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Intuitionistic Fuzzy Dot $\beta-$Sub Algebra of $\beta-$ Algebras

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Abstract: In this paper, we introduce the notion of intuitionistic fuzzy dot $\beta-$sub algebras on $\beta-$algebras and investigate some of their properties.

Keywords: BCK/BCI algebras, B-algebras, fuzzy dot $\beta-$subalgebra, intuitionistic fuzzy dot $\beta-$subalgebras on $\beta-$algebras.

I. INTRODUCTION

In 1996, Y.Imai and K.Iseki ([5],[6],[7]) introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of BCI algebras. In 2002, J. Neggers and H.S. Kim [12] introduced the notion of B$-$algebras which is another generalization of BCK algebras. Also they introduced the notion of $\beta-$algebras[13] where two operations are coupled in such a way as to reflect the natural coupling, which exists between the usual group operation and its associated B$-$algebras. In 2012, Y.H.Kim [10] investigated some properties of $\beta-$algebras.

The important point in the evaluation of the modern concept of uncertainty was the paper by Lofti A. Zadeh [16] that introduced the theory of fuzzy sets. The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups in 1997, by Rosenfeld [14].The concept of intuitionistic fuzzy subset was introduced by Atanassov [17] in 1986, which is a generalization of the notion of fuzzy sets. Fuzzy sets give a degree of membership of an element in a given set , while Intuitionistic fuzzy sets give both a degree of membership and a non-membership. OG. Xi [15] applied the concept of fuzzy sets to BCK algebras and got some results in 1991. In 1993, Y.B. Jun [8] applied it to BCI algebras. In their paper [9] , the authors introduced the notion of fuzzy dot sub algebras of BCK/BCI algebras as a generalization of a fuzzy subalgebra, and then investigated several basic properties which are related to fuzzy dot sub algebras. In [2] AI-Shehrie introduced the notion of fuzzy dot SU-Sub algebras. In [11], K.H.Kim introduced the notion of fuzzy dot sub algebras of d-algebras in[4]. In[1] M.Abu Ayub Ansari and M.Chandramouleeswaran introduced the notion of fuzzy dot $\beta-$subalgebra of $\beta-$algebras.

This motivated us to study the intuitionistic fuzzy dot $\beta-$subalgebra of $\beta-$algebras. In this paper, we Introduce the notion of intuitionistic fuzzy dot $\beta-$sub algebras on $\beta-$algebras and investigate some of their properties.

II. PRELIMINARIES

Definition 2.1: BCK$-$algebra
A BCK$-$algebra is a non$-$empty set X with a costant '0' and a binary operation $'*'$satisfying the following axioms

BCK1: $\{(x*y)*(x*z)\}*(z*y)=0$
BCK2: $\{x*(x*y)\}*y=0$
BCK3: $x*x=0$
BCK4: $x*y=0$ and $y*x=0 \Rightarrow x=y$
BCK5: $0*x=0 \quad \forall\ x,y,z\in X$

Definition 2.2: BCI$-$algebra
A BCI$-$algebra is a non$-$empty set X with a constant '0' and a binary operation $'*'$satisfying the following axioms

BCI1: $\{(x*y)*(x*z)\}*(z*y)=0$
BCI2: $\{x*(x*y)\}*y=0$
BCI3: $x*x=0$
BCI4: $x*y=0$ and $y*x=0 \Rightarrow x=y$

Definition 2.3: B$-$algebra
A B$-$algebra is a non$-$empty set X with a costant '0' and a binary operation $'*'$satisfying the following axioms

B1: $x*x=0$
B2: $x*0=0$
B3: $(x*y)*z=x*\{z*(0*y)\} \quad \forall\ x,y,z\in X$

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Definition 2.4: B − Subalgebra

A non-empty subset S of a B − algebra X is called a B − Subalgebra of X if $x * y \in S$ for any $x, y \in S$

Definition 2.5: B − Homomorphism

A mapping $f : X \to Y$ of a B − algebra X is called B − homomorphism if

$f(x * y) = f(x) * f(y) \quad \forall\ x, y \in X$

Note:1: In B − homomorphism $f(0) = 0$

Definition 2.6: β − algebra

A β − algebra is a non − empty set X with a costant '0' and a binary operations '+' and '−' satisfying the following axioms

