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Performance of Two Sample Scale Tests: An Empirical Study

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Abstract: For the two samples scale problem, we have considered the ten existing tests viz. F-test, Mood test, Ansari-Bradley test, Siegel-Tukey test, Lepage type test, Cramer-Von-Mises test, Kamat test, Wald-W test, Wald-R test and Levene test. To study the performance of these tests we generate data from the distributions viz. Normal, Exponential, Double Exponential, Logistic and Cauchy. We have calculated the empirical level under null situations and power under alternative situations taking various combinations of scale parameters and sample sizes using simulation technique. We also simulated critical values for Kamat test, Wald-W test and Wald-R test for some sample sizes. Results are displayed in various tables and graphs. Discussion and conclusion are made on the basis of results obtained.

Keywords: Mood test, Ansari-Bradley test, Siegel-Tukey test, Lepage type Test, Cramer –von Mises test, Kamat test, Wald-W test, Wald-R test, Levene test, Simulation study, Symmetrical and Asymmetrical Distributions.

1. INTRODUCTION

The study of variance difference differences in two or more populations is one of the important problem in statistical inference. There are many parametric, nonparametric tests are developed till now. In the early years, Lehmann (1951), Mood (1954) and Shukhatme (1957) developed some nonparametric tests of variance differences. The procedure suggested by Lehmann however, has been shown not to be distribution free. The test proposed by Mood assumes knowledge of the means of the two populations, whereas in practice such knowledge rarely exists. The procedure suggested by Shukhatme also assumes knowledge regarding the location of the two populations and in addition this procedure is not as efficient as the one proposed by Mood. Procedures proposed by Kamat (1956) and Rosenbaum (1953) seem likely to lack power. Ansari and Bradley (1960) developed a rank test for dispersion. They have shown the equivalence of two rank tests for comparing dispersion, one test due to Barton and David (1958), the other to Ansari and Freund, and have provided tables of the exact distribution. They observed that Siegel and Tukey (1960) have proposed a similar test which permits use of existing tables. They also exhibit the mean of the limiting normal distribution under the alternative hypothesis .Assuming normality of the data the t test and F test are uniformly most powerful unbiased test in the case of location and scale alternatives respectively, and it is well known that the t test is α- robust for non normal distributions, expect for those with very long tails, but not β- robust, whereas F test is extremely not α robust for non normal data. Perng Littell (1976) proposed a test Q of equality of means and variances in the case of normal population where Q is mixture of t test and F test. As the F test Q test, however is not α robust for non normal data. So in this connection Buning and Thadewald (2000) looked for tests which are generally powerful for non normal data and for location and scale alternative. One of the most powerful tests for this problem is the test of Lepage, which is a combination of the Wilcoxon test for location and Ansari Bradley test for scale. For General alternatives including different shapes of distributions of the X and Y variables the tests of Kolmogorov – Smirnov, Cramer – Von Mises and Wald-Wolfowitz might be the most popular ones, where Cramer-Von-Mises test seems to be the best one among these non parametric tests over a broad class of distributions. Hall and Padmanabhan (1997) have proposed some tests, which are Flinger – Killeen (F –K: med) type modification of the Ansari –Bradley, Mood and Klotz test and estimated their quantities by means of the smooth bootstrap. In 2011, Padmanabhan, Othman and Yin proposed variants of the above tests, called refined robust test or refinements, for short and showed that they are superior to the standard test and in addition their performance improves with increasing sample sizes.

Allingham and Rayner(2011) introduced a new test for equality of variances based on the nonparametric analogue of a very natural Wald test and call it the R Test. In an indicative empirical study they showed that, for moderate sample sizes when assuming normality the R test is nearly as powerful as the F test and nearly as robust as Levene's test. It is an appropriate test for equality of variances without the assumption of normality, and so it can be strongly recommended.

Our aim of study is to compare the ten existing tests viz.F-test Mood test, Ansari-Bradley test, Siegel-Tukey test, Lepage Type Test (Lepage Test), Cramer-von-Mises test, Kamat test, Wald-W test, Wald-R test and Levene test for testing difference of scale parameters. We here use Monte Carlo simulation technique for find the results by generating observations for symmetric and asymmetric

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distributions (normal, exponential, double exponential, logistic and cauchy). In this paper as we observed that Wald-W test is not able to satisfy the specified significance level under Cauchy, exponential and double exponential distributions and so we are not calculating power of this test under these distributions. In section 2 we describe the test statistics and section 3 contains some simulation power of the tests considered. Discussion and conclusion are given in section 4 and section 5. The simulated critical values of Wald-W test and Wald – R test have shown in Table - 3

2. TEST STATISTICS

Let $X_1, X, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be two independent random sample from population with absolutely continuous distribution function $F\left(\frac{X-\mu_1}{\sigma_1}\right)$ and $F(\frac{Y-\mu_2}{\sigma_2})$, respectively. We wish to test the null hypothesis H_0 : $\sigma_1 = \sigma_2, \ \ \mu_1 = \mu_2$ Vs $H_1 : \sigma_1 > \sigma_2, \ \ \mu_1 = \mu_2$

Mood Test (M):
In closer analogy with classical test for scale, Mood has proposed a test using as test statistic

$$M = \sum_{i=1}^{n}\left(R_i - \frac{n+m+1}{2}\right)^2$$

Where R_i is the size rank of X_i in the pooled sample (the smallest of whose observations receives a rank of 1, the next smallest a 2, etc., and largest a rank of n + m) and (n + m + 1)/2 is the average rank of all observations comprising the pooled sample. The null distribution of M has a mean of [n (n + m +1) (n +m -1)]/12 and a variance of [nm (n + m + 1)(n + m + 2)(n + m -2)]/180, and the test apparently consists of treating M as a normal deviate.

