

Mathematical Estimation of Production Performance of Fish Population

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Abstract: Fish population dynamics demonstrates the quantitative changes in the number of individuals in which a given fish population grows and shrinks over time as controlled by some specific factors, and is generally used in the fisheries science in order to determine sustainable yields. The aim of this article is to present and analyze a generic mathematical formula of a single-region size structured model which is useful for the fish production estimation. With the intention to estimate the final fish size, the von Bertalanffy's growth equation is modified and utilized based on the initial size of the fish species. For fish population calculation, initial size, birth, growth, mortality rates and the arbitrary constant of modified von Bertalanffy's growth equation are considered as input variables. The number of total fish population and the fish size at different time spans are calculated. The results coming from the mathematical calculation are compared to the experimental results of freshwater mud eel fish production. Comparing to these values, it reveals that there is no significant difference between the result obtained by implementing the proposed mathematical model and the experimental outcomes obtained. Finally, it is expected that the modified model can be used for the practical applications of fish production.

Keywords: Mathematical estimation; Fish population; Fish size; Mortality rates; von Bertalanffy's growth equation.

I. INTRODUCTION

Bangladesh is a densely populated country, currently with a population of around 160 million people [1]. This country is an agro-based developing country being endowed with natural fisheries resources. It is fortunate in having an extensive water resource in the form of ponds, natural depressions (i.e., haors and beels), lakes, canals, rivers and estuaries. The consumption of fish food is increasing as the population is increasing geometrically whereas the land and water are decreasing at the same rate. Therefore, to meet the growing demand, it is necessary to establish sustainable fisheries, or to increase the number of artificial fisheries which is practically impossible due to the consumption of lands by overpopulation. At present the fisheries sector in Bangladesh plays a significant role for fulfilling the demand of protein, nutrition, employment, poverty alleviation of a large number of unemployed population and foreign exchange earnings. Aquaculture produces about 3.46 million tons of fish, of which about 2 million tons were farmed in the 2013–14 [2]. Bangladesh ranks third among the world's largest inland fish producing countries after China and India. Around three quarters of rural households practice some form of freshwater aquaculture covering some 10 million ponds and most of which measure less than 400 m² [3].

Fisheries in Bangladesh are diverse, there are about 795 native species of fish and shrimp in the fresh and marine waters of Bangladesh, and 12 exotic species that have been introduced [4]. To meet the present demand and considering future potentials, a large number of fisheries

have been established in different parts of the country. Although there seems a huge success in producing large quantity of fish population, the major tasks for fisheries management is to regulate their fishery in such a way as to obtain the maximum benefit from it. However, the fishery is a complex system and it is not easy to interpret the wide range of data that can be obtained about such diverse features as growth rates, harvesting with respect to fishing gear, mortality, immigration and emigration, etc., nor is it easy to predict the effect on the fisheries management. Hence, several mathematical models and techniques are existing for the study of fish population in order to get an idea how much fish be produced by using these parameters.

Fish population dynamics describes the quantitative changes in the number of individuals in a population or the vital rates of a population as a result of various different processes i.e., growth, natural mortality, mortality due to fishing, emigration or immigration, sexual differentiation over time. A number of authors worked on the fish population dynamics based on modelling the growth processes. Dubey et al. [5] presented a dynamic model for a single-species fishery with optimal harvesting policy that depends partially on a logistically growing resource. Later on, the same authors proposed a non-linear mathematical model to study the fisheries resource system dynamics using the Pantryagin's maximum principle [6]. Faugeras and Maury [7] established an advection-diffusion size-structured fish population dynamics model to simulate the

skipjack tuna population in the Indian Ocean. The proposed model is fully spatialized, and the movements are parameterized with oceanographical and biological data.

Kar [8] proposed a nonlinear mathematical model to study the dynamics of a fishery resource system in an aquatic environment that consists of two zones; a free fishing zone and a reserve zone where fishing is strictly prohibited. He observed that in the absence of any predator, even under continuous harvesting, fish population may be maintained at an appropriate equilibrium level. Later on, Kooten et al. [9] proposed a mathematical model to study the relation between hatching size and response to harvesting mortality. The result shows that the hatching size determines dynamics through its effect on the relative strength of cannibalistic mortality. However, most of the models are of weakly coupled hyperbolic partial differential equations with non-local boundary conditions [10]. These models are setup to preserve the fish species disappearing or to provide assessment of the fish abundance and fishery exploitation in order to determine the sustainable yield of fish population [11]. A recent article has been published on the fish population dynamics in which both size and time are taken as structure variables to account for growth, mortality, movements of fish, environmental variability and variable distribution of fishing effort within the multi-regions. This model is established considering a system of hyperbolic partial differential equations where both linear and nonlinearities boundary conditions are carefully discussed [12].