$\beta 1$: $x - 0 = x$

$\beta 2$: $(0 - x) + x = 0$

$\beta 3$: $(x - y) - z = x - (z + y) \quad \forall\ x, y, z \in X$

Example 2.6:

Let $X = \{0, 1, 2, 3\}$ be a set with constant '0' and two binary operations '+' and '−' are defined on X with the Cayley table

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

−	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Then $(X, +, -, 0)$ is a β − algebra

Definition 2.7: β − Homomorphism

Let $(X, +, -, 0)$ and $(Y, +, -, 0')$ be two β − algebras. A mapping $f : X \to Y$ is said to be a β − homomorphism if it satisfies the following conditions $f(x + y) = f(x) + f(y)$ and $f(x - y) = f(x) - f(y)$ $\forall\ x, y \in X$

Note:2: In a β − Homomorphism $f(0) = 0'$

Definition 2.8: Fuzzy Set

Let X be a set of universal discourse. A fuzzy set μ in X is defined as a function $\mu : X \to [0,1]$. For each element x in X, $\mu(x)$ is called the membership value of x in X

Definition 2.9: Intersection of two Fuzzy Sets

If μ_1 and μ_2 are two fuzzy sets of X then the intersection $\mu_1 \cap \mu_2$ of μ_1 and μ_2 is defined as $(\mu_1 \cap \mu_2)(x) = \text{Min}\{\mu_1(x), \mu_2(x)\}$

Definition 2.10: Union of two Fuzzy Sets

If μ_1 and μ_2 are two fuzzy sets of X then the union $\mu_1 \cup \mu_2$ of μ_1 and μ_2 is defined as $(\mu_1 \cup \mu_2)(x) = \text{Max}\{\mu_1(x), \mu_2(x)\}$

In general $(\cap \mu_i)(x) = \text{Min}\{\mu_i(x) / i = 1, 2, 3 ...\}$

Definition 2.10: Union of two Fuzzy Sets

If μ_1 and μ_2 are two fuzzy sets of X then the union $\mu_1 \cup \mu_2$ of μ_1 and μ_2 is defined as $(\mu_1 \cup \mu_2)(x) = \text{Max}\{\mu_1(x), \mu_2(x)\}$

In general $(\cup \mu_i)(x) = \text{Max}\{\mu_i(x) / i = 1, 2, 3 ...\}$

Note:3: If μ_1 and μ_2 are two fuzzy sets of X then $\mu_1 \subseteq \mu_2 \Leftrightarrow \mu_1(x) \le \mu_2(x)$

Note:4: If μ is a fuzzy set on X, then $\mu^c(x) = 1 - \mu(x)$

Definition 2.11: Direct product of two Fuzzy Sets

If μ_1 and μ_2 are two fuzzy sets of X_1 and X_2 respectively.

Then the direct product $\mu_1 \times \mu_2$ of μ_1 and μ_2 is defined as the fuzzy set of $X_1 \times X_2$ $(\mu_1 \times \mu_2)(x_1, x_2) = \text{Min}\{\mu_1(x_1), \mu_2(x_2)\} \quad \forall (x_1, x_2) \in X_1 \times X_2$

Definition 2.12: Level Fuzzy Subset

Let μ be a fuzzy set on X. For $t \in [0,1]$, the set $\mu_t = \{x \in X / \mu(x) \ge t\}$ is called level fuzzy subset of μ

Proposition 2.13:

If $t_1 \le t_2$, then $\mu_{t_2} \subseteq \mu_{t_1}$ where μ_{t_2} and μ_{t_1} are any two level fuzzy subsets of μ where μ be a fuzzy set on X

Definition 2.13: Fuzzy Dot $\beta-$Subalgebra of $\beta-$algebra

Let μ be a fuzzy set in a $\beta-$algebra X. Then μ is called a fuzzy dot $\beta-$Subalgebra of X if it satisfies the following conditions

