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Dynamic Modelling and validation of Continuous Beam with Free-Free Boundary Conditions

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Abstract: The study of vibration plays a significant role in many engineering problem. Free vibration analysis of a continuous beam with free-free boundary condition is carried out in this work Free-free end condition is generally encountered during operating condition of aeroplanes, missiles, submarine etc. where the structure is not supported at the both ends as such they are floating in space. Two significant parameters associated with a vibrating body are natural frequency and mode shape. In this work, the mathematical model of a beam is developed with the help of Euler-Bernoulli beam theory, which depicts the natural frequency of the beam and associated mode shape of the beam. FEM model of the beam has been created and simulation work is carried out on ANSYS 15.0 software. Moreover, experimental analysis is also performed through OROS software. Results obtained through mathematical model are further compared with experimental model and computational model, which provide a considerable agreement with each other.

Keywords: Free-Free beam, FEA, Ansys, OROS.

I. INTRODUCTION

The ultimate aim of vibration analysis is to deviate the excitation forcing frequency from the natural frequency of the structure as far as possible so that the condition of resonance does not take place. Another important term used in the vibration of the structure is the mode shape. A mode shape is a specific pattern of vibration executed by a mechanical system at a specific exciting frequency. The mode shape always describes the curvature of vibration at all points in time but the magnitude of the curvature will change. Radice J.J [1] studied the effect of local boundary condition on the natural frequencies of simply-supported beams. A comsol finite element model is made to examine the changes in the natural frequencies of a beam with respect to the location of the pin supports through thickness of the structure.

Kumar et al. [2] investigated an influence of mechanical properties of material on frequency and mode shape analysis of transmission gearbox. The objective of this research work was to examine the effect of mechanical properties of the material on mode shapes and natural frequency of heavy vehicle gearbox transmission casing. Baroudi et al. [3] discussed transverse vibration analysis of Euler-Bernoulli beam carrying point mass submerged in fluid media. He developed an analytical method to examine the effects of adding mass on natural frequency and mode shape of Euler-Bernoulli beams carrying concentrated mass at any arbitrary position submerged in a fluid media. Rao et al. [4] studied experimental and analytical modal analysis of welded structure used for damage identification in vibration. Han et al. [5] studied dynamics of transversely vibrating beams using four engineering theories i.e. Euler-Bernoulli, Rayleigh, shear and Timoshenko. Palej et al. [6] studied modal analysis of multi-degree of freedom systems with repeated frequency. Modal analysis of mechanical systems comprising of n identical masses coupled with springs in such a way that the stiffness matrix takes the form of a n order symmetric matrix. Luay [47] calculates the natural frequency of stepped cantilever beam. Three models are used to determine the natural frequency of stepped cantilever beam. The models used to calculate natural frequency are Rayleigh model, Finite elements model (ANSYS model) and modified Rayleigh model Rayleigh model calculates the stiffness at each point of the beam. Simsek et al. [8] explained the vibration analysis of a beam in free vibration analysis of beam by using a third-order Shear deformation theory. In this, free vibration of the beam having different boundary conditions is investigated within the framework of the third order shear deformation theory.

II. MATHEMATICAL FORMULATION

Consider a cantilever beam subjected to a transverse load as shown in a figure 1. The free body diagram of an element of the beam is as shown in figure 2.

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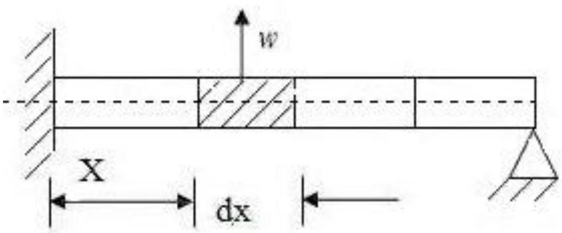

Fig. 1 A beam under transverse vibration

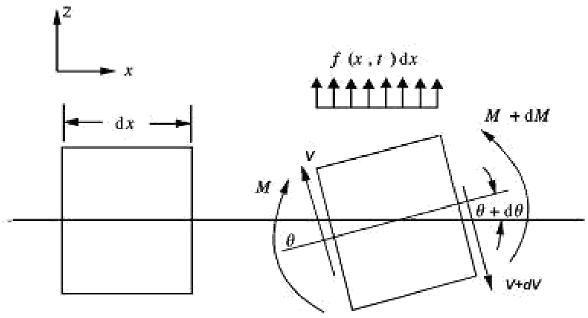

Fig. 2 Free body diagram of a section of a beam in transverse vibration

From the references [9,10,11,12], the equation of motion for the forced transverse vibration of the beam may be expressed as,

$$\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A(x)dx\frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) \quad (1)$$

Where, first term of eq (1) may be expressed as a bending of the beam, second term denotes an inertia force acting on the element of the beam. $f(x,t)$ is the external applied force.

