},k{d}= k{a, d}=k{a, b, d}={a, b, d},k{b, d}=k{c, d}= k{b, c, d}=k{a, c, d}=k X=X
Čech T -closed set of X={ϕ, X, {a}, {b}, {c}, {a, b},{a, c}, {b, c} ,{b, d},{a, b, c},{a, b, d}{b, c, d}}
Čech MP-closed set of X={ϕ, X,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c}{b, d}, {c, d},{a, b, c},{a, c, d},{a, b, d},{b, c, d}}
Then A={d} is Čech MP-closed set but not in Čech T-closed set.

Example 3.21: Let X={a, b, c, d} and define the closure operator on X by k{ϕ}=ϕ, k{a}=k{d}=k{a, b}= k{a, d}=k{a, b, d}={a, b, d},k{b}={b}, k{c}=k{b, d}= k{c, d}=k{b, c, d}= {b, c ,d},k{a, c}=k{a, c, d}={a, c, d}, k{b, c}=k{a, b, c}=k X=X
Čech D-closed of X={ϕ, X,{b},{d},{a, b},{a, c},{a, d},{ b, c},{b, d},{a ,b, c}, {a, c, d},{a, b, d},{b, c, d}}
Čech MP-closed set of X={ϕ, X,{b},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d}, {a, b, c},{a, c, d},{a, b, d},{b, c, d}}
Then A={c, d} is Čech MP-closed set but not in Čech D-closed set.

Remark 3.22: From the above results we have the following implications.



Theorem 3.23: Let (X,k) be a closure space. If A and B are Čech MP-closed subsets of (X,k) then A∪B is Čech MP-closed set.
Proof: Let G be a π-open subset of (X,k) such that A∪B⊆G ,then A⊆G,B⊆G. Since A and B are Čech MP-closed sets, k_{β}(A)⊆G and k_{β}(B)⊆G that implies k_{β}(A)⋃ k_{β}(B)⊆G. But k_{β}(A∪B)= k_{β}(A)∪ k_{β}(B)⊆G. Therefore (A∪B) is Čech MP-closed set.
Remark 3.24: The intersection of Čech MP-closed sets need not be Čech MP-closed set.

Theorem 3.25: If A is a Čech MP-closed set, then k_{β}(A)-A contains no non empty Čech π-closed set.
Proof: Let A be Čech MP-closed set. Let F be a non empty Čech π-closed set ⊆ k_{β}(A)-A. That implies F⊆ k_{β}(A)⋂A^c. (i.e.,) F⊆k_{β}(A) and F⊆A^c.F⊆A^c⇒ A⊆F^c. Since F is Čech π-closed, F^c is Čech π-open. Thus we have k_{β}(A)⊆ F^c. Consequently F⊂[$k_{\beta}(A)$]^c.Hence we get F⊆ k_{β}(A)⋂ [$k_{\beta}(A)$]^c=ϕ. Hence k_{β}(A)-A contains no non empty Čech π-closed set.

Corollary 3.26: Let A be a Čech MP-closed set. Then A is Čech β-closed if and only if K_{β}(A)-A is Čech MP-closed set.
Proof: Suppose that A is Čech MP-closed set and Čech β-closed set. Since A= k_{β} (A) we have k_{β}(A)-A=ϕ, which is Čech π-closed. Conversely, Suppose that A is Čech MP-closed set and k_{β}(A)-A contains no non empty Čech π-closed set. Then k_{β}(A)-A is itself Čech π-closed⇒ k_{β}(A)-A=ϕ. Hence A is Čech MP-closed.

Proposition 3.27: Let (X,k) be a čech closure space. If A is čech MP- closed and F is čech π- closed in (X,k) then A⋂F is čech MP- closed.

Proof : Let G be a čech π-open subset of (X,k) such that A⋂F⊆G, Then A⊆G⋃(X-F).and so, since A is čech MP-closed,k_β(A)⊆G⋃(X,F), Then k_β(A)⋂F⊆G, since F is čech π-closed, k_β(A⋂F)⊆G. Therefore A⋂F is čech MP-closed.

Proposition 3.28: Let (Y,l) be a closed subspace of (X,k). If F is a čech MP-closed subset of (Y,l),then F is a čech MP-closed subset of (X,k).

Proof: Let G be čech π-open set of (X,k) such that F⊆G. Since F is čech MP-closed and G⋂Y is čech π-open k_β(F)⋂Y⊆G, But Y is closed subset of (X,k) and k_β(F)⊆G, where G is a čech π-open set. Therefore F is a čech MP-closed set of (X,k).

