

On Intuitionistic Fuzzy Slightly π gb-Continuous Functions

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Abstract: In this paper we introduce and study the concepts of intuitionistic fuzzy slightly π gb-continuous functions, we investigate some of their properties. By using intuitionistic fuzzy slightly π gb-continuous-function some properties of separation axioms are discussed.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy π gb-closed set, intuitionistic fuzzy clopen set, intuitionistic fuzzy slightly π gb-continuous, 2010 AMS Classification: 54A40, 03E72

I. INTRODUCTION

The concept of fuzzy set was introduced by L. A. Zadah [13]. The fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological space was introduced and developed by C. L. Chang [3]. Atanasov [2] was introduced The concept of intuitionistic fuzzy set, as a generalization of fuzzy set. This approach provided a wide field to the generalization of various concepts of fuzzy mathematics.

In 1997 Coker[6] defined intuitionistic fuzzy topological spaces. Recently many concepts of fuzzy topological space have been extended in intuitionistic fuzzy (IF) topological spaces. Ravi, Margaret parimalam, Murugesan and Pandi [9] introduced the concept of slightly π gb-continuous functions. In this paper we introduce and study the concepts of intuitionistic fuzzy slightly π gb-continuous in intuitionistic fuzzy topological space. Also we apply that concept to study some properties in separation axioms.

II. PRELIMINARIES

Definition 2.1 [2] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$

, where the function $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A + \nu_A \leq 1$ for each $x \in X$. Denote by $IFS(X)$, the set of all intuitionistic fuzzy sets in X .

Definition 2.2 [2] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ and

$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$ Then :

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$
- (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$
- (e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$
- (f) $0_{\sim} = \{ \langle x, 0, 1 \rangle : x \in X \}$ and $1_{\sim} = \{ \langle x, 1, 0 \rangle : x \in X \}$
- (g) $0_{\sim}^c = 1_{\sim}$ and $1_{\sim}^c = 0_{\sim}$

Definition 2.3 [4] Let $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy

point (IFP) $p_{(\alpha, \beta)}$ is intuitionistic fuzzy set

$$p_{(\alpha, \beta)} = \begin{cases} (\alpha, \beta) & \text{if } x = p \\ (0, 1) & \text{if otherwise} \end{cases}$$

Clearly an intuitionistic fuzzy point can be represented by an ordered pair offuzzy point as follows:

$$p_{(\alpha, \beta)} = (p_{\alpha}, p_{(1-\beta)})$$

In this case, p is called the support of $p_{(\alpha, \beta)}$ and α, β are called the value and no value of $p_{(\alpha, \beta)}$ respectively.

An IFP $p_{(\alpha, \beta)}$ is said to belong to an IFS $A = \{ \langle x, \mu_A, \nu_A \rangle : x \in X \}$ denoted by $p_{(\alpha, \beta)} \in A$, if $\alpha \leq \mu_A(x)$ and $\beta \geq \nu_A(x)$.

Definition 2.4 [2] Let X and Y be two nonempty sets and $f : X \rightarrow Y$ be a function. Then

(a) If $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle : y \in Y \}$ is an IFS in Y , then the preimage of B under f denoted by

$f^{-1}(B)$ is the IFS in X defined by

$$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) \rangle : x \in X \} \quad (b)$$

If $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ is an IFS in X , then the image of A under f denoted by $f(A)$ is the IFS in Y defined by

$$f(A) = \{ \langle y, f(\mu_A)(y), 1 - f(1 - \nu_A)(y) \rangle : y \in Y \}$$

Where,

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if otherwise} \end{cases}$$

$$1 - f(1 - \nu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if otherwise} \end{cases}$$

Replacing fuzzy sets [13] by intuitionistic fuzzy sets in Chang definition of fuzzy topological space [3] we get the following.