Siegel Tukey Test (ST):
This test replaces the pooled data from the two sample with a reordering of the ranks (i) from 1 to N. to illustrate the ranking procedure, note the following table when N is assumed to be an even number

Ordered Score	V^1	V^2	V^3…. $V^{N/2}$…..V^{N-3}	V^{N-2} V^{N-1} V^N
Rank Replacement	1	4	5 …..	N …… 7 6 3 2

At the left end of the ordered set of scores, V^1 is assigned a rank of 1. The test development now requires a move to the extreme right end of the ordered scores where V^N is replaced by the rank 2 and V^{N-1} by the rank 3. Now move back to the left and substituted the ranks 4 for V^2 and 5 for V^3. Operating the pairs, this process is repeated until all ordered scores are replaced by their appropriate ranks. If N is an odd number, throw out the middle score. This will enable the adjacent ranks to sum to the same number and thus achieve a desired symmetry to the test. If there is no difference in scale between the populations from which the two samples are drawn, then the sum of the ranks associated with each sample should be approximately equal. On the other hand, if the score spread is not homogeneous for the two groups, then the rank sum of the sample with the greatest spread will be significantly smaller than the rank sum of the more compressed sample. Using an indicator variable, Z_i, let $Z_i = 1$ if the i^{th} replacement score is associated with the X sample and $Z_i = 0$ if the i^{th} score is tagged to the Y sample. This indicator variable is useful in setting up the test statistic which is defined as

$$ST = \sum_{i=1}^{N} iZ_i$$

The null distribution of the ST test is exactly the same as that of the Wilcoxon test. Thus, Wilcoxon table can be used to determine the significance of ST for $N < 20$. Equivalent table have been developed by Siegel and Tukey (1960). For $N < 20$ the distribution of ST approximates a normal distribution with $E(ST) = n(N+1)/2$ and $Var(ST) = nm(N+1)/12$. The Test statistics become

$$Z = \frac{(ST) - E(ST)}{\sqrt{Var(ST)}}$$

where Z follows standard normal distribution.

Cramer – Von Mises Test (CVM):
Let $X_{(1)}, X_{(2)}, \ldots, X_{(m)}$ and $Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}$ be the order statistics of $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ respectively and let $\tilde{F}_m$ and $\tilde{G}_n$ be the usual empirical distribution function for the X and Y samples. Furthermore, let be $Z_{(1)}, Z_{(2)}, \ldots, Z_{(N)}$ the order statistics of the combined two samples.
Note that

$$\tilde{F}_m\left(Y_{(j)}\right) = \frac{\left[R\left(Y_{(j)}\right) - j\right]}{m}, \quad j = 1, 2, \ldots, n$$

and

$$\tilde{G}_n\left(X_{(i)}\right) = \frac{\left[R\left(X_{(i)}\right) - i\right]}{n}, \quad i = 1, 2, \ldots, m$$

Where $R(X_{(i)})$ and $R(Y_{(j)})$ are rank of $X_{(i)}$ and $Y_{(j)}$ among $Z_{(1)}, Z_{(2)}, \ldots Z_{(N)}$, respectively. The Cramer – Von –Mises statistics (CM) is defined as follows,

$$CM = \frac{mn}{N^2}\sum_{i=1}^{N}\left[\tilde{F}_m\left(Z_{(i)}\right) - \tilde{G}_n\left(Z_{(i)}\right)\right]^2$$

CM can be rewritten as

$$CM = \frac{\sum_{i=1}^{m}\left[R\left(X_{(i)} - i\right)\right]^2}{nN} + \frac{\sum_{j=1}^{n}\left[R\left(Y_{(j)} - j\right)\right]^2}{mN} - \frac{4mn-1}{6N}$$

The Corresponding test reject H_0, if $CM \geq C_{1-\alpha}(m,n)$. Exact and asymptotically critical values $C_{1-\alpha}$ can be found in Burr (1963).

Lepage Test (L):
The test of Lepage (1971) is a combination of the Wilcoxon test W for location alternatives and the Ansari-Bradley test AB for scale alternatives. The W – test is locally most powerful for the logistic distribution and AB – test for a special Mielke distribution. The Wilcoxon statistic W is defiened by

$$\mathrm{W} = \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{R}\left(\mathrm{X}_{(\mathrm{i})}\right)$$

where $\mathrm{R}\left(\mathrm{X}_{(\mathrm{i})}\right)$ is the rank of $\mathrm{X}_{(\mathrm{i})}$ in the combined two samples. The Wilcoxon test corresponds to the location model $\mathrm{G}(\mathrm{z}) = \mathrm{F}(\mathrm{z} - \theta)$ with the hypothesis $\mathrm{H}_0: \theta = 0$ versus $\mathrm{H}_1: \theta < 0,\ \theta > 0$ or $\theta \neq 0$
Under H_0, we have $(\mathrm{N} = \mathrm{m} + \mathrm{n})$:

$$\mu_{w} = \mathrm{E(W)} = \frac{\mathrm{m(N+1)}}{2},$$

$$\sigma_w^2 = Var(W) = \frac{mn(N+1)}{12}$$

The Ansari-Bradley test corresponds to the scale model G (z) = F (z/τ) ,where τ is a scale parameter, with the hypotheses $H_0: \tau = 1$ versus $H_1: \tau < 1, \tau > 1$ or $\tau \neq 1$. The Corresponding test statistics is defined by

$$AB = \sum_{i=1}^{m} \left({}^{(N+1)}/_{2} - \left| R\left(X_{(i)}\right) - {}^{(N+1)}/_{2} \right| \right)$$

$$\mu_{AB} = E(AB) = \begin{cases} \dfrac{m(N+2)}{4} & if\ N\ is\ even \\ \dfrac{m(N+1)^2}{4N} & if\ N\ is\ odd \end{cases}$$

$$\sigma_{AB}^2 = Var(AB) = \begin{cases} \dfrac{mn(N^2-4)}{48(N-1)} & if\ \ N\ is\ even \\ \dfrac{mn(N+1)(N^2+3)}{48N^2} & if\ N\ \ is\ odd \end{cases}$$

Then the Statistics LP of Lepage is defined by

$$LP = \left(\frac{W - \mu_w}{\sigma_w}\right)^2 + \left(\frac{AB - \sigma_{AB}}{\sigma_{AB}}\right)^2$$

In Lepage (1971) it is shown that, under H_0, the statistics W and AB are uncorrelated for all m, n and as a consequence, the asymptotic distribution of LP is, under H_0, χ^2 with 2 degree of freedom, that is H_0 is rejected if $LP \geq \chi^2_{1-\alpha}(2)$.