Proposition3.29: Let (X,k) be a čech closure space and let k be idempotent. If A is a čech MP-closed subset of (X,k) such that A⊆B⊆k_β(A),then B is a čech MP-closed subset of (X,k)

Proof: Let G be a čech π-open subset of (X,k) such that B⊆G. Then A⊆G, since A is čech MP-closed, k_β(A)⊆G. As k is idempotent, k_β(B)⊆ k_β(k_β(A))= k_β(A)⊆G, Hence B is čech MP-closed.

Proposition 3.30: Let (X,k) be a čech closure space and A⊆X, then the following are true:

a. If A is čech closed then A is čech MP-closed.
b. If A is čech g-closed then A is čech MP-closed.
c. If A is čech β-closed then A is čech MP-closed.
d. If A is čech π-closed then A is čech MP-closed.
e. If A is čech π-open and čech MP-closed then A is čech β-closed.

Proof: Given A is čech closed implies k(A)=A. But if A⊆G and G is π-open. Then k_β(A)⊆k(A)=A⊆G which implies A is čech MP-closed. Thus if A is čech closed then A is čech MP-closed.
The Proof of the remaining statements are obvious.

IV. ČECH MP-OPEN SETS

Definition 4.1: A subset A in Čech closure space (X,k) is called Čech MP-open set if its complement is Čech MP-closed set.

Theorem 4.2: A subset A in Čech closure space (X,k) is called Čech MP-open set if and only if F⊆X- k_β(X-A) whenever F is π-closed and F⊆A.

Proof: Suppose that A is Čech MP-open and F be a π-closed subset of (X,k) such that A⊆F then X-A⊆X-F. But X-A is Čech MP-closed set and X-F is π-open. That implies k_β(X-A)⊆X-F (i.e.,) F⊆X- k_β(X-A) Conversely, Let F be a π-closed set, F⊆A and F⊆X- k_β(X-A) that implies k_β(X-A)⊆X-F,X-F is π-open that implies X-A is Čech MP-closed set and so A is Čech MP-open.

Theorem 4.3: If A and B are Čech MP-open subsets of (X,k) then A∩B is Čech MP-open set.

Proof: Let F be a π-closed subset of (X,k) such that F⊆A∩B .Then X-(A∩B)⊆X-F This implies that (X-A)∪(X-B)⊆X-F. (X-A)∪(X-B) is Čech MP-closed set. (By proposition 3.23) Thus k_β[(X-A)∪(X-B)]⊆X-F Hence k_β[X-(A∩B)]⊆X-F that implies F⊆X- k_β[X-(A∩B)] that implies A∩B is Čech MP-open (by Theorem 4.2).

Theorem 4.4: Let (X, g) be a closure space and let (Y ,h) be a closed subspace of (X, g).If G is Čech MP-open subset of (X, g) then G∩Y is Čech MP-open subset of (Y ,h).

Proof: Let G be a Čech MP-open subset of (X,g). then X-G is a Čech MP-closed subset of (X, g). Since Y is a closed subset of (X, g),(X-G)∩Y is a Čech MP-closed set of (X, g). But (X-G)∩Y=Y-(G∩Y). Therefore Y-(G∩Y) is a Čech MP-closed subset of (X, g). Hence G∩Y is a Čech MP-open subset of (X, g).

REFERENCES

[1] D. Andrijevic, "On β-open sets", Mat vesnik 48(1996), 59-64.
[2] I.Arockiarani and J.Martina jency. "On J-Čech closed sets in closure spaces, Kerala Mathematical Association.
[3] C.Boonpok, "Between closed sets and generalized closed in closure spaces , Acta Math Univ, Ostra 16(2008) 3-14.
[4] C.Boonpok, "Generalized closed sets in Čech closed spaces" Acta Universitatis Apulensis.
[5] E.cech, " Topological Spaces", Inter science publications John wiley and son, New York (1996).
[6] K. Chandrasekhara Rao and R.Gowri "On closure Spaces", Varahamihir journal of Mathematical Science 5(2) (2005), 375-378.
[7] R.Devi and M.Parimala "On αψ-closed Sets in Bi-Čech Closure Spaces" Int. Journal of Math. Analysis, Vol. 4, 2010, no. 33, 1599-1606.
[8] A.Francina Shalini and I.Arockiarani," Čech πgβ-closed sets in closure spaces, Journal of Global Research in Mathematical Archives, Volume1, No: 4, April 2013.
[9] C.Janaki and I. Arockiarani "A new class of open functions in Čech spaces", Actaciencia India, 36M, 4(2010), 649-653.
[10] N.Levine," Generalized closed sets in topology, Rend, circ, Mat palemo,19(1970),89-96.
[11] D. Sasikala and I.Arockiarani "Čech w-closed sets in closure spaces", International Journal of Mathematical Archive-4(10), 013, 30-35.