For free vibration f (x, t) =0 and above equation becomes

$$c^2\frac{\partial^4 w(x,t)}{\partial x^4} + \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \quad (2)$$

Where, $C=\sqrt{(\frac{EI}{\rho A})}$

The solution of the free vibration problem can be found using the method of separation of variable

$$w(x,t) = W(x)T(t) \quad (3)$$

where w (x, t) is variation of displacement of the element with spatial coordinate and time W(x) is variation of displacement of the element with spatial coordinate only and T(t) is variation of displacement of the element with time only.

Substituting this equation in the final equation of motion and rearranging leads to,

$$\frac{c^2 d^4 w(x)}{w(x)dx^4} = \frac{1}{T(t)}\frac{d^2T(t)}{dt} = a = \omega^2 \quad (4)$$

Where $a=\omega^2$ is a positive constant.

Above equation can be written as

$$\frac{d^4 w(x)}{dx^4} - \beta^4 w(x) = 0 \quad (5)$$

$$\frac{d^2 T(t)}{dt^2} + \omega^2 T(t) = 0 \quad (6)$$

Where

$$\beta^4 = \frac{\omega^2}{c^2} = \frac{\rho A\omega^2}{EI} \quad (7)$$

The solution to time dependent equation can be expressed as

$$T(t) = A\cos\omega t + B\sin\omega t \quad (8)$$

where, A and B are constant that can be determined from the initial conditions.

Hence the solution of the equation becomes:

$$W(x) = C_1 e^{\beta x} + C_2 e^{\beta x} + C_3 e^{i\beta x} + C_4 e^{-i\beta x} \quad (9)$$

where $C_{1,}C_{2,}C_{3,}C_{4,}$ are constant. Above equation can also be expressed as:

The constant $C_{1,}C_{2,}C_{3,}C_{4,}$can be found out from boundary conditions. The natural frequencies of the beam can be computed from:

$$\omega^2 = \frac{EI}{\rho A}\beta^4 \quad (10)$$

The spatial part can be written as

$$W(x) = C_1\sin(\beta x) + C_2\cos(\beta x) + C_3\sinh(\beta x) + C_4\cosh(\beta x) \quad (11)$$

The function W (x) is normal mode or characteristic function of the beam and ω is the natural frequency of vibration. For a given beam, there is infinite number of normal modes with each natural frequency associated with normal mode. The unknown constant$C_{1,}C_{2,}C_{3,}C_{4,}$ and value of β can be determined from the boundary conditions of the beam.

Applying free-free beam boundary conditions

$$w''(0) = 0, w'''(0) = 0, w''(L) = 0, w'''(L) = 0 \quad (12)$$

One obtains

$$-C_2 + C_4 = 0 \quad (13)$$

$$-C_2 + C_4 = 0 \quad (14)$$

$$-C_1\sin(\beta L) - C_2\cos(\beta L) + C3\sinh\beta L + C4\cosh\beta L = 0 \quad (15)$$

$$-C_1\cos(\beta L) - C_2\sin(\beta L) + C_3\cosh(\beta L) + C4\sinh\beta L = 0 \quad (16)$$

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The eqs. (13) to(16) can be arranged in matrix form:

$$\begin{bmatrix} \sinh(\beta L) - \sin(\beta L) & \cosh(\beta L) - \cos(\beta L) \\ \cosh(\beta L) - \cos(\beta L) & \sin(\beta L) + \sinh(\beta L) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For non-trivial solution, the determinant of a matrix has to be vanished to obtain:

$$\cosh(\beta L)\cos(\beta L) = 1 \qquad (17)$$

The above transcendental equation has an infinite number of solutions, it may be solved numerically, the first five values are written:

TABLE IVALUE OF B_NL WITH VARIOUS MODE ORDER

Mode order n	$\beta_n L$
1	4.73
2	7.85
3	10.99
4	14.13
5	17.27

Putting these values in above Eq. 18, gives the mode shapes corresponding to the natural frequency ωthat can be calculated from the characteristic Eq.10 The mode shapes are given by the following expression:

$$W(x) = [cos(\beta_n x) + cosh(\beta_n x)] - \frac{cos(\beta_n l) - cosh(\beta_n l)}{sin(\beta_n l) - sinh(\beta_n l)}[sin(\beta_n x) + sinh(\beta_n x)] \qquad (18)$$

III. EXPERIMENTAL SETUP AND PROCEDURE

In this work, vibration analysis of a continuous beam with free fee boundary conditions has been carried out through vibration analyzer OROS 36, which operates with NV Gate 8.30 version® software, and records the signals of the beam in the form of acceleration, velocity and displacement. Experimental arrangement for fre-free beam is shown in fig 3.

Fig. 3 Experimental setup

The OROS set up is placed and the accelerometer is rigidly fixed on the bar with the help of an adhesive material. The bar is freely hanging at the end point with the help of elastic cable. Accelerometer measure the vertical vibrations which are generated on bar.

Fig. 4 Experimental equipment for vibration analysis

OROS36 is made for high channel count capacity without comprising the analyzer geographies. All the channels are handled in real time: FFT, 1/3rd Octave, CPB or Synchronous order analysis. OR36 & OR38 keep these real-time capabilities upto 20 kHz. There are LCD screen controls on the OR36 and OR38 hardware that allow you to run, stop the analyzer, change the fan speed etc.

Steps to be followed in the experimental modal analysis are:

1. A bar of a particular material (mild steel) with dimensions (L, r) and transducer (i.e.,measuring device, accelerometer) was chosen.
2. Both ends of the bar are given specific boundary condition (i.e. free-free condition).
3. An accelerometer (having magnetic base) was placed at the centre of the bar to observe the free vibration response. (i.e. acceleration).

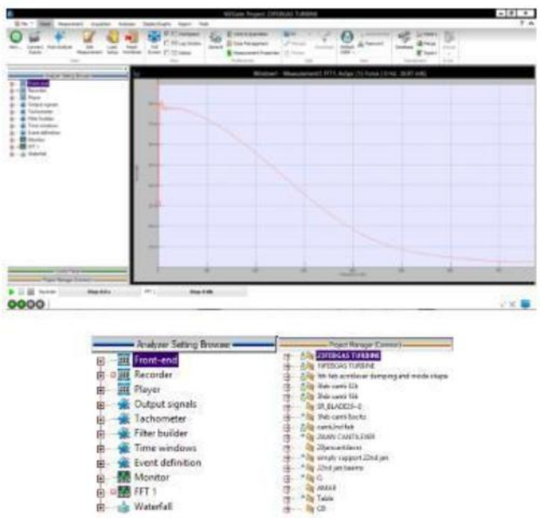

Fig. 5 Graphic User Interface of NV Gate

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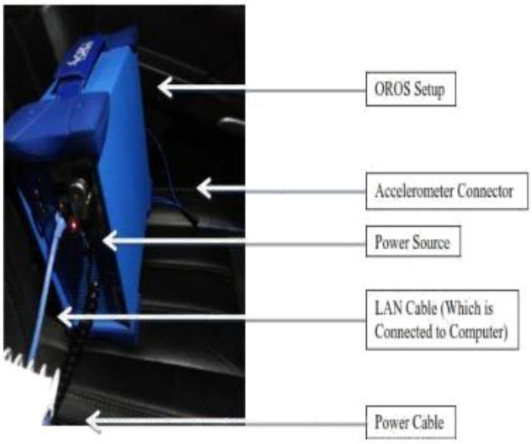

Fig. 6(a) OROS setup (Rear view)

4. An initial deflection was given to the bar and is allowed to oscillate on its own. To get the higher frequency it is advised to give initial displacement at an arbitrary position (e.g. at the mid span). This can also be done by bending the bar from its equilibrium position by application of a small static force at the centre of the bar and suddenly releasing it, so that the bar oscillates on its own without any external applied force during the oscillation.
5. The data obtained from the accelerometer is recorded in the form of graph showing variation of the vibration response with time.
6. The procedure was repeated for 5 to 10 times to check the repeatability of the experiment.
7. The whole set of data was recorded in a data base to obtain the desired result.