Kamat Test (K):
Let R_n and R_m be the range of ranks of x and y respectively. The test statistic proposed is

$$D_{n,m} = R_n - R_m + m$$

Where $D_{n,m}$ can take values 0, 1, 2, . . ., m+n. Large or small values of $D_{n,m}$ will indicate possible divergence from the hypothesis that the parameters of dispersion of the populations from which the samples have been drawn are equal. Percentage points of $D_{n,m}$ are available in Kamat (1956). For some values of m and n we have calculated by simulation.

Wald Test:
A Wald test statistics for testing $H: \theta = \sigma_2^2 - \sigma_1^2 = 0$ against $K: \theta \neq 0$ is as follows

$$W = \frac{(S_1^2 - S_2^2)^2}{2S_1^4/(n_1+1) + 2S_2^4/(n_2+1)}$$

Being a Wald test, the asymptotic distribution of W is χ_1^2, while its exact distribution is not obvious. However, W is a one to one function of F, and so the two test are equivalent. Since the exact distribution of F is known, the F test is the more convenient test.
The variances $var\left(S_j^2\right)$ used in W are estimated optionally using the Rao-Blackwell theorem. This depends very strongly on the assumption of normality. If normality is in doubt then we can estimate $var\ (S_1^2 - S_2^2)$.
A robust alternative to W is thus

$$R = \frac{(S_1^2 - S_2^2)^2}{(m_{14} - S_1^4)/n_1 + (m_{24} - S_2^4)/n_2}$$

In which $m_{i4},\ i = 1, 2$, are the fourth central sample moments for the i[th] sample, $S^4 = nm_2^2/(n-1)$ and S^2 is the unbiased sample variance, . We call the test based on R the R Test. In large samples the denominator in R will approximate $var\ (S_1^2 - S_2^2)$ and R will have asymptotic distribution χ_1^2.

Levene Test:
The statistics of Levene is just the square of the t – statistic applied on transformed x's and y's. Let be transformations $X_i^{'} = |X_i - \bar{X}|$ and $Y_i^{'} = |Y_i - \bar{Y}|$, $i = 1, \dots, m,$ $j = 1, \dots, n,$ of the random variables $X_1, \dots, X_m$ and $Y_1, \dots, Y_n$, respectively. Furthermore, let be $\bar{X'}$ and $\bar{Y'}$ the arithmetic means of the $X_i^{'}$ and $Y_j^{'}$ variables. The Levene statistics is then defined by

$$LV = \frac{\left(\bar{X'} - \bar{Y'}\right)^2}{s'^2\left({}^1/_m + {}^1/_n\right)} \text{ with the pooled variance}$$

$$s'^2 = \frac{\sum_{i=1}^{m}(X_i^{'} - \bar{X'})^2 + \sum_{j=1}^{n}(Y_i^{'} - \bar{Y'})^2}{m+n-2}$$

The statistics LV has, underH_0, approximately a F distribution with $(1, m+n-2)$ degrees of freedom. Thus, the Levene test reject H_0 if $LV \geq F_{1-\alpha}(1, m+n-2$.

F- Test:
We assume independent random sample of size m and n from normal population, $N(\mu_i, \sigma_i^2)$ for i = 1, 2. We wish to test $Ho: \sigma_1^2 = \sigma_2^2$ against the alternative $K: \sigma_1^2 \neq \sigma_2^2$. If $S_i^2, i = 1, 2$ are the unbiased sample variance, then the so called F test is equivalent to the likelihood ratio test and is based on the quotient of the sample variance, $\mathrm{S}_2^2/_{\mathrm{S}_1^2} = \mathrm{F}$.

The null distribution of F namely $\mathrm{F}_{\mathrm{m}-1,\ \mathrm{n}-1}$is sensitive to

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departures from normality. If the cumulative distribution function of this distribution is $F_{m-1,\ n-1}(x)$, and if c_p is such that $F_{m-1,\ n-1}(c_p) = p$, then the F test rejects Ho at the $100_{\alpha\%}$ level when $F \le c_{\alpha/2}$ and when $F \ge c_{1-\alpha/2}$.

3. SIMULATION

To determine the significance level and power of the tests, we generate random sample from the normal, exponential, double exponential, logistic and Cauchy distribution. The null hypothesis of equal variance is studied against the alternatives (introducing scale parameter with different values and equal location parameter in Y population) such as: (1.5,1), (2,1), (2.5,1) and (3,1). Here Table-1(a) to 1(e) gives the estimates of significance level of the considered tests for normal, exponential, double exponential, logistic and Cauchy distribution respectively. Table-2(a) to 2(e) gives the estimates of power of the tests for normal. Exponential, double exponential, logistic and Cauchy distribution respectively.

Table -1(a): Empirical level of tests under Normal Distribution based on 10, 000 replications.

n_1, n_2	Mood	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald W	Wald R	Levene	F
5, 10	0.107 0.054 0.028	0.112 0.042 0.023	0.112 0.063 0.025	0.089 0.037 0.003	0.101 0.052 0.011	0.103 0.042 0.011	0.100 0.050 0.010	0.100 0.050 0.010	0.092 0.043 0.005	0.131 0.066 0.013
10, 10	0.104 0.048 0.022	0.099 0.042 0.022	0.107 0.055 0.029	0.100 0.046 0.005	0.103 0.047 0.009	0.108 0.051 0.020	0.100 0.050 0.010	0.100 0.050 0.010	0.080 0.038 0.005	0.097 0.046 0.009
10, 15	0.104 0.052 0.026	0.097 0.049 0.023	0.101 0.052 0.024	0.099 0.047 0.005	0.100 0.050 0.010	0.179 0.099 0.015	0.100 0.050 0.010	0.100 0.050 0.010	0.073 0.033 0.004	0.113 0.056 0.012
15, 15	0.102 0.052 0.023	0.101 0.050 0.023	0.101 0.047 0.023	0.099 0.049 0.007	0.102 0.051 0.010	0.119 0.063 0.013	0.100 0.050 0.010	0.100 0.050 0.010	0.091 0.045 0.008	0.108 0.055 0.012
15, 20	0.107 0.053 0.026	0.094 0.046 0.023	0.099 0.046 0.025	0.097 0.049 0.007	0.100 0.051 0.010	0.127 0.067 0.012	0.100 0.050 0.010	0.100 0.050 0.010	0.101 0.049 0.009	0.091 0.043 0.006
20, 20	0.104 0.051 0.025	0.102 0.047 0.023	0.100 0.050 0.024	0.097 0.046 0.007	0.104 0.049 0.009	0.123 0.071 0.019	0.100 0.050 0.010	0.100 0.050 0.010	0.084 0.041 0.006	0.090 0.050 0.011