1. $\mu(x+y) \geq \mu(x) \circ \mu(y)$
2. $\mu(x-y) \geq \mu(x) \circ \mu(y) \quad \forall \; x, y \in X$

Example:2.13

Consider the $\beta-$algebra $(X,+,-,0)$ where $X=\{0, 1, 2, 3\}$

Define $\mu : X \to [0,1]$ such that

$$\mu(x)=\begin{cases} 0.6 & \text{if } x=0 \\ 0.7 & \text{if } x=1 \\ 0.3 & \text{if } x=2,3 \end{cases}$$

Then μ is a fuzzy dot $\beta-$subalgebra of X

Theorem:2.1

Every fuzzy $\beta-$subalgebra of X is a fuzzy dot $\beta-$subalgebra of X

Theorem:2.2

If μ_1 and μ_2 are two fuzzy dot $\beta-$subalgebra of X then $\mu_1 \cap \mu_2$ is also a fuzzy dot $\beta-$subalgebra of X

Corollary:2.2

If $\{\mu_i / i=1,2,3....\}$ be a family of fuzzy dot $\beta-$subalgebra of X then $\cap\mu_i$ is also a fuzzy dot $\beta-$subalgebra of X

Theorem:2.3

If μ_1 and μ_2 are two fuzzy dot $\beta-$subalgebra of X then the direct product $\mu_1 \times \mu_2$ is defined by $(\mu_1 \times \mu_2)(x,y)=\mu_1(x) \circ \mu_2(y)$ is also a fuzzy dot $\beta-$subalgebra of $X \times X$

Theorem:2.4

Let $f : X \to Y$ be a homomorphism of a $\beta-$algebra of X into a $\beta-$algebra of Y. If μ is a fuzzy dot $\beta-$algebra of Y then the pre-image of μ, denoted by $f^{-1}(\mu)$ isdefined as $f^{-1}\{\mu(x)\}=\mu\{f(x)\}, \forall x \in X$ is a fuzzy dot $\beta-$subalgebra of X

Theorem:2.5

Let $f : X \to X$ be an endomorphism on a $\beta-$subalgebra of X .If μ is a fuzzy dot $\beta-$algebra of X. Define a fuzzy set $\mu_f : X \to [0,1]$ by $\mu_f(x)=\mu(f(x)) \;\forall x \in X$. Then μ_f is a fuzzy dot $\beta-$algebra of X

Theorem:2.6 For a fuzzy set A of a $\beta-$algebra of X. Let μ_A be a fuzzy relation defined by $\mu_A(x+y)=A(x) \circ A(y)$. Then A is a fuzzy dot $\beta-$subalgebra of X if and only if μ_A is a fuzzy dot $\beta-$subalgebra of $X \times X$

Theorem:2.7

Let X and Y be $\beta-$algebras. Let μ be a fuzzy dot $\beta-$subalgebra of $X \times X$. Define a fuzzy set $P_x(\mu)(x)=\mu(x,0), \forall x \in X$. Then $P_x(\mu)$ is a fuzzy dot $\beta-$subalgebra of X . Also define a fuzzy set $P_y(\mu)$ of Y by $P_y(\mu)(y)=\mu(0,y), \forall y \in Y$ Then $P_y(\mu)$ is a fuzzy dot $\beta-$subalgebra of Y

III. CHAPTER

Intuitionistic Fuzzy dot $\beta-$subalgebras of a $\beta-$algebra

In this section we introduce the notion of Intuitionistic fuzzy dot $\beta-$subalgebras of a $\beta-$algebra and prove some simple theorems

Definition:3.1 Intuitionistic Fuzzy Set

An Intuitionistic fuzzy set A over X is an object having the form $A=\{\langle x, \mu(x), \gamma(x)\rangle / x \in X\}$ where $\mu(x): X \to [0,1]$ and $\gamma(x): X \to [0,1]$ with the condition $0 \leq \mu(x)+\gamma(x) \leq 1, \;\forall x \in X$. The numbers $\mu(x)$ and $\gamma(x)$ denote, respectively , the degree of membership and non-membership of the element $x \in A$. Obviously, when $\gamma(x)=1-\mu(x), \;\forall\; x \in X$, the set A becomes a fuzzy set. For the sake of simplicity, we shall use the symbol $A=(\mu,\gamma)$ for the intuitionistic fuzzy set $A=\{\langle x, \mu(x), \gamma(x)\rangle / x \in X\}$