Definition 2.5 [5] An intuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms

(T₁) $0_{\sim}, 1_{\sim} \in \tau$

(T₂) If $G_1, G_2 \in \tau$, then $G_1 \cap G_2 \in \tau$

(T₃) If $G_i \in \tau$ for each i in I , then $\bigcup_{i \in I} G_i \in \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFT in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . the

complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.6 [5] Let $p(\alpha, \beta)$ be an IFP in IFTS X . An IFS A in X is called an IF neighborhood (IFN) of $p(\alpha, \beta)$ if there exists an IFOS B in X such that $p(\alpha, \beta) \in B \subseteq A$.

Definition 2.7 [6] A subset of an intuitionistic fuzzy space X is said to be clopen if it is intuitionistic fuzzy open set and intuitionistic fuzzy closed set.

Definition 2.8 [3] Let (X, τ) be an IFTS and $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ be an IFS in X . Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \bigcup \{ G : G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$$

$$\text{cl}(A) = \bigcap \{ K : K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$$

Definition 2.9 An IFS A of an IFTS (X, τ) is an :

- (1) Intuitionistic fuzzy regular open set (IFROS in short) [6] if $\text{int}(\text{cl}(A)) = A$.
- (2) Intuitionistic fuzzy regular closed set (IFRCS in short) [6] if $\text{cl}(\text{int}(A)) = A$.
- (3) intuitionistic fuzzy π -open set (IF π OS in short)[11] if the finite union of intuitionistic fuzzy regular open sets.
- (4) intuitionistic fuzzy π -closed set (IF π CS in short)[11] if the finite intersection of intuitionistic fuzzy regular closed sets.
- (5) intuitionistic fuzzy generalized open set (IFGOS in short)[12] if $F \subseteq \text{int}(A)$ whenever $F \subseteq A$ and F is an IFCS in X .
- (6) intuitionistic fuzzy generalized closed set (IFGCS in short) [12] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .
- (7) intuitionistic fuzzy b-open set (IFbOS in short)[7] if $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$.
- (8) intuitionistic fuzzy b-closed set (IFbCS in short)[7] if $\text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A)) \subseteq A$.

Definition 2.10 [8] Let (X, τ) be an IFTS and A be an IFS in X . Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by :

$$\text{bint}(A) = \bigcup \{ G : G \text{ is an IFbOS in } X \text{ and } G \subseteq A \}$$

$$\text{bcl}(A) = \bigcap \{ K : K \text{ is an IFbCS in } X \text{ and } A \subseteq K \}$$

Definition 2.11 An IFS A of an IFTS (X, τ) is an :

- (1) intuitionistic fuzzy gb-open set (IFGbOS in short)[8] if $F \subseteq \text{bint}(A)$ whenever $F \subseteq A$ and F is an IFCS in X .
- (2) intuitionistic fuzzy gb-closed set (IFGbCS in short)[8] if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .
- (3) intuitionistic fuzzy π g-open set (IF π gOS in short) if $F \subseteq \text{int}(A)$ whenever $F \subseteq A$ and F is an IF π CS in X .
- (4) intuitionistic fuzzy π g-closed set (IF π gCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF π OS in X .

Definition 2.12 An IFS A of an IFTS (X, τ) is an

- (1) intuitionistic fuzzy π gb-open set (IF π gbOS in short) if $F \subseteq \text{bint}(A)$ whenever $F \subseteq A$ and F is an IF π CS in X .
- (2) intuitionistic fuzzy π gb-closed set (IF π gbCS in short) if $\text{bcl}(A) \subseteq U$, whenever $A \subseteq U$ and U is an IF π OS in X .