Table-1(b): Empirical Levels of Tests under Exponential Distribution based on 10, 000 replications.

n_1, n_2	Moods	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald W	Wald R	Levene	F
5, 10	0.108 0.055 0.029	0.108 0.039 0.021	0.108 0.060 0.023	0.090 0.034 0.003	0.105 0.053 0.009	0.111 0.033 0.007	0.235 0.145 0.052	0.119 0.067 0.017	0.096 0.044 0.007	0.222 0.155 0.074
10, 10	0.100 0.053 0.023	0.100 0.038 0.013	0.107 0.052 0.024	0.096 0.040 0.003	0.095 0.045 0.009	0.099 0.055 0.029	0.322 0.236 0.111	0.129 0.075 0.022	0.090 0.042 0.006	0.217 0.158 0.072
10, 15	0.104 0.051 0.026	0.095 0.049 0.022	0.098 0.054 0.024	0.101 0.044 0.006	0.100 0.048 0.010	0.196 0.107 0.020	0.323 0.229 0.091	0.125 0.064 0.021	0.098 0.047 0.008	0.216 0.159 0.082
15, 15	0.103 0.051 0.023	0.104 0.054 0.024	0.105 0.048 0.025	0.100 0.048 0.006	0.100 0.049 0.010	0.118 0.055 0.016	0.341 0.259 0.133	0.124 0.067 0.019	0.100 0.047 0.007	0.242 0.184 0.097
15, 20	0.099 0.048 0.025	0.099 0.051 0.027	0.105 0.051 0.028	0.102 0.048 0.006	0.102 0.052 0.009	0.126 0.064 0.012	0.343 0.249 0.113	0.115 0.061 0.014	0.101 0.049 0.007	0.210 0.150 0.069
20, 20	0.100 0.050 0.024	0.104 0.053 0.026	0.102 0.054 0.028	0.100 0.050 0.007	0.107 0.052 0.010	0.111 0.063 0.013	0.347 0.262 0.132	0.110 0.057 0.015	0.100 0.047 0.008	0.216 0.169 0.089

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Table-1(c): Empirical Levels of Tests under Double Exponential Distribution based on 10, 000 replications.

n_1, n_2	Moods	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald W	Wald R	Levene	F
5, 10	0.108 0.055 0.029	0.108 0.039 0.021	0.108 0.060 0.023	0.090 0.034 0.004	0.105 0.053 0.009	0.111 0.033 0.007	0.176 0.104 0.029	0.071 0.031 0.006	0.126 0.062 0.016	0.193 0.126 0.052
10, 10	0.101 0.053 0.023	0.100 0.038 0.018	0.107 0.052 0.024	0.096 0.041 0.003	0.095 0.045 0.009	0.099 0.055 0.029	0.245 0.163 0.066	0.086 0.041 0.007	0.135 0.070 0.017	0.178 0.121 0.043
10, 15	0.104 0.051 0.026	0.095 0.049 0.022	0.098 0.054 0.024	0.102 0.044 0.007	0.100 0.048 0.010	0.196 0.107 0.020	0.243 0.157 0.052	0.086 0.035 0.005	0.139 0.078 0.015	0.188 0.127 0.054
15, 15	0.103 0.051 0.023	0.104 0.054 0.024	0.105 0.048 0.025	0.101 0.048 0.006	0.100 0.049 0.010	0.118 0.055 0.016	0.262 0.177 0.074	0.093 0.041 0.007	0.118 0.058 0.012	0.194 0.134 0.057
15, 20	0.099 0.048 0.025	0.099 0.051 0.027	0.105 0.051 0.028	0.102 0.048 0.006	0.102 0.052 0.009	0.126 0.064 0.012	0.257 0.168 0.060	0.089 0.043 0.006	0.123 0.062 0.013	0.172 0.110 0.041
20, 20	0.100 0.050 0.024	0.104 0.053 0.026	0.102 0.054 0.028	0.101 0.050 0.007	0.107 0.052 0.010	0.122 0.067 0.017	0.267 0.182 0.074	0.091 0.044 0.007	0.144 0.079 0.019	0.177 0.129 0.058

Table-1(d): Empirical Levels of Tests under Logistic Distribution based on 10, 000 replications.