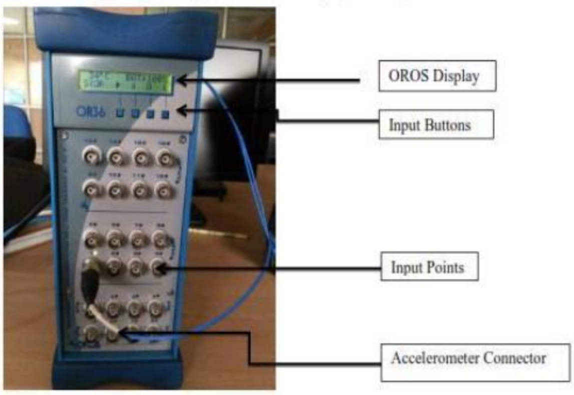

Fig. 6(b) OROS Setup (Front View)

IV. COMPUTATIONAL ANALYSIS

The finite element analysis (FEA) also called as finite element method (FEM), is based on the concept of building a complex object with simple parts, or, dividing a complicated object into smaller and pieces. Application of this idea is found everywhere in everyday life, as well as in engineering [13,14]. For example, children playing with LEGO toys by using many smaller pieces, each of very simple geometry, to form various objects such as ships, trains, or buildings. With more and more small number of pieces, these objects will look more realistic.

In mathematical term, this is a simple use of the concept, that is, to approach or represent a smooth object with a finite number of simple small pieces and increasing the number of such pieces in order to improve the accuracy of this representation. In engineering terms, the finite element method is a mathematical technique for finding approximate solution to boundary value problems for partial differential equation. FEM divides a large problem into smaller and simpler parts, called finite elements. The simple equations that model these finite elements are assembled into a larger system of equations that will model the entire problem. Then FEM uses different methods from the calculus to approximate a solution by minimizing an error function. Rapid engineering analyses can be done because the structure is represented using the known properties of geometric shapes, that is finite elements. Efficient, general-purpose computer codes exist with suitable matrix assembler and equation solvers for calculating the following structural properties:

The following procedure has been adopted for FEA analysis for the beam;

- Divide the geometric model into pieces to generate a "mesh" (a collection of elements having nodes)
- Define the behaviour of the physical quantities on each and every element.
- Connect the elements at the nodes to obtain the system of equations for the entire model.
- Specify the load and the boundary conditions.
- Solve the system of equations constituting unknown quantities at the nodes (such as the displacements).
- Calculate the required quantities (e.g., strains and stresses) at each elements or nodes.

The FEA analysis has been carried out on ANSYS 15.0 software. ANSYS is a general purpose finite element modelling tool for mathematically solving a different types of mechanical problems. These problems comprise static/dynamic, structural analysis, fluid problems, heat transfer as well as electromagnetic and acoustic problems.

V. RESULTS AND DISCUSSION

In this section, comparison between the values of natural frequency obtained analytical, experimentally and by the ANSYS 15.0 software. The expression for the natural frequency of the bar with free-free boundary condition as

$$\omega_n^2 = \frac{EI}{\rho A}\beta^4 \qquad (19)$$

Where E=Young modulus
ρ (rho)=density of material

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A=cross section area of bar
L=length of the bar
I=Area moment of inertia of the bar
ω_n= natural frequency of vibration of the beam in nth mode

Value of β can be from frequency equation (17) of free-free beam as discussed in section 2.
Figure shows experimental results of the uniform cross section with free-free boundary condition, which was obtained through OROS-36 with NV-Gate software. Table represents each value of mode at respective frequency.

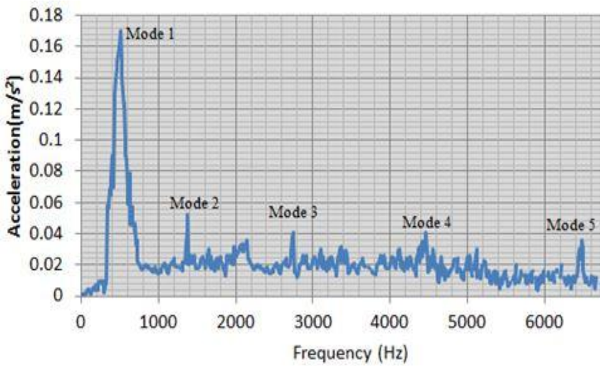

Fig. 7 : Experimental results

TABLE III EXPERIMENTAL NATURAL FREQUENCY

Mode order	Natural frequency (Hz)
1	510.2
2	1380.8
3	2740.6
4	4549.8
5	6450.7

Comparison of the mode shape obtained by ANSYS software and those obtained analytically can also be done. Analytically obtained mode shape have been shown in Figure 8.