Properties of Intuitionistic Fuzzy Set

If $A=\{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ an $B=\{\langle x, \mu_B(x), \gamma_B(x)\rangle / x \in X\}$ are any two intuitionistic fuzzy sets of a set X, then

(a). $A \subseteq B \Leftrightarrow$ for all $x \in X$, $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$

(b). $A=B \Leftrightarrow \mu_A(x)=\mu_B(x)$ and $\gamma_A(x)=\gamma_B(x)$

(c). $A \cap B=\{\langle x, (\mu_A \cap \mu_B)(x), (\gamma_A \cap \gamma_B)(x)\rangle\}$ where

$(\mu_A \cap \mu_B)(x) = \text{Min}\{\mu_A(x), \mu_B(x)\}$ and
$(\gamma_A \cap \gamma_B)(x) = \text{Max}\{\gamma_A(x), \gamma_B(x)\}$

(d). $A \cup B = \{\langle x, (\mu_A \cup \mu_B)(x), (\gamma_A \cup \gamma_B)(x)\rangle\}$ where
$(\mu_A \cup \mu_B)(x) = \text{Max}\{\mu_A(x), \mu_B(x)\}$ and
$(\gamma_A \cup \gamma_B)(x) = \text{Min}\{\gamma_A(x), \gamma_B(x)\}$

Definition:3.2
An Intuitionistic fuzzy set A of a $\beta-$algebra X is said to be an intuitionistic fuzzy dot $\beta-$subalgebra of X if it satisfies the following axioms:

IFDβSA1: $\mu(x+y) \geq \mu(x) \circ \mu(y)$

IFDβSA2: $\mu(x-y) \geq \mu(x) \circ \mu(y)$

IFDβSA3: $\gamma(x+y) \leq \gamma(x) \circ \gamma(y)$

IFDβSA4: $\gamma(x-y) \leq \gamma(x) \circ \gamma(y)$

Example:3.2
Consider the $\beta-$algebra $X = \{0,1,2,3\}$. Define $\mu: X \to [0,1]$ and $\gamma: X \to [0,1]$ such that

$$\mu(x) = \begin{cases} 0.6 & \text{if } x = 0 \\ 0.7 & \text{if } x = 1 \\ 0.3 & \text{if } x = 2,3 \end{cases} \quad \& \quad \gamma(x) = \begin{cases} 0.4 & \text{if } x = 0 \\ 0.3 & \text{if } x = 1 \\ 0.7 & \text{if } x = 2,3 \end{cases}$$

Then $A = (\mu, \gamma)$ is a IFDβSA of X

Theorem:3.1
Every intuitionistic fuzzy $\beta-$subalgebra of X is a intuitionistic fuzzy dot $\beta-$subalgebra of X

Proof:
Let $A = (\mu, \gamma)$ be a intuitionistic fuzzy $\beta-$subalgebra of X . Then

$\mu(x+y) \geq \text{Min}\{\mu(x), \mu(y)\} \geq \mu(x) \circ \mu(y)$

$\gamma(x+y) \leq \text{Max}\{\gamma(x), \gamma(y)\} \leq \gamma(x) \circ \gamma(y)$

$\mu(x-y) \geq \text{Min}\{\mu(x), \mu(y)\} \geq \mu(x) \circ \mu(y)$

$\gamma(x-y) \leq \text{Max}\{\gamma(x), \gamma(y)\} \leq \gamma(x) \circ \gamma(y)$

Therefore $A = (\mu, \gamma)$ is a a intuitionistic fuzzy dot $\beta-$subalgebra of X

Theorem: 3.2
If $A = (\mu_1, \gamma_1)$ and $B = (\mu_2, \gamma_2)$ be any two intuitionistic fuzzy dot $\beta-$subalgebra of X then $A \cap B$ is also a intuitionistic fuzzy dot $\beta-$subalgebra of X Proof: For any $x, y \in X$,