n_1, n_2	Moods	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald W	Wald R	Levene	F
5, 10	0.108 0.053 0.029	0.113 0.038 0.021	0.108 0.060 0.023	0.090 0.035 0.004	0.105 0.053 0.009	0.111 0.033 0.007	0.122 0.068 0.014	0.082 0.037 0.006	0.093 0.041 0.008	0.164 0.091 0.029
10, 10	0.101 0.053 0.023	0.102 0.039 0.019	0.107 0.052 0.024	0.096 0.041 0.003	0.095 0.045 0.009	0.099 0.055 0.029	0.163 0.094 0.026	0.093 0.046 0.008	0.095 0.043 0.007	0.134 0.079 0.021
10, 15	0.104 0.051 0.026	0.095 0.050 0.023	0.098 0.054 0.024	0.102 0.044 0.007	0.100 0.048 0.010	0.196 0.107 0.020	0.157 0.089 0.023	0.091 0.041 0.006	0.101 0.047 0.009	0.148 0.087 0.026
15, 15	0.103 0.051 0.023	0.100 0.055 0.025	0.105 0.048 0.025	0.101 0.048 0.006	0.100 0.049 0.010	0.118 0.055 0.016	0.167 0.103 0.031	0.097 0.048 0.010	0.102 0.047 0.008	0.147 0.090 0.028
15, 20	0.099 0.048 0.025	0.093 0.048 0.024	0.105 0.051 0.028	0.102 0.048 0.006	0.102 0.052 0.009	0.126 0.064 0.012	0.166 0.093 0.025	0.092 0.047 0.008	0.096 0.047 0.008	0.126 0.071 0.019
20, 20	0.101 0.050 0.024	0.103 0.050 0.024	0.102 0.054 0.028	0.101 0.050 0.007	0.107 0.052 0.010	0.122 0.067 0.0174	0.173 0.103 0.029	0.094 0.049 0.009	0.101 0.051 0.009	0.130 0.084 0.026

Table-1(e): Empirical Levels of Tests under Cauchy Distribution based on 10, 000 replications.

n_1, n_2	Moods	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald W	Wald R	Levene	F
5, 10	0.110 0.056 0.028	0.113 0.038 0.021	0.113 0.063 0.022	0.087 0.035 0.003	0.096 0.051 0.011	0.110 0.035 0.007	0.549 0.362 0.251	0.033 0.017 0.004	0.069 0.031 0.005	0.283 0.248 0.193
10, 10	0.101 0.050 0.023	0.102 0.039 0.019	0.109 0.052 0.027	0.091 0.038 0.005	0.092 0.040 0.008	0.107 0.043 0.020	0.677 0.622 0.504	0.046 0.016 0.002	0.053 0.018 0.002	0.378 0.345 0.280
10, 15	0.105 0.050	0.095 0.050	0.099 0.054	0.097 0.045	0.099 0.048	0.178 0.092	0.695 0.629	0.039 0.011	0.005 0.022	0.338 0.309

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	0.026	0.023	0.024	0.006	0.011	0.017	0.480	0.001	0.002	0.260
15, 15	0.103 0.053 0.025	0.100 0.055 0.025	0.103 0.051 0.025	0.102 0.048 0.007	0.099 0.049 0.010	0.120 0.071 0.010	0.729 0.681 0.584	0.036 0.012 0.001	0.057 0.015 0.001	0.407 0.379 0.324
15, 20	0.103 0.050 0.023	0.093 0.048 0.024	0.097 0.048 0.025	0.093 0.043 0.007	0.095 0.045 0.007	0.126 0.064 0.011	0.749 0.697 0.587	0.035 0.011 0.001	0.058 0.022 0.002	0.357 0.329 0.279
20, 20	0.104 0.053 0.026	0.103 0.050 0.024	0.099 0.052 0.025	0.099 0.048 0.007	0.105 0.049 0.008	0.143 0.085 0.022	0.771 0.727 0.642	0.035 0.010 0.000	0.061 0.019 0.001	0.397 0.378 0.334

Table-2(a): Empirical Power of Tests under Normal Distribution

n_1, n_2	σ_1, σ_2	Mood	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald –W	Wald - R	Levene	F
(5, 10)	(1.5, 1)	0.201	0.140	0.189	0.089	0.075	0.171	0.179	0.105	0.045	0.297
	(2, 1)	0.356	0.248	0.326	0.176	0.105	0.322	0.365	0.178	0.112	0.544
	(2.5, 1)	0.487	0.357	0.447	0.272	0.130	0.458	0.557	0.234	0.209	0.720
	(3, 1)	0.584	0.448	0.543	0.357	0.152	0.566	0.715	0.274	0.317	0.828
(10, 10)	(1.5, 1)	0.245	0.188	0.228	0.095	0.056	0.236	0.215	0.157	0.122	0.304
	(2, 1)	0.476	0.385	0.431	0.209	0.074	0.476	0.521	0.337	0.309	0.628
	(2.5, 1)	0.650	0.546	0.591	0.348	0.096	0.675	0.755	0.490	0.500	0.837
	(3, 1)	0.758	0.658	0.701	0.475	0.118	0.801	0.887	0.596	0.648	0.933
(10, 15)	(1.5, 1)	0.296	0.248	0.260	0.134	0.071	0.422	0.314	0.150	0.166	.401
	(2, 1)	0.561	0.486	0.502	0.299	0.105	0.699	0.665	0.347	0.432	0.745
	(2.5, 1)	0.747	0.672	0.688	0.468	0.141	0.859	0.876	0.509	0.667	0.906
	(3, 1)	0.846	0.794	0.803	0.602	0.178	0.929	0.958	0.614	0.808	0.968
(15, 15)	(1.5, 1)	0.340	0.308	0.293	0.144	0.066	0.378	0.309	0.258	0.158	0.452
	(2, 1)	0.653	0.596	0.576	0.357	0.104	0.703	0.717	0.565	0.467	0.823
	(2.5, 1)	0.840	0.775	0.763	0.567	0.162	0.887	0.914	0.762	0.727	0.957
	(3, 1)	0.923	0.878	0.866	0.714	0.222	0.959	0.977	0.858	0.871	0.992
(15, 20)	(1.5, 1)	0.392	0.319	0.319	0.178	0.078	0.427	0.387	0.262	0.213	0.457
	(2, 1)	0.723	0.646	0.645	0.439	0.137	0.763	0.800	0.584	0.580	0.843
	(2.5, 1)	0.884	0.829	0.829	0.653	0.218	0.916	0.957	0.774	0.826	0.963
	(3, 1)	0.952	0.920	0.919	0.791	0.306	0.970	0.992	0.862	0.933	0.992
(20, 20)	(1.5, 1)	0.421	0.372	0.377	0.191	0.076	0.478	0.397	0.341	0.308	0.526
	(2, 1)	0.781	0.709	0.712	0.483	0.139	0.825	0.836	0.729	0.719	0.900
	(2.5, 1)	0.928	0.877	0.880	0.720	0.245	0.957	0.971	0.900	0.917	0.986
	(3, 1)	0.976	0.946	0.947	0.857	0.371	0.988	0.995	0.954	0.979	0.998