Definition 2.13 Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an :

- (1) intuitionistic fuzzy π -irresolute if $f^{-1}(F)$ is intuitionistic fuzzy π -closed in X for every intuitionistic fuzzy π -closed set F of Y .
- (2) intuitionistic fuzzy π gb-continuous if $f^{-1}(F)$ is π gb-closed in X for every intuitionistic fuzzy closed set F of Y .
- (3) intuitionistic fuzzy π gb-continuous if the inverse image of every intuitionistic fuzzy clopen set in Y is an intuitionistic fuzzy π gb-open (resp. IF π gb-clopen) set in X .
- (4) intuitionistic fuzzy totally continuous if inverse image of every intuitionistic fuzzy open set in Y is an intuitionistic fuzzy clopen set in X .

III. MAIN RESULTS

This section is devoted to introduce and investigate of intuitionistic fuzzy slightly π gb-continuous.

Definition 3.1 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy slightly π -generalized b-continuous (briefly IF slightly π gb-continuous) if the inverse image of every IF clopen set in Y is IF π gb-open in X .

Definition 3.2 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ from IFTS (X, τ) to another IFTS (Y, σ) is said to be an IF slightly π gb-continuous if for each IFP $p(\alpha, \beta) \in X$ and each IF clopen set B in Y containing $f(p(\alpha, \beta))$, there exists an IF π gb-open set A in X such that $f(A) \subseteq B$.

Theorem 3.3 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function from an IFTS (X, τ) to another IFTS (Y, σ) then the following statements are equivalent

1. f is an IF slightly π gb-continuous.
2. Inverse image of every IF clopen set in Y is an IF π gb-open in X .
3. Inverse image of every IF clopen set in Y is an IF π gb-closed in X .
4. Inverse image of every IF clopen set in Y is an IF π gb-clopen in X .

Proof. (1) \Rightarrow (2) Let B be an IF clopen set in Y and let $(p(\alpha, \beta)) \in f^{-1}(B)$. Since $f(p(\alpha, \beta)) \in B$ by (1) there exists an IF π gb-open set A in X containing $p(\alpha, \beta)$ such that $A_{p(\alpha, \beta)} \subseteq f^{-1}(B)$ we obtain that

$f^{-1}(B) = \bigcup_{p(\alpha, \beta) \in f^{-1}(B)} A_{p(\alpha, \beta)}$, which is an IF π gb-open in X .

(2) \Rightarrow (3) Let B be an IF clopen set in Y , then B^c is IF clopen. By (2) $f^{-1}(B^c) = (f^{-1}(B))^c$ is an IF π gb-open, thus $f^{-1}(B)$ is an IF π gb-closed set.

(3) \Rightarrow (4) Let B be an IF clopen set in Y . Then by (3) $f^{-1}(B)$ is IF π gb-closed set. Also B^c is an IF clopen and

(3) implies $f^{-1}(B^c) = (f^{-1}(B))^c$ is an IF π gb-closed set. Hence $f^{-1}(B)$ is an IF π gb-clopen set.

(4) \Rightarrow (1) Let B be an IF clopen set in Y containing $f(p(\alpha, \beta))$. By (4), $f^{-1}(B)$ is an IF π gb-open. Let us take $A = f^{-1}(B)$, then $f(A) \subseteq B$. Hence f is an IF slightly π gb-continuous. ■

Definition 3.4 The intersection of all IF π gb-closed sets containing an IF set A is called an IF π gb-closure of A and denoted by π gbcl(A), and the union of all IF π gb-open sets contained in an IF set A is called an IF π gb-interior of A and denoted by π gbint(A).

Remark 3.5 If $A = \pi$ gbcl(A), then A need not be an IF π gb-closed.

