

# Tuning the Group Velocity of Light for 1D Photonic Crystal with Defect

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**Abstract:** An exact representation of dispersion relation for one dimensional periodic system composed of dielectric layers with a defect is derived by means of transfer matrix, in order to calculate the group velocity of the defect modes in the photonic band gap. It is found that slow group velocities can be tuned by modifying size of the defect layer, as moving towards the band edge the frequency values get lower shifts. Effect of the different defect dielectric constants on the group velocity is also presented.

**Keywords:** photonic crystal, defect mode, dispersion relation, group velocity, slow light.

## I. INTRODUCTION

A major interest of photonics is to provide some basic strategies to reach novel approaches to new technology including the concepts of slow light which has advantageous of controlling the optical signals for data transmission [1]. Many of the approaches to achieve the slow light rely on electromagnetically induced transparency [2], coupled resonator structures [3] and photonic crystals (PhCs) [4] which require good theoretical foundations. In particular, the PhCs at the defect frequency represent ultra-slow group velocities [2,3]. There are many studied examples of PhCs that are focused on light propagating in periodic arrays of dielectric scatterers [5,6]. Theoretical and numerical analysis for dispersion properties in which the periodic medium is one dimensional (1D) [7,8], two dimensional (2D) [9,10] or three dimensional (3D) [11,12] are extensively well known. When these structures consist of defects such as cavities [13], line defects [14] the investigation to solutions of Maxwell equations are now more difficult. However, controlling the slow light requires a reduction of the group velocities which can be obtained by means of dispersion properties. In this paper, it is purposed to examine some optical properties in PhC structure with defect for exploration the tuning the group velocity.

In Section II it is derived an analytic expression for band structure with defect states in 1D-PhC consisting of dielectric layers with a defect as a single layer. In Section III numerical results are presented and discussed for dispersion relation and the affected group velocity by different defect layer thickness and dielectric constant. Section IV evaluates these results for the applications.

## II. THEORY

In order to study optic properties of 1D defect-PC, John and Wang model [15] is used which permits to investigate light propagation in a periodic medium. In particular, it is considered electromagnetic wave propagating in a 1D medium that has uniform dielectric properties in one

direction which is taken to be x axis. The structure composed of alternating dielectric layers is illustrated in Fig.1.

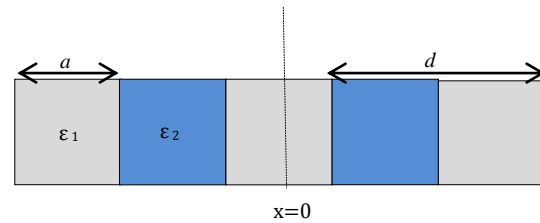


Fig.1 Schematic diagram of an ideal 1D PhC of dielectric layers, a and d represent thickness of layer with the dielectric constant of  $\epsilon_1$  and lattice constant, respectively.

The wave equation for electromagnetic modes in a 1D system may be written as

$$\frac{\partial^2}{\partial x^2} E(x, t) = \frac{\epsilon(x)}{c^2} \frac{\partial^2}{\partial t^2} E(x, t) \quad (1)$$

with

$$\epsilon(x) = \begin{cases} \epsilon_1 & a/2 < x < (a/2 + d) \\ \epsilon_2 & a/2 < x < a/2 \end{cases}$$

where a and (d-a) are the lengths of dielectric layers with dielectric constants  $\epsilon_1$  and  $\epsilon_2$ , respectively. d is also lattice constant and c is the velocity of light in the vacuum. The solutions of Equation (1) for a certain region in the structure are superpositions of left- and right-travelling waves. Boundary conditions require these solutions and their derivatives to be continuous at two interfaces between dielectric media. It is assumed that time dependence of the electromagnetic wave of angular frequency  $\omega$  has the form of  $E(x, t) = E(x) \exp[-i\omega t]$  and dielectric layers are periodically positioned, namely,  $\epsilon(x + d) = \epsilon(x)$ . As a result of the periodicity of the lattice, the Bloch's theorem implies that bounded solutions must satisfy

$$E(x + d) = e^{i\beta d} E(x)$$

where  $\beta$  is Bloch wavevector along the x axis which is normal to 1D-PhC. Smith et al. [6] are obtained an exact solution for PhCs for one dimension using an analytic method which can be expanded to higher dimensions or defect structures. It was discussed in detail by several authors, e.g., Ojha et al. [16].