$(\mu_1 \cap \mu_2)(x+y) = \text{Min}\ \{\mu_1(x+y), \mu_2(x+y)\}$

$\geq \text{Min}\ \{\mu_1(x) \circ \mu_1(y), \mu_2(x) \circ \mu_2(y)\}$

$\geq [\text{Min}\{\mu_1(x), \mu_2(x)\}] \circ [\text{Min}\{\mu_1(y), \mu_2(y)\}]$

$= (\mu_1 \cap \mu_2)(x) \circ (\mu_1 \cap \mu_2)(y)$

Hence

$(\mu_1 \cap \mu_2)(x+y) \geq (\mu_1 \cap \mu_2)(x) \circ (\mu_1 \cap \mu_2)(y)$ …………(1)

$(\mu_1 \cap \mu_2)(x-y) = \text{Min}\ \{\mu_1(x-y), \mu_2(x-y)\}$

$\geq \text{Min}\ \{\mu_1(x) \circ \mu_1(y), \mu_2(x) \circ \mu_2(y)\}$

$\geq [\text{Min}\{\mu_1(x), \mu_2(x)\}] \circ [\text{Min}\{\mu_1(y), \mu_2(y)\}]$

$= (\mu_1 \cap \mu_2)(x) \circ (\mu_1 \cap \mu_2)(y)$

Hence $(\mu_1 \cap \mu_2)(x-y) \geq (\mu_1 \cap \mu_2)(x) \circ (\mu_1 \cap \mu_2)(y)$ …………(2)

$(\gamma_1 \cap \gamma_2)(x+y) = \text{Max}\ \{\gamma_1(x+y), \gamma_2(x+y)\}$

$\leq \text{Max}\ \{\gamma_1(x) \circ \gamma_1(y), \gamma_2(x) \circ \gamma_2(y)\}$

$\leq [\text{Max}\{\gamma_1(x), \gamma_2(x)\}] \circ [\text{Max}\{\gamma_1(y), \gamma_2(y)\}]$

$= (\gamma_1 \cap \gamma_2)(x) \circ (\gamma_1 \cap \gamma_2)(y)$

Hence $(\gamma_1 \cap \gamma_2)(x+y) \leq (\gamma_1 \cap \gamma_2)(x) \circ (\gamma_1 \cap \gamma_2)(y)$ ………….(3)

$(\gamma_1 \cap \gamma_2)(x-y) = \text{Max}\ \{\gamma_1(x-y), \gamma_2(x-y)\}$

$\leq \text{Max}\ \{\gamma_1(x) \circ \gamma_1(y), \gamma_2(x) \circ \gamma_2(y)\}$

$\leq [\text{Max}\{\gamma_1(x), \gamma_2(x)\}] \circ [\text{Max}\{\gamma_1(y), \gamma_2(y)\}]$

$= (\gamma_1 \cap \gamma_2)(x) \circ (\gamma_1 \cap \gamma_2)(y)$

Hence $(\gamma_1 \cap \gamma_2)(x-y) \leq (\gamma_1 \cap \gamma_2)(x) \circ (\gamma_1 \cap \gamma_2)(y)$ ………(4)

From (1),(2) and (3), (4)

$A \cap B$ is also a intuitionistic fuzzy dot $\beta-$subalgebra of X

Corollary:3.2
If $A = \{(\mu_i, \gamma_i) / i = 1,2,3...\}$ be a family of intuitionistic fuzzy dot $\beta-$subalgebra of X, then $\mu_i \cap \gamma_i$ is also a intuitionistic fuzzy dot $\beta-$subalgebra of X

Theorem: 3.3
Let $A = (\mu_1, \gamma_1)$ and $B = (\mu_2, \gamma_2)$ be any two intuitionistic fuzzy dot $\beta-$subalgebra of X then $(A \times B)(x, y) = A(x) \circ B(y)$ is also a intuitionistic fuzzy dot $\beta-$subalgebra of $X \times X$

Proof:

Let $X = X \times X$ and let $\mu = \mu_1 \times \mu_2$, $\gamma = \gamma_1 \times \gamma_2$