Remark 3.6 The union of two IF π gb-closed sets is generally not an IF π gb-closed set and the intersection of two IF π gb-open sets is generally not an IF π gb-open set

Example 3.7 Let $X = \{a, b, c\}$ and let $\tau = \{0, 1, \sim, A, B, C\}$ is IF Ton X , where

$A = \{\langle a, 0, 1 \rangle, \langle b, 1, 0 \rangle, \langle c, 0, 1 \rangle\}$, $B = \{\langle a, 0, 1 \rangle, \langle b, 0, 1 \rangle, \langle c, 1, 0 \rangle\}$ and $C = \{\langle a, 0, 1 \rangle, \langle b, 1, 0 \rangle, \langle c, 1, 0 \rangle\}$. Then the IFSSs A^c, B^c are IF π gbOSs but $A^c \cap B^c = C^c$ is not an IF π gbOS of X , since $C^c \subseteq C^c$ and $C^c \notin \text{bint}(C^c) = 0$. And the IFSSs A, B are IF π gbCSs but $A \cup B = C$ is not an IF π gbCS of X , since $C \subseteq C$ and $\text{bcl}(C) = 1 \sim \notin C$.

Proposition 3.8 Every intuitionistic fuzzy π gb-continuous is an intuitionistic fuzzy slightly π gb-continuous. But the converse need not be true.

Example 3.9 Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$A = \{\langle a, 1, 0 \rangle, \langle b, 0, 1 \rangle\}$,
 $B = \{\langle a, 0, 1 \rangle, \langle b, 0.8, 0.2 \rangle\}$,
 $C = \{\langle a, 1, 0 \rangle, \langle b, 0.8, 0.2 \rangle\}$,
 $D = \{\langle u, 0.7, 0.3 \rangle, \langle v, 0.5, 0.5 \rangle\}$.

Then $\tau = \{0, 1, \sim, A, B, C\}$ and $\sigma = \{0, 1, \sim, D\}$ are IFTS on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF slightly π gb-continuous but not an IF π gb-continuous.

Since $f^{-1}(D^c) = \{\langle a, 0.3, 0.7 \rangle, \langle b, 0.5, 0.5 \rangle\} \subseteq C$ (π -open set) and $\text{bcl}(f^{-1}(D^c)) = 1 \sim \notin C$

Proposition 3.10 Every intuitionistic fuzzy π gb-irresolute function is an intuitionistic fuzzy slightly π gb-continuous. But the converse need not be true.

Theorem 3.11 If $f: X \rightarrow Y$ is an IF slightly π gb-continuous and $g: Y \rightarrow Z$ is an IF totally continuous then $g \circ f$ is an intuitionistic fuzzy π gb-continuous.

Proof. Let B be an IFOS in Z , since g is an IF totally continuous, $g^{-1}(B)$ is an IF clopen set in Y . Now $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$. Since f is an IF slightly π gb-continuous, $f^{-1}(g^{-1}(B))$ is an IF π gbOS in X . Hence $g \circ f$ is an intuitionistic fuzzy π gb-continuous. ■

Lemma 3.12 Let $f: X \rightarrow Y$ be bijective, IF π -irresolute and IF strongly b-closed. Then for every IF π gb-closed set A of X , $f(A)$ is an IF π gb-closed in Y .

Proof. Let A be any IF π gb-closed set of X and V an IF π gb-open set of Y containing $f(A)$. Since $f^{-1}(V)$ is an IF π -open set of X containing A , $\text{bcl}(A) \subseteq f^{-1}(V)$ and hence $f(\text{bcl}(A)) \subseteq V$. Since f is an IF strongly b-closed and $\text{bcl}(A)$ is an IF b-closed in X , $f(\text{bcl}(A))$ is an IF b-closed in

Y. Since $\text{bcl}(f(A)) \subseteq \text{bcl}(f(\text{bcl}(A))) \subseteq V$, $\text{bcl}(f(A)) \subseteq V$. Therefore $f(A)$ is an IF π gb-closed in Y. ■

Theorem 3.13 Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be an intuitionistic fuzzy functions. If f is a bijective, IF π -irresolute and IF strongly b-closed and if $g \circ f: X \rightarrow Z$ is an IF slightly π gb-continuous, then g is an IF slightly π gb-continuous.