Table-2(b): Empirical power of Tests under Exponential Distribution

n_1, n_2	σ_1, σ_2	Mood	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald –R	Levene
(5, 10)	(1.5, 1)	0.083	0.053	0.080	0.050	0.093	0.061	0.100	0.047
	(2, 1)	0.104	0.054	0.082	0.084	0.166	0.079	0.137	0.080
	(2.5, 1)	0.120	0.051	0.075	0.128	0.250	0.088	0.178	0.126
	(3, 1)	0.129	0.042	0.065	0.172	0.335	0.091	0.208	0.173
(10, 10)	(1.5, 1)	.080	0.064	0.081	0.081	0.101	0.090	0.122	0.110
	(2, 1)	0.094	0.079	0.098	0.173	0.222	0.132	0.199	0.244
	(2.5, 1)	0.096	0.081	0.102	0.283	0.350	0.172	0.270	0.385
	(3, 1)	0.096	0.084	0.105	0.391	0.468	0.206	0.335	0.509
(10, 15)	(1.5, 1)	0.093	0.074	0.081	0.093	0.123	0.163	0.106	0.105
	(2, 1)	0.127	0.087	0.095	0.202	0.268	0.217	0.188	0.246
	(2.5, 1)	0.150	0.086	0.094	0.330	0.432	0.256	0.267	0.401
	(3, 1)	0.166	0.079	0.087	0.462	0.573	0.283	0.333	0.544
(15, 15)	(1.5, 1)	0.091	0.089	0.083	0.115	0.143	0.130	0.142	0.159
	(2, 1)	0.111	0.123	0.112	0.272	0.326	0.216	0.270	0.377
	(2.5, 1)	0.120	0.138	0.128	0.453	0.523	0.292	0.379	0.592

	(3, 1)	0.117	0.142	0.133	0.611	0.671	0.355	0.461	0.740
(15, 20)	(1.5, 1)	0.101	0.094	0.094	0.128	0.160	0.137	0.125	0.160
	(2, 1)	0.142	0.115	0.114	0.308	0.377	0.217	0.251	0.408
	(2.5, 1)	0.174	0.119	0.119	0.511	0.591	0.286	0.369	0.627
	(3, 1)	0.198	0.116	0.116	0.680	0.750	0.344	0.458	0.780
(20, 20)	(1.5, 1)	0.101	0.098	0.099	0.152	0.188	0.158	0.149	0.220
	(2, 1)	0.135	0.134	0.137	0.376	0.444	0.280	0.313	0.523
	(2.5, 1)	0.154	0.153	0.157	0.605	0.667	0.387	0.451	0.756
	(3, 1)	0.155	0.166	0.169	0.773	0.816	0.478	0.548	0.882

Table-2c: Empirical power of Tests under Double Exponential Distribution

n_1, n_2	σ_1, σ_2	Mood	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald –R	Levene
(5, 10)	(1.5, 1)	0.151	0.105	0.148	0.072	0.048	0.131	0.065	0.147
	(2, 1)	0.261	0.184	0.248	0.138	0.057	0.238	0.108	0.249
	(2.5, 1)	0.367	0.262	0.339	0.224	0.070	0.326	0.148	0.346
	(3, 1)	0.458	0.331	0.421	0.317	0.084	0.411	0.178	0.428
(10, 10)	(1.5, 1)	0.178	0.141	0.171	0.074	0.069	0.168	0.084	0.145
	(2, 1)	0.338	0.277	0.321	0.124	0.089	0.315	0.171	0.272
	(2.5, 1)	0.487	0.405	0.455	0.187	0.110	0.456	0.255	0.396
	(3, 1)	0.609	0.519	0.565	0.249	0.126	0.577	0.321	0.505
(10, 15)	(1.5, 1)	0.213	0.186	0.196	0.101	0.061	0.303	0.071	0.189
	(2, 1)	0.411	0.361	0.373	0.199	0.083	0.513	0.162	0.371
	(2.5, 1)	0.583	0.512	0.531	0.323	0.109	0.676	0.253	0.539
	(3, 1)	0.705	0.636	0.651	0.439	0.137	0.788	0.329	0.667
(15, 15)	(1.5, 1)	0.239	0.234	0.218	0.107	0.060	0.239	0.131	0.178
	(2, 1)	0.472	0.451	0.433	0.235	0.082	0.449	0.291	0.389
	(2.5, 1)	0.663	0.625	0.611	0.382	0.110	0.648	0.427	0.568
	(3, 1)	0.792	0.749	0.734	0.517	0.144	0.788	0.534	0.707
(15, 20)	(1.5, 1)	0.265	0.248	0.248	0.123	0.068	0.274	0.111	0.219
	(2, 1)	0.527	0.492	0.492	0.280	0.100	0.515	0.266	0.466
	(2.5, 1)	0.730	0.687	0.687	0.459	0.142	0.700	0.419	0.664
	(3, 1)	0.849	0.805	0.805	0.607	0.192	0.824	0.534	0.798
(20, 20)	(1.5, 1)	0.289	0.270	0.273	0.132	0.065	0.304	0.151	0.236
	(2, 1)	0.589	0.548	0.553	0.318	0.101	0.578	0.373	0.508
	(2.5, 1)	0.795	0.742	0.746	0.518	0.151	0.768	0.561	0.716
	(3, 1)	0.901	0.858	0.862	0.682	0.214	0.877	0.680	0.842