The dynamic model approach, therefore, is a widely applied technique to a number of environmental management issues particularly to the fisheries management problems [13–15]. To devise these models, a considerable amount of work of recording data is required, extending over a prolonged period of time in order to obtain reliable results. Thus, the data collected for the fisheries management to make it possible, in particular, to establish growth and mortality rates. A considerable number of researchers works on the von Bertalanffy’s growth equation in order to estimate the both growth and reproduction of fish population [16–20]. Hart and Chute [21] introduced a mathematical formula for estimating von Bertalanffy growth parameters from growth increment data using a linear mixed-effects model that lack explicit age information. Although this approach produces unbiased estimates, it is sometimes difficult to implement and compute the growth and size of fish population in different time spans.

From the aforementioned literatures it is found that the fish population dynamics plays an important role in the fishing gear considering the various controlled parameters i.e., growth, natural mortality, mortality due to fishing, emigration or immigration, harvesting time, setting catch quotas, restricting the legal size of fishing, etc. In this work, a generic mathematical formula of a single-region size structured model which is useful for the different fish

species has been used and analyzed for the fish production. In order to estimate the final fish size, the von Bertalanffy’s growth equation has been modified and utilized based on the initial size of the fish species. During the fish population calculation, birth, initial size, growth, mortality rates and the constant ‘K’ of modified von Bertalanffy’s growth equation are considered as input variables within the single-region. The outcomes of this article are calculated in order to find out the total population and the fish size in different time horizon. At the end of the work, the results coming from the mathematical equation are compared to the experimental results of fish population production performance.

II. EXPERIMENTAL

In this article, a hypothetical mathematical model is developed and data coming from the field of the fish population are compared considering the linear model approach. Therefore, among the parameters, only birth, growth and mortality rates are considered as input factors and the total population, size on time are considered as output factors. On compare, the results are summarized.

A. Mathematical Model Used

Refereeing to the published article [12], let, P be the total fish-population in the domain, Ω . The number of individuals of length between 0 and L, in the domain Ω_k at time t is given by

$$P^k(t) = \int_0^L p^k(x,t) dx \quad (1)$$

where $p^k(x, t)$ is the density of fish population of length x at time t in the zero Ω_k . The basic model of population dynamic of fisheries is

$$P^k(t + \delta t) = P^k(t) + \text{birth rate} + \text{migration rate} - \text{mortality rate} - \text{emigration rate} \quad (2)$$

where $P_k(t + \delta t)$ is the number of fish at time t + δt , δt is the time variation. The mortality rate includes both the fishing mortality and natural mortality. The mathematical equation is

$$\begin{cases} \frac{\partial p^k(x,t)}{\partial t} + \frac{\partial (v^k(x,t)p^k(x,t))}{\partial x} = -g^k(x,t)p^k(x,t) + (Mp)^k & (x,t) \in \mathcal{D} \\ p^k(x,0) = p_0^k & x \in (0,L) \\ v^k(0,t)p^k(x,0) = \int_0^L \beta(x,t)p^k(x,t)dx & t \in (0,T) \end{cases} \quad (3)$$

where,

$$(Mp)^k = \begin{pmatrix} -\sum_{i \neq 1}^{nk} m_{1 \rightarrow i} & m_{2 \rightarrow 1} & m_{3 \rightarrow 1} & \cdots & m_{nk \rightarrow 1} \\ m_{1 \rightarrow 2} & -\sum_{i \neq 2}^{nk} m_{2 \rightarrow i} & m_{3 \rightarrow 2} & \cdots & m_{nk \rightarrow 2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m_{1 \rightarrow nk} & m_{2 \rightarrow nk} & m_{3 \rightarrow nk} & \cdots & -\sum_{i \neq nk}^{nk} m_{k \rightarrow i} \end{pmatrix} \begin{pmatrix} p^1 \\ p^2 \\ \vdots \\ p^{nk} \end{pmatrix}$$

Modification of von Bertalanffy’s Fish Size Equation

One of the most widely used models for quantifying growth in fish is the von Bertalanffy’s growth curve which is most widely used models for quantifying growths and is especially important in fisheries studies. He derived this equation from simple physiological arguments. It assumes that the growth rate of an organism declines with size so that the rate of change in length, l , may be described by:

$$\frac{dl}{dt} = K(L_{\infty} - l) \tag{4}$$

where t is time, l is the length (or some other measure of size), K is the growth rate and L_{∞} is the final fish size. After simplification, the equation reduces to:

$$l_t = L_{\infty}(1 - e^{-K(t-t_0)}) \tag{5}$$

The parameter t_0 is included to adjust the equation for the initial size of the fish and is defined as age at which the fish population would have had zero size.