Proof. Let V be an IF clopen subset of Z . Then $(g \circ f)^{-1}(V) = (f^{-1} \circ g^{-1})(V) = f^{-1}(g^{-1}(V))$ is IF π gb-closed in X . Then by (3.12), $g^{-1}(V) = f(f^{-1}(g^{-1}(V)))$ is an IF π gb-closed in Y . Therefore g is an IF slightly π gb-continuous. ■

Theorem 3.14 A mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ from an IFTS (X,τ) to another IFTS (Y,σ) is an IF slightly π gb-continuous if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IF clopen set B in Y such that $f(p_{(\alpha,\beta)}) \in B$, $\text{cl}(f^{-1}(B))$ is an IFN of IFP $p_{(\alpha,\beta)}$ in X .

Proof. Let f be any IF slightly π gb-continuous mapping, $p_{(\alpha,\beta)}$ be an IFP in X and B be any IF clopen set in Y such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) \subseteq \text{bcl}(f^{-1}(B)) \subseteq \text{cl}(f^{-1}(B))$. Hence $\text{cl}(f^{-1}(B))$ is an IFN of $p_{(\alpha,\beta)}$ in X .

Conversely, let B be any IF clopen set in Y and $p_{(\alpha,\beta)}$ be an IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B)$. According to assumption $\text{cl}(f^{-1}(B))$ is an IFN of IFP $p_{(\alpha,\beta)}$ in X . So, $p_{(\alpha,\beta)} \in f^{-1}(B) \subseteq \text{cl}(f^{-1}(B))$, and by (definition of IF slightly π gb-continuous) there exists an IF π gb-open A in X such that $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(B)$. Therefore f is an IF slightly π gb-continuous. ■

IV. SOME APPLICATION THEOREMS

Definition 4.1 An IFTS (X, τ) is called $(\pi$ gb - $T_0)$ (co - T_0) if and only if for each pair of distinct intuitionistic fuzzy points, $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X there exist an intuitionistic fuzzy π gb-open set (IF clopen set) U , $\in X$ such that $x_{(\alpha,\beta)} \in U, y_{(\nu,\delta)} \notin U$.

Theorem 4.2 If $f:(X,\tau) \rightarrow (Y,\sigma)$ is an IF slightly π gb-continuous, bijection and Y is co- T_0 , then X is an IF π gb- T_0 .

Proof. Suppose that Y is IF co - T_0 , For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X , there exists an IF clopen set A in Y such that $f(x_{(\alpha,\beta)}) \in A$ and $f(y_{(\nu,\delta)}) \notin A$. Since f is an IF slightly π gb-continuous an bijection $f^{-1}(A)$ is an IF π gb-open sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A)$,

$y_{(\nu,\delta)} \notin f^{-1}(A)$. This shows that X is an IF π gb- T_0 . ■

Definition 4.3 An IFTS (X,τ) is called $(\pi$ gb- $T_1)$ (co- T_1) if and only if for each pair of distinct intuitionistic fuzzy points, $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X there exists an intuitionistic fuzzy π gb-open sets (IF clopen sets) $U, V \in X$ such that $x_{(\alpha,\beta)} \in U, y_{(\nu,\delta)} \notin U$ and, $y_{(\nu,\delta)} \in V, x_{(\alpha,\beta)} \notin V$.

Theorem 4.4 If $f: (X,\tau) \rightarrow (Y,\sigma)$ is an IF slightly π gb-continuous, injection and Y is co- T_1 , then X is an IF π gb- T_1 .

Proof. Suppose that Y is an IF co - T_1 , For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X , there exists an IF clopen sets A, B in Y such that $f(x_{(\alpha,\beta)}) \in A, f(y_{(\nu,\delta)}) \notin A, f(x_{(\alpha,\beta)}) \notin B, \text{ and } f(y_{(\nu,\delta)}) \in B$.