Table-2d: Empirical power of Tests under Cauchy distribution

n_1, n_2	σ_1, σ_2	Mood	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald –R	Levene
(5, 10)	(1.5, 1)	0.117	0.093	0.133	0.059	0.063	0.085	0.028	0.063
	(2, 1)	0.183	0.149	0.201	0.089	0.075	0.137	0.038	0.103
	(2.5, 1)	0.250	0.205	0.275	0.131	0.090	0.180	0.047	0.141
	(3, 1)	0.311	0.255	0.331	0.167	0.102	0.219	0.057	0.181
(10, 10)	(1.5, 1)	0.135	0.115	0.141	0.057	0.042	0.110	0.023	0.033
	(2, 1)	0.229	0.206	0.242	0.103	0.051	0.191	0.037	0.059
	(2.5, 1)	0.323	0.292	0.336	0.157	0.060	0.268	0.049	0.092
	(3, 1)	0.412	0.378	0.424	0.213	0.068	0.336	0.061	0.121
(10, 15)	(1.5, 1)	0.155	0.145	0.155	0.076	0.057	0.178	0.016	0.054
	(2, 1)	0.276	0.267	0.279	0.139	0.072	0.265	0.026	0.101
	(2.5, 1)	0.395	0.384	0.395	0.216	0.087	0.344	0.037	0.151
	(3, 1)	0.496	0.481	0.494	0.298	0.103	0.414	0.049	0.205
(15, 15)	(1.5, 1)	0.177	0.176	0.166	0.084	0.057	0.145	0.019	0.035
	(2, 1)	0.319	0.328	0.312	0.169	0.069	0.249	0.035	0.076
	(2.5, 1)	0.459	0.466	0.448	0.261	0.087	0.326	0.050	0.120
	(3, 1)	0.570	0.586	0.568	0.361	0.108	0.405	0.064	0.161
(15,	(1.5, 1)	0.190	0.181	0.181	0.096	0.061	0.145	0.017	0.056

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20)	(2, 1) (2.5, 1) (3, 1)	0.360 .516 0.641	0.356 0.515 0.634	0.356 0.513 0.633	0.196 0.313 0.428	0.081 0.107 0.139	0.240 0.323 0.404	0.028 0.039 0.054	0.108 0.166 0.228
(20, 20)	(1.5, 1) (2, 1) (2.5, 1) (3, 1)	0.206 0.399 0.561 0.692	0.208 0.405 0.573 0.702	0.212 0.412 0.579 0.708	0.105 0.216 0.356 0.485	0.060 0.082 0.116 0.153	0.194 0.294 0.390 0.472	0.020 0.034 0.051 0.063	0.044 0.097 0.147 0.200

Table-2e Empirical power of Tests under Logistic Distribution

n_1, n_2	σ_1, σ_2	Mood	Ansari Bradley	Siegel Tukey	Lepage	CVM	Kamat	Wald –W	Wald - R	Levene
(5, 10)	(1.5, 1) (2, 1) (2.5, 1) (3, 1)	0.180 0.325 0.451 0.553	0.121 0.223 0.326 0.416	0.171 0.299 0.412 0.507	0.084 0.156 0.243 0.325	0.075 0.103 0.124 0.144	0.158 0.296 0.410 0.516	0.203 0.387 0.560 0.698	0.084 0.145 0.197 0.235	0.146 0.313 0.463 0.585
(10, 10)	(1.5, 1) (2, 1) (2.5, 1) (3, 1)	0.219 0.425 0.605 0.726	0.173 0.346 0.506 0.627	0.207 0.395 0.556 0.671	0.085 0.181 0.308 0.423	0.049 0.064 0.083 0.101	0.209 0.899 0.584 0.718	0.247 0.515 0.723 0.851	0.116 0.256 0.383 0.475	0.145 0.334 0.529 0.677
(10, 15)	(1.5, 1) (2, 1) (2.5, 1) (3, 1)	0.262 0.512 0.701 0.815	0.224 0.447 0.625 0.747	0.238 0.461 0.639 0.761	0.123 0.264 0.425 0.557	0.066 0.098 0.135 0.169	0.367 0.624 0.798 0.892	0.346 0.646 0.838 0.933	0.109 0.253 0.391 0.489	0.212 0.476 0.689 0.827
(15, 15)	(1.5, 1) (2, 1) (2.5, 1) (3, 1)	0.297 0.585 0.787 0.891	0.283 0.553 0.736 0.846	0.276 0.535 0.722 0.836	0.132 0.313 0.507 0.663	0.062 0.096 0.142 0.196	0.300 0.588 0.789 0.897	0.344 0.682 0.877 0.955	0.191 0.433 0.621 0.731	0.217 0.517 0.750 0.882
(15, 20)	(1.5, 1) (2, 1) (2.5, 1) (3, 1)	0.330 0.651 0.844 0.927	0.310 0.610 0.796 0.894	0.309 0.611 0.795 0.893	0.154 0.380 0.590 0.746	0.074 0.122 0.186 0.264	0.342 0.639 0.828 0.919	0.421 0.772 0.929 0.979	0.173 0.424 0.620 0.737	0.270 0.617 0.842 0.938
(20, 20)	(1.5, 1) (2, 1) (2.5, 1) (3, 1)	0.367 0.725 0.897 0.962	0.335 0.663 0.849 0.931	0.341 0.668 0.851 0.933	0.168 0.428 0.667 0.817	0.070 0.127 0.209 0.317	0.374 0.718 0.890 0.961	0.423 0.798 0.943 0.986	0.237 0.561 0.766 0.860	0.280 0.665 0.882 0.962

Table-3 Simulated Critical Values for Wald – W Test and Wald-R Test

Sample Size		Wald Test (W)			Wald Test (R)		
5	10	3.1892	4.0945	5.2790	8.1734	12.5656	32.9237
10	10	2.5581	3.1708	4.2124	4.7939	6.5617	11.8474
10	15	2.7347	3.5266	5.1450	4.4941	6.4015	11.1498
15	15	2.6283	3.3485	4.7258	3.8587	5.2409	8.6041
15	20	2.6942	3.6037	5.3406	3.7735	5.2283	9.0331
20	20	2.6555	3.5032	5.1984	3.5842	4.9308	8.2530
20	25	2.6874	3.5553	5.4277	3.4807	4.8885	8.0067
25	25	2.6784	3.6216	5.3316	3.3674	4.6840	7.8428
30	30	2.6317	3.6175	5.4279	3.1931	4.4862	7.5974