Thus, in order to fit into the model developed by von Bertalanffy it is required to estimate the three parameters such as L_{∞} , the final fish size, K the growth rate, and t_0 , the initial fish size. Therefore, it is observed that plotted curve starts from the final size and moves towards the initial size of the organism, which resembles a concave downward curve (i.e., exponential decay) in nature. Moreover, it is must to know the final size of the organism in order to estimate the size of species in time periods.

The theory behind various growth models is reviewed, however, this experimental research is carried out on the basis of an idea that the curve shows an inverse relationship to the von Bertalanffy’s model. Therefore, the von Bertalanffy’s growth equation is modified which reduced to:

$$L(t_{n+1}) = L(t_n) + L(t_n) \times [1 - e^{-K(t-t_0)}], \tag{6}$$

$$n = 0, 1, 2, \dots$$

where $L(t_0)$ is the initial fish size and ‘ K ’ is said to be arbitrary constant which is positive. The value of ‘ K ’ is to be estimated as the similar way to estimate the values of mortality, growth, birth rate etc. from experimental sample environment.

The modified equation bears the advantages that the resulted value starts from an initial size of the fish species. Besides, it is not necessary to know the final size of the fish at different time periods. The proposed model starts from an initial size and will be continue exponentially which resembles as an exponential upward curve. Moreover, it is possible to estimate the fish size in different time span.

B. Sample Collection and Presentation

The experiment was conducted in three tanks in the Fish Breeding House under the Department of Genetic Engineering and Biotechnology, Shahjalal University of Science and Technology (SUST), Sylhet, Bangladesh. In addition, two tanks and one backyard tank were selected in a rural house, Chatak, Sunamgonj. Experimentation with a plastic tank was designed in a pond of tilapia hatchery at Kamal Bazar, Sylhet. Collected fish species were divided into three categories and placed in different places such as two house tanks and one backyard tank. To compare the significance of the findings, paired sample statistics of mean and standard deviation, paired sample correlations, and paired sample significance test were conducted for the growth of experimental fish considering the length (size) of fish population. After the collection of primary data from different environments, Table 1 is prepared [22].

Refereeing to the article [12], it was said that the considered domain was a multi-region, domain Ω_k , and the model developed was valid for both linear and nonlinear cases. However, the article [22] revealed the data only for a single environment (single domain, Ω_k , $k = 1$). Hence the present work has been followed a single domain, which implies the model is linear case only.

TABLE I Stocking rates of fish population in different Environments

C/E	FLS (cm)	FS (cm)	TN	SF	GR	MR	RP
Tank 1	15	30.6	40	35	0.7070	0.125	6
Tank 2	15.5	25.05	40	30	0.4215	0.25	6
BT	15.7	25.4	40	31	0.4315	0.225	6

*C/E = Culture or Environment, BT = Backyard tank, FLS = Fingerline size, FS = Final size, TN = Total number of fish, SF = Survival fish, GR = Growth rate, MR = Mortality rate, RP = Rearing period (month).

III. RESULT AND DISCUSSION

For this work only one site is under the influence of fishery. The case is one sided, therefore, there is no harvesting, and no migration and emigration takes place in the system. Referring to the Table 1, the side uses only the initial size, final size, growth rate, and mortality rate. On the application of Eq. 3, the value of birth rate, β must be considered as small as possible but not zero.

A. Tank 1

For this case the value of β is 0.000213. Taking the values of growth and mortality rate form Table 1 and putting these values in Eq. 3, it is found that after 6 months or 0.5 year, the total population is 34.79, shown in Fig. 1, which is very close to the value found in the published article [22]. Since they have used the initial fish size or newborn fingerlings fish started on the time being, i.e. $t_0 = -0.2$ year for each cases. Following the similar way, taking the

initial fish length is 15 cm and putting the value of $K=0.305$ per year, using the Eq. 6, it is found that after 6 months, the size of fish population is 30.5956 cm, shown in Fig. 2, which is very close to the fish size found in the same article.