On the other hand, if the 1D-PhC mentioned above has a defect which consists of a single layer, its dispersion properties change because the translation symmetry is broken. Such a system is shown in Fig. 2 considering the three regions I, II, and III centered about the origin, it can be identified defect region as region II, a semi-infinite lattice which its solution increases exponentially to the left as region I and the other semi-infinite lattice which its solution decreases exponentially to the right as region III.

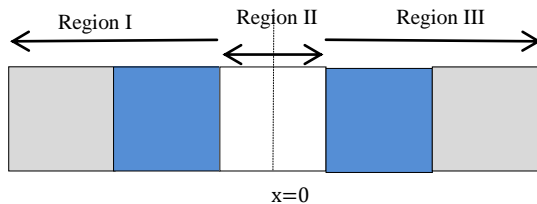


Fig. 2 The same lattice as Fig. 1. exact removed part at the center as a defect in air.

The general solutions in regions I and III are

$$E_I(x) = Ae^{ikx} + Be^{-ikx}$$

$$E_{III}(x) = Ce^{ikx} + De^{-ikx}$$

where  $k = \sqrt{\epsilon_1} \omega / c$ . Now, Bloch's theorem requires that  $E(x) = e^{\pm k d} E(x + d)$  with real and positive wavevector  $k$ . The general solution for region II is

$$E_{II}(x) = e^{k d} (Ae^{iq(x+d)} + Be^{-ik(q+d)})$$

where  $q = \sqrt{\epsilon_2} \omega / c$ . By applying of the boundary conditions to the  $E_I$  and  $E_{II}$ , at  $x = -\frac{a}{2}$ , a matrix equation for unknown coefficients A and B, which determines the eigenfunctions and eigenvalues is given by

$$(TM - e^{k d} I) \begin{bmatrix} A \\ B \end{bmatrix} = 0 \quad (2)$$

where

$$T = \begin{bmatrix} e^{ikd} & 0 \\ 0 & e^{-ikd} \end{bmatrix}$$

and matrix elements of M are

$$M_{11} = e^{-ika} \left( \cos qa + \frac{i}{2} \left( \frac{k}{q} + \frac{q}{k} \right) \sin qa \right)$$

$$M_{12} = \frac{i}{2} \left( \frac{q}{k} - \frac{k}{q} \right) \sin qa$$

$$M_{21} = \frac{i}{2} \left( \frac{q}{k} - \frac{k}{q} \right) \sin qa$$

$$M_{22} = e^{ika} \left( \cos qa - \frac{i}{2} \left( \frac{k}{q} + \frac{q}{k} \right) \sin qa \right).$$

Solution to the Equation (2) gives the relation between  $k$  and  $\omega$ , exactly,

$$= \left[ \cosh(\kappa a) \cos \left( \frac{\sqrt{\epsilon_1} \omega (d-a)}{c} \right) \cos \left( \frac{\sqrt{\epsilon_2} \omega a}{c} \right) - \frac{1}{2} \left( \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_2}} + \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}} \right) \sin \left( \frac{\sqrt{\epsilon_1} \omega (d-a)}{c} \right) \sin \left( \frac{\sqrt{\epsilon_2} \omega a}{c} \right) \right]. \quad (3)$$

Because bound states of 1D systems are nondegenerate and the defect-1D system has a reflection symmetry, symmetric or antisymmetric defect modes are introduced in the gap of the PhC [6]. By demanding that  $E_{II}$  and  $E_{III}$  obey the conditions at  $x = \frac{a}{2}$ , involving the coefficients C and D, a second matrix equation can be obtained with negative  $\kappa$  which yields Equation (3).

The group velocity of electromagnetic waves in dispersion material is given as [17]

$$V_g = \frac{d\omega}{dk} = \left[ \frac{dk}{d\omega} \right]^{-1} \quad (4)$$

Equation (3) leads directly to calculate  $V_g$  by the inverse of the first-order dispersion.