$\mu(x+y) = \mu\{(x_1, x_2) + (y_1, y_2)\}$

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$$
\begin{aligned}
&= \mu(x_1+y_1, x_2+y_2) \\
&= (\mu_1\times\mu_2)(x_1+y_1, x_2+y_2) \\
&= \mu_1(x_1+y_1)\circ\mu_1(x_2+y_2) \\
&\geq \mu_1(x_1)\circ\mu_2(x_2)\circ\mu_1(y_1)\circ\mu_2(y_2) \\
&= \mu_1(x_1)\circ\mu_2(x_2)\circ\mu_1(y_1)\circ\mu_2(y_2) \\
&= (\mu_1\times\mu_2)(x_1,x_2)\circ(\mu_1\times\mu_2)(y_1,y_2) \\
&= \mu(x)\circ\mu(y)
\end{aligned}
$$

$$
\begin{aligned}
\gamma(x+y) &= \gamma\{(x_1,x_2)+(y_1,y_2)\} \\
&= \gamma(x_1+y_1, x_2+y_2) \\
&= (\gamma_1\times\gamma_2)(x_1+y_1, x_2+y_2) \\
&= \gamma_1(x_1+y_1)\circ\gamma_1(x_2+y_2) \\
&\leq \gamma_1(x_1)\circ\gamma_2(x_2)\circ\gamma_1(y_1)\circ\gamma_2(y_2) \\
&= \gamma_1(x_1)\circ\gamma_2(x_2)\circ\gamma_1(y_1)\circ\gamma_2(y_2) \\
&= (\gamma_1\times\gamma_2)(x_1,x_2)\circ(\gamma_1\times\gamma_2)(y_1,y_2) \\
&= \gamma(x)\circ\gamma(y)
\end{aligned}
$$

$$
\begin{aligned}
\mu(x-y) &= \mu\{(x_1,x_2)-(y_1,y_2)\} \\
&= \mu(x_1-y_1, x_2-y_2) \\
&= (\mu_1\times\mu_2)(x_1-y_1, x_2-y_2) \\
&= \mu_1(x_1-y_1)\circ\mu_1(x_2-y_2) \\
&\geq \mu_1(x_1)\circ\mu_2(x_2)\circ\mu_1(y_1)\circ\mu_2(y_2) \\
&= \mu_1(x_1)\circ\mu_2(x_2)\circ\mu_1(y_1)\circ\mu_2(y_2) \\
&= (\mu_1\times\mu_2)(x_1,x_2)\circ(\mu_1\times\mu_2)(y_1,y_2) \\
&= \mu(x)\circ\mu(y)
\end{aligned}
$$

$$
\begin{aligned}
\gamma(x-y) &= \gamma\{(x_1,x_2)-(y_1,y_2)\} \\
&= \gamma(x_1-y_1, x_2-y_2) \\
&= (\gamma_1\times\gamma_2)(x_1-y_1, x_2-y_2) \\
&= \gamma_1(x_1-y_1)\circ\gamma_1(x_2-y_2) \\
&\leq \gamma_1(x_1)\circ\gamma_2(x_2)\circ\gamma_1(y_1)\circ\gamma_2(y_2) \\
&= \gamma_1(x_1)\circ\gamma_2(x_2)\circ\gamma_1(y_1)\circ\gamma_2(y_2) \\
&= (\gamma_1\times\gamma_2)(x_1,x_2)\circ(\gamma_1\times\gamma_2)(y_1,y_2) \\
&= \gamma(x)\circ\gamma(y)
\end{aligned}
$$

Hence $A\times B$ is also a intuitionistic fuzzy dot $\beta-$ subalgebra of $X\times X$

Theorem: 3.4

Let $f: X\to Y$ be a homomorphism of a $\beta-$algebra of X into a $\beta-$algebra of Y. If A is a intuitionistic fuzzy dot $\beta-$algebra of Y, then the pre-image of A, denoted by $f^{-1}(A)$ is defined as $f^{-1}\{A(x)\} = A\{f(x)\}$, $\forall\ x\in X$, is a intuitionistic fuzzy dot $\beta-$subalgebra of X