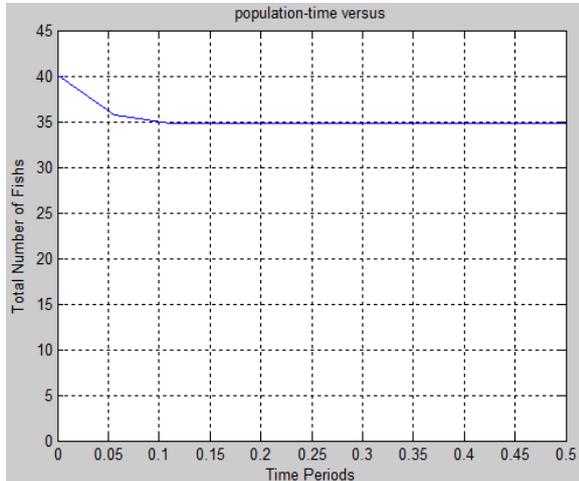


Fig. 1. Total fish population vs time periods for Tank 1

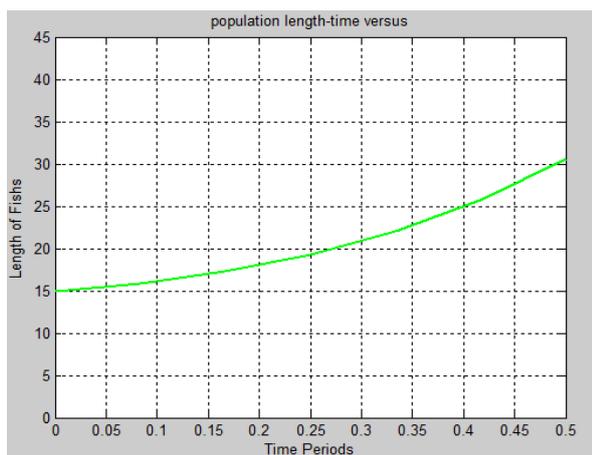


Fig. 2. The final fish size vs time periods for Tank 1

B. Tank 2

For this case the value of β is 0.0002. Taking the values of growth and mortality rate from Table 1 and putting these values in Eq. 3, it is found that after 6 months or 0.5 year, the total population is 30.02, shown in Fig. 3, which is very close to the value 30, found in the reference article. Following the similar approach as stated for Tank 1, taking the initial fish length is 15.5 cm and putting the value of $K=0.196$ per year, using the Eq. 6, it is found that after 6 months, the size of fish population is 25.075 cm, shown in Figure 4. Comparing to the reference article it is observed that the final fish sizes are approximately same.

C. Backyard Tank

In this case the value of β is 0.00019. Taking the values of growth and mortality rate from Table 1 and putting these values in Eq. 3, it is found that after 6 months or 0.5 year, the total population is 31.075, shown in Fig. 5, which is very close to the value 31 found in the reference article.