### III. RESULTS AND DISCUSSION

It is clear that dispersion is a basic concept for calculating properties of the light in a medium, thus, it is firstly calculated the variation of the normalized frequency as a function of the normalized wave vector by using Equation (3). It is considered that the 1D-defect-PhC consists of the alternating dielectric layers with  $\epsilon_1 = 1.5$  and  $\epsilon_2 = 3.46$  in the case of  $a/d = 0.5$ . Fig. 3 shows typical band diagram of the analytic results. The first band gap is formed as a region by splitting of the bottom and the upper band edge states. As it is seen, the 1D-defect-PhC supports only one defect state in the first PBG.

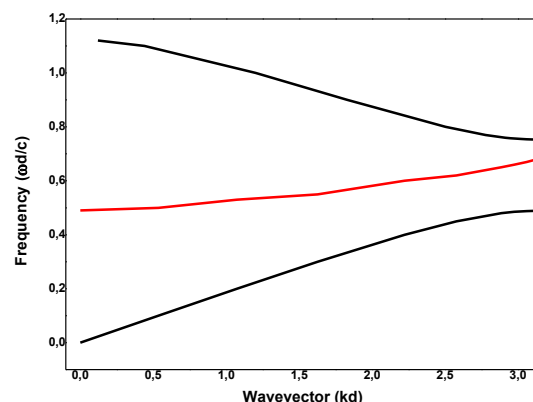


Fig. 3 The photonic band diagram for 1D-defect PhC with  $f=0.5$ . Dielectric constants  $\epsilon_1$  and  $\epsilon_2$  are 1.5 and 3.46, respectively. Red line shows defect mode introduced in the gap.

In order to investigate the effect of the dielectric layers thicknesses to the group velocity, three cases are

considered in which the filling factors are  $f = a/d = 0.3, 0.4$  and  $0.5$ , respectively. Fig. 4 represents the group velocity  $V_g$  as expressed in Equation (4) for different values of the layer thickness. It can be seen that the maximum value of the group velocity of defect modes is about  $0.3c$  for  $f = 0.5$ . By decreasing the filling factor it can be achieved the lower group velocity. However, small shift can be seen near the band edge frequencies.

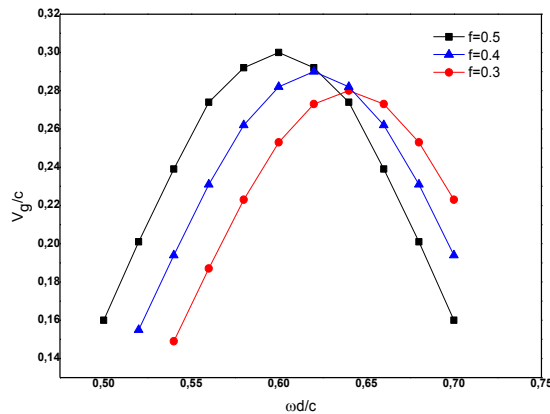


Fig. 4 The group velocity as a function of the normalized frequency for 1D-defect PhC with different filling factors.

The effect of the dielectric constant of the defect layer on the group velocity in middle of the photonic band gap with the fixed value  $f = 0.5$  is depicted in Fig. 5. It is clearly seen that  $V_g$  decreases when the defect dielectric constant increases.

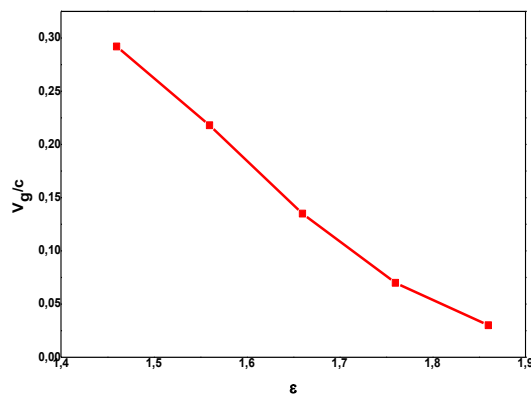


Fig. 5 Effect of the defect dielectric constant on group velocity

#### IV. CONCLUSION

In conclusion, it is theoretically calculated the dispersion relation and the group velocity in the 1D-defect-PhC with the dielectric lattice of the layers. It is found that the group velocity can be tuned by modifying the size of defect layers or alternating the defect layer dielectric properties. The results present a foresight that one can choose suitable slow light region by disregarding nonlinear optical properties.

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