Since f is an IF slightly π gb-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are IF π gb-open sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A), y_{(\nu,\delta)} \notin f^{-1}(A), x_{(\alpha,\beta)} \notin f^{-1}(B)$ and $y_{(\nu,\delta)} \in f^{-1}(B)$. This shows that X is an IF π gb- T_1 . ■

Definition 4.5 An IFTS X is said to be π gb - T_2 or π gb-Hausdorff (co- T_2 or co-Hausdorff) if for all pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X there exists an IF π gb-open sets (IF clopen sets) $U, V \in X$ such that $x_{(\alpha,\beta)} \in U, y_{(\nu,\delta)} \in V$ and $U \cap V = 0$.

Theorem 4.6 If $f:(X,\tau) \rightarrow (Y,\sigma)$ is an IF slightly π gb-continuous, injection and Y is co- T_2 , then X is an IF π gb- T_2 .

Proof. Suppose that Y is IF co - T_2 space then for any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}, y_{(\nu,\delta)}$ in X , there exists an IF clopen sets A, B in Y such that $f(x_{(\alpha,\beta)}) \in A, \text{ and } f(y_{(\nu,\delta)}) \in B$. Since f is IF slightly π gb-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are IF π gb open sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A)$, and $y_{(\nu,\delta)} \in f^{-1}(B)$. Also we have $f^{-1}(A) \cap f^{-1}(B) = 0$. ■

Definition 4.7 An IFTS X is said to be co-regular (respectively IF slightly π gb-regular) if for each clopen (respectively IF π gb-clopen) set C and each IF $x_{(\alpha,\beta)} \notin C$, there exist an intuitionistic fuzzy open sets A and B such that $C \subseteq A, x_{(\alpha,\beta)} \in B$ and $A \cap B = 0$.

Theorem 4.8 If f is an IF slightly π gb-continuous injective IF open function from an IF strongly π gb-regular space X onto an IF space Y , then Y is IF co-regular.

Proof. Let D be an IF clopen set in Y and $y_{(\gamma, \delta)} \notin D$. Take $y_{(\gamma, \delta)} = f(x_{(\alpha, \beta)})$. Since f is an IF slightly π gb-continuous, $f^{-1}(D)$ is an IF π gb-closed set in X . Let $C = f^{-1}(D)$. So $x_{(\alpha, \beta)} \notin C$. Since X is IF strongly π gb-regular, there exists IFOS's A and B such that $C \subseteq A$, $x_{(\alpha, \beta)} \in B$ and $A \cap B = 0$. Hence we have $D = f(C) \subseteq f(A)$ and $y_{(\gamma, \delta)} = f(x_{(\alpha, \beta)}) \in f(B)$ such that $f(A)$ and $f(B)$ are disjoint IF open sets. Hence Y is an IFco-regular. ■

Definition 4.9 An IFTS X is said to be an IF co-normal (respectively IF strongly π gb-normal) if for each IF clopen (respectively IF π gb-closed) sets C_1 and C_2 in X such that $C_1 \cap C_2 = 0$, there exist an intuitionistic fuzzy open sets A and B such that $C_1 \subseteq A$ and $C_2 \subseteq B$ and $A \cap B = 0$.

Theorem 4.10 If f is an IF slightly π gb-continuous injective IF open function from an IF strongly π gb-normal space X onto an IF space Y , then Y is an IFco-normal.

Proof. Let C_1 and C_2 be disjoint IF clopen sets in Y . Since f is an IF slightly π gb-continuous, $f^{-1}(C_1)$ and $f^{-1}(C_2)$ are IF π gb-closed sets in X . Let us take $C = f^{-1}(C_1)$ and $D = f^{-1}(C_2)$. We have $C \cap D = 0$. Since X is an IF strongly π gb-normal, there exist disjoint IF open sets A and B such that $C \subseteq A$ and $D \subseteq B$. Thus $C_1 = f(C) \subseteq f(A)$ and $C_2 = f(D) \subseteq f(B)$ such that $f(A)$ and $f(B)$ disjoint IF open sets. Hence Y is an IFco-normal. ■

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