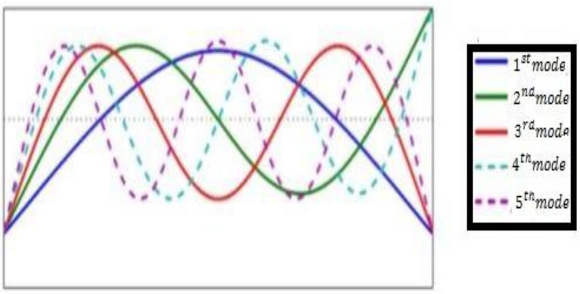

Fig. 8 : First five mode shape of beam with free-free boundary condition

In modal analysis of the mild steel beam using ANSYS 15.0, different modal frequency and modes shapes are shown in Figs (9-13).

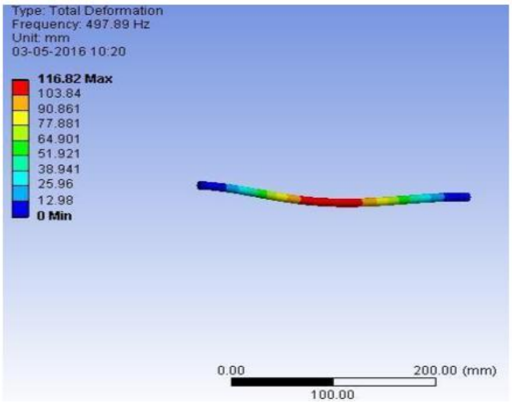

Fig. 9 : First mode shape

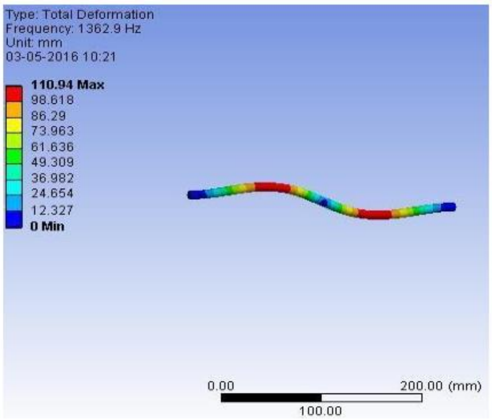

Fig. 10: Second mode shape

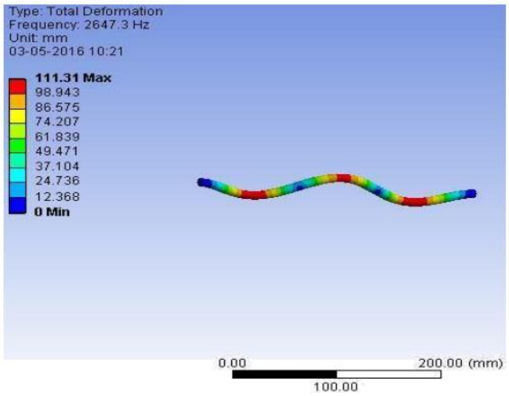

Fig. 11 : Third mode shape

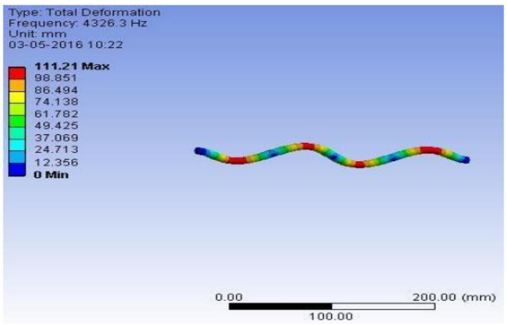

Fig. 12 : Fourth mode shape

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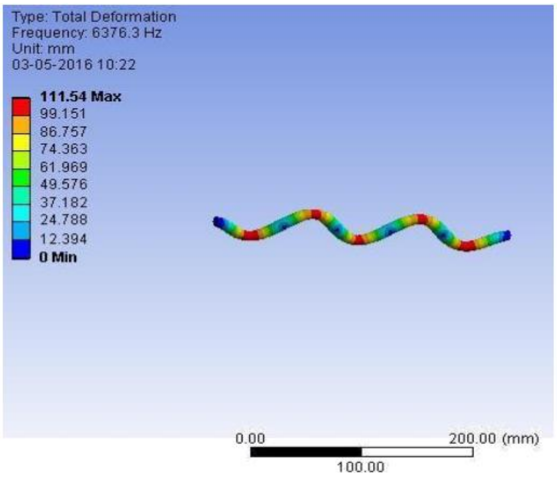

Fig. 13 : Fifth mode shape

The value of mode shapes are presented in table 3.

TABLE IIIII NATURAL FREQUENCY OBTAINED BY ANSYS 15.0

Mode order	Natural frequency (Hz)
1	497.89
2	1362.9
3	2647.3
4	4326.3
5	6367.6

It can be easily seen that mode shape obtained by analytical formula and those obtained by ANSYS 15.0 are quite similar. It is observed that for n mode order, there is n-1 beam vibrating at a particular frequency.
Let us compare the natural frequency value (shown in table 4) obtain analytically, experimentally and by ANSYS 15.0 software.

TABLE IVV COMPARISON BETWEEN NATURAL FREQUENCIES OBTAINED BY DIFFERENT METHODS

Mod e order	Natural frequen cy (Hz)	Experimental natural frequency (Hz)		Computational natural frequency (Hz) (ANSYS)	
		Measu red value	Percen tage Error	Measu red value	Percent age Error
1	498.84	500.2	.27	497.89	.19
2	1374	1380.8	.49	1362.9	.8
3	2693.98	2740.6	1.73	2647.3	1.68
4	4453.31	4549.8	2.16	4326.3	2.85
5	6652.48	6450.7	3.03	6376.3	4.15