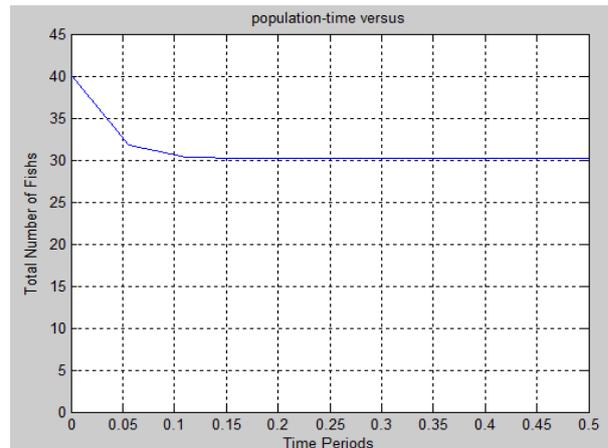


Fig. 3. Total fish population vs time periods for Tank 2

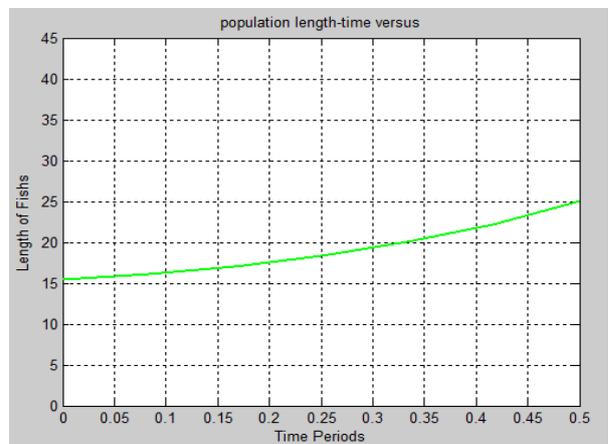


Fig. 4. The final fish size vs time periods for Tank 2

Following the similar approach as stated for Tank 1 and 2, taking the initial fish length is 15.7 cm and putting the value of $K=0.196$ per year, using the Eq. 6, it is found that after 6 months, the size of fish population is 25.3988 cm, shown in Fig. 4. Comparing to the reference article it is observed that the final fish sizes are approximately same. The comparison between the values computed by mathematical modelling considering a single region, and the result computed for the production performance of freshwater mud eel, *Monopterus Cuchia*, are given in Table 2.

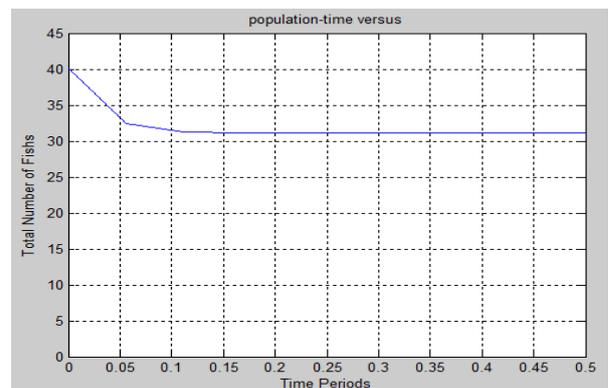


Fig. 5. Total fish population vs time periods for Backyard Tank

Comparing to the values, i.e., total fish population and the final fish size obtained by mathematical modelling and experimental data reveals that the model developed is justified for practical applications, and the model can be used for fish population production sector.

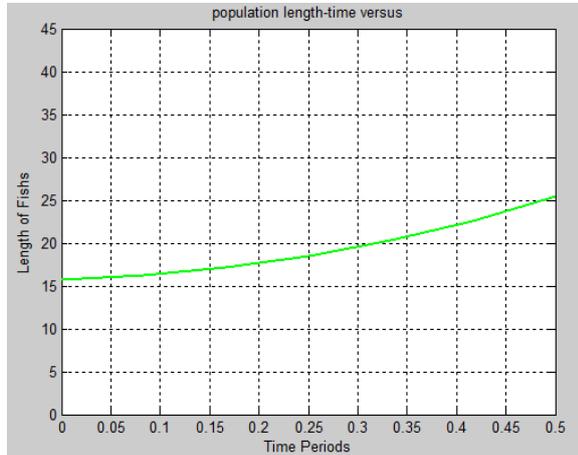


Fig. 6. The final fish size vs time periods for Backyard Tank

TABLE 2 Comparison between the computed value and the values form reference article

C/E	FLS (cm)	Reference Article		Computed value	
		TNFS	FS (cm)	Total fish Population	FS (cm)
Tank 1	15.0	35	30.6	34.79	30.5956
Tank 2	15.5	30	25.05	30.02	25.075
BT	15.7	31	25.4	31.07	25.3988

*C/E = Culture or Environment, BT = Backyard tank, FLS = Fingerline size, FS = Final size, TNFS = Total number of fish survived.

IV. CONCLUSION

In this article, a single-region linear size structured fish population model has been developed and investigated under various environments. The model is formulated in the generic way so that it can be used for different types of fish species. Furthermore, the final fish size model developed by von Bertalanffy is modified based on the initial size of the fish species. The proposed model starts from an initial size and continue exponentially upward direction, which is considered to estimate the fish size in different time span.

A commercial software, Mathlab[®], is used to calculate the values of total population and the final size of fish population. The values obtained by mathematical modelling are compared to the experimental data for the production performance of freshwater mud eel. Comparing to these values, it reveals that there is no significant difference between the result obtained by implementing

the proposed mathematical model and the experimental outcomes obtained.

As it is considered that the fishery is a complex system and it is not easy to interpret the wide range of data for its diverse features, nor is it easy to predict the long term effect on the fisheries management, the management can decide for long term investment. Therefore, the fisheries management can predict how much yield will be after a certain period of time. In the same time the management can estimate the future value of fish population, as well as the profit gain from the production. Finally, it is expected that the modified model can be used for the practical applications, and will be able to estimate the fish production.

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the fields of numerical methods i.e. finite difference methods and finite element method. Currently, he is supervising a good number of MS, MPhil and PhD scholars. He has published several national and international journals.



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