Proof:

Let $A=(\mu,\gamma)$ be aintuitionistic fuzzy dot $\beta-$ subalgebra of Y and let $x,y\in X$. Then

$$
\begin{aligned}
\{f^{-1}(\mu)\}(x+y) &= \mu\{f(x+y)\} \\
&= \mu(f(x)+f(y)) \\
&\geq \mu(f(x))\circ\mu(f(y)) \\
&= \{f^{-1}(\mu)(x)\}\circ\{f^{-1}(\mu)(y)\}
\end{aligned}
$$

Also $\{f^{-1}(\mu)\}(x-y) = \mu\{f(x-y)\}$

$$
\begin{aligned}
&= \mu(f(x)-f(y)) \\
&\geq \mu(f(x))\circ\mu(f(y)) \\
&= \{f^{-1}(\mu)(x)\}\circ\{f^{-1}(\mu)(y)\}
\end{aligned}
$$

$$
\begin{aligned}
\{f^{-1}(\gamma)\}(x+y) &= \gamma\{f(x+y)\} \\
&= \gamma(f(x)+f(y)) \\
&\leq \gamma(f(x))\circ\gamma(f(y)) \\
&= \{f^{-1}(\gamma)(x)\}\circ\{f^{-1}(\gamma)(y)\}
\end{aligned}
$$

Also $\{f^{-1}(\gamma)\}(x-y) = \gamma\{f(x-y)\}$

$$
\begin{aligned}
&= \gamma(f(x)-f(y)) \\
&\leq \gamma(f(x))\circ\gamma(f(y)) \\
&= \{f^{-1}(\gamma)(x)\}\circ\{f^{-1}(\gamma)(y)\}
\end{aligned}
$$

Hence $f^{-1}(A)$ is a intuitionistic fuzzy dot $\beta-$ subalgebra of X

Theorem:3.5

Let $f: X\to Y$ be an endomorphism on a $\beta-$algebra of X .If A be a intuitionistic fuzzy dot $\beta-$ subalgebra of X .Define a intuitionistic fuzzy set $\mu_f: X\to[0,1]$ by $\mu_f(x)=\mu(f(x))$ and $\gamma_f: X\to[0,1]$ by $\gamma_f(x)=\gamma(f(x))$, $\forall\ x\in X$ .Then $A=(\mu_f,\gamma_f)$ is a intuitionistic fuzzy dot $\beta-$subalgebra of X

Proof:

Let $x,y\in X$. Then

$$
\begin{aligned}
\mu_f(x+y) &= \mu(f(x+y)) \\
&= \mu(f(x)+f(y)) \\
&\geq \mu(f(x))\circ\mu(f(y)) \\
&= \mu_f(x)\circ\mu_f(y)
\end{aligned}
$$

Also, $\mu_f(x-y) = \mu(f(x-y))$

$$
= \mu(f(x)-f(y))
$$

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$$\geq \mu\left(f(x)\right)\circ\mu\left(f(y)\right)$$
$$=\mu_f(x)\circ\mu_f(y)$$
$$\gamma_f\left(x+y\right)=\gamma\left(f(x+y)\right)$$
$$=\gamma\left(f(x)+f(y)\right)$$
$$\leq\gamma\left(f(x)\right)\circ\gamma\left(f(y)\right)$$
$$=\gamma_f(x)\circ\gamma_f(y)$$

Also,
$$\gamma_f\left(x-y\right)=\gamma\left(f(x-y)\right)$$
$$=\gamma\left(f(x)-f(y)\right)$$
$$\leq\gamma\left(f(x)\right)\circ\gamma\left(f(y)\right)$$
$$=\gamma_f(x)\circ\gamma_f(y)$$

Hence $A=\left(\mu_f,\gamma_f\right)$ is a intuitionistic fuzzy dot $\beta-$sub algebra of X

IV. CONCLUSION

In this chapter we introduce the concept of intuitionistic fuzzy dot $\beta-$sub algebra of $\beta-$algebras and investigate some of their useful properties. In my opinion, these definitions and results can be extended to other algebraic systems also.

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