It can be observed that as the mode order increases, there is greater variation or error between natural frequency obtained analytically and natural frequency obtained experimentally or by software. This may be due to the fact that as frequency is increased, range on which software have to operate rises. As ANSYS works on approximate method (FEM) which in turn increases the chances of error in the measured value over wide operating range. As for the case in OROS software, structural damping may the reason for the error to exist in higher mode.
Fundamental natural frequency obtained by analytically with the help of above equation is 498.84 Hz whereas experimentally obtained with the help of OROS software equation is around 510Hz. The difference in the values obtained is due to some assumption made in deriving the equation theoretically such as elastic, isotropic, homogenous material. However in practice, there exists some degree of anisotropic, non-homogeneity in the material which is reason of difference of the two values. Graphical representation of natural frequencies at various dynamic modes is presented in figure 14.

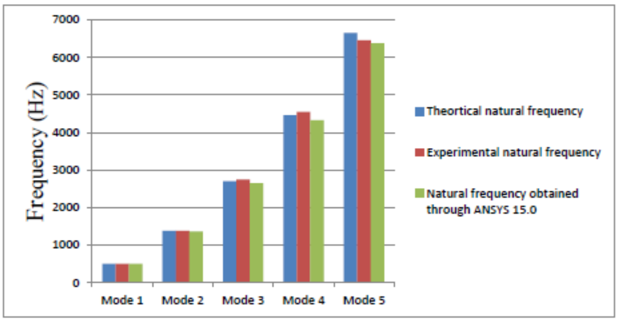

Fig. 13 : Graphical representation of natural frequency values

VI. CONCLUSIONS

In this work, analytical, experimental and computational modal analysis of a bar with a free-free boundary condition is studied and its characteristics parameter such natural frequency and mode shape of the bar is discussed.
Following conclusion can be made:

- Boundary condition of the bar plays a very significant role in the vibration response of the bar.
- Simulation results were compared with experimental data which was evaluated through OROS system.
- Error observed in experimental value is because thin cable used as a supporting element must have some value of elasticity, i.e. not an ideal material.
- Error observed between experimental value and ANSYS value is because a real beam material is not perfectly homogenous, elastic and isotropic as is considered by ANSYS.

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