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Fig : 1

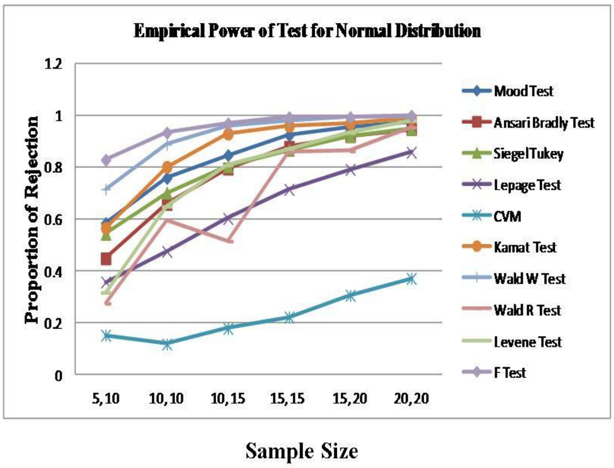


Fig : 2

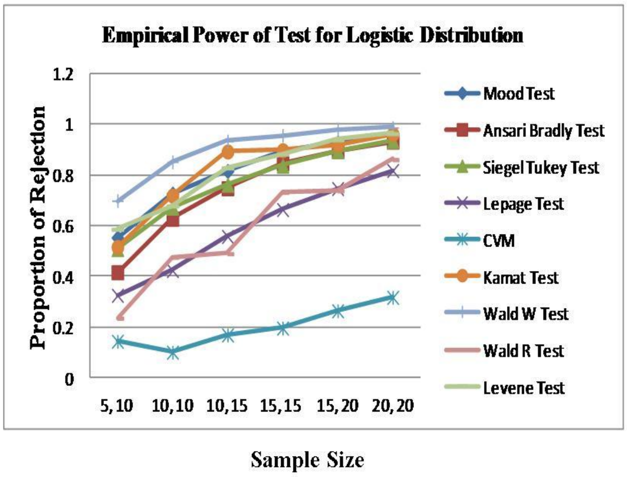


Fig : 3

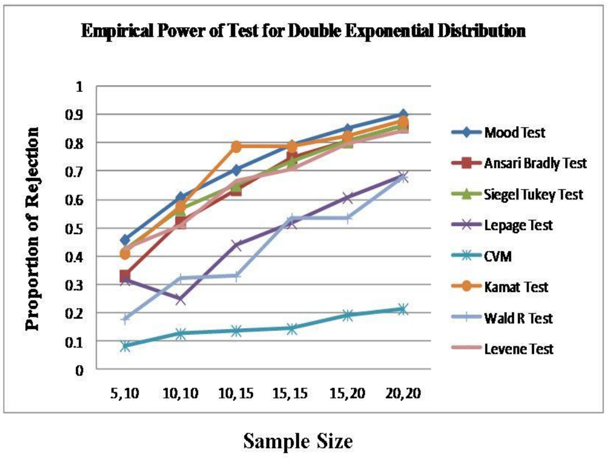
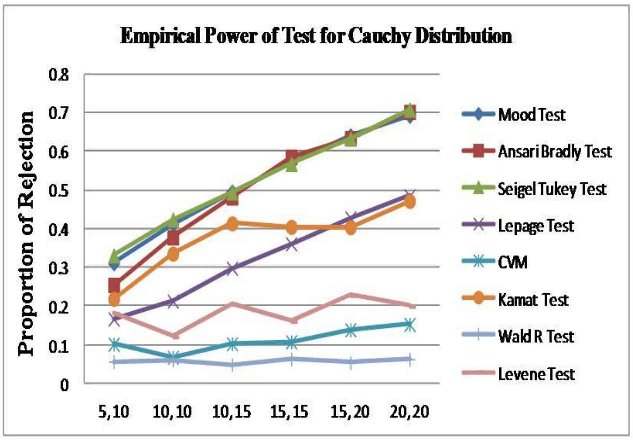


Fig : 4



4. DISCUSSION

Tables and graphs above show the empirical level and power of test statistics under different distributions, various combinations of scale parameters and sample sizes.

As we observed in normal distribution , F-test gives the highest power and simultaneously the performance of Wald-W, Kamat, Mood, Ansari-Bradley, Siegel-Tukey, Lepage and Levene test are also quite good although slightly less than the F-test. Under this distribution CVM test shows the least power of all.

In exponential distribution, CVM test gives the more power against the small sample sizes and Levene test gives the highest power for the large sample sizes. Empirical power of Lepage also lies neck to neck with these two tests. Power of Kamat and Wald are less than these three tests but higher than Siegel-Tukey, Ansari-Bradley and Mood test. The last three tests give the almost similar power but less other tests.

Under double exponential distribution, except CVM and Wald tests, the performance of Mood, Ansari-Bradley, Siegel-Tukey, Kamat and Levene tests are more or less similar in power, so we may choose any one of them.

In Cauchy distribution, Empirical power of Ansari-Bradley, Mood and Siegel-Tukey are found to be more than other tests. CVM and Wald-R test shows the lowest power of all. Power of Kamat, Lepage and Levene test are lies between these two groups. We may say that under Cauchy distribution the performance of Mood, Ansari-Bradley and Siegel-Tukey are the best.

Under logistic distribution, Wald-W gives the highest power. Mood, Ansari-Bradley, Seigel-Tukey and Levene tests exhibits the similar power but slightly less that Wald W test. CVM test shows the smallest power in this distribution. Power of Wald-R and Lepage tests are found to be less but higher than the CVM test.

5. CONCLUSION

From the above discussion, it may be conclude that for testing equality of variance in two samples F-test being the more powerful in normal distribution. CVM and Lepage tests are more powerful in exponential distribution. Except F, CVM and Wald tests and any other tests discuss above may be used in double exponential distribution. Mood, Ansari-Bradley and Siegel-Tukey are found to be powerful in Cauchy distribution. Finally, Wald-W test found to be more powerful in logistic distribution. Accordingly one may choose the tests statistics for testing equality of variance of two populations.

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