IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology
Vol. 3, Issue 3, March 2016

Approach for the analysis of FGM plate under sinusoidal varying pressure load by RBF method

Rahul Kumar[1], S.C Srivastava [2], Jeeoot Singh[3]

PG Student, Department of Mechanical Engineering, B.I.T., Mesra, India[1]

Associate professor, Department of Production Engineering, B.I.T., Mesra, India [2]

Associate professor, Department of Mechanical Engineering, B.I.T., Mesra, India[3]

Abstract: In this technical paper, analysis of functionally graded plates is presented. The bending response of simply supported functionally graded plate subjected to mechanical sinusoidal varying load is evaluated using higher order shear deformation theory in association with meshless approach. The governing differential equation of the plate is obtained using energy principle. Multiquadric radial basic function based on meshless method is applied for discreatization of the equation. The effects of material gradient index, span to thickness ratio on the sinusoidal varying pressure load of functionally graded material plates are highlighted.

Keywords: FGM, plates, Meshless method, MQ RBF, bending, sinusoidal varying load.

I. INTRODUCTION

The past few decades have seen composite materials taken the emerging role in the fields of structural applications. One of the most demanded composite material is functionally graded material. The term FGMs was originated in the mid-1980s by a group of scientists in Japan. FGMs attain the multistructural status from their property gradation. By gradually varying the volume fraction of constituent materials, their material properties exhibit a smooth and continuous change from one layer to another layer. The ceramic and metal constituents of FGMs are able to withstand high-temperature environments due to their better thermal resistance characteristics and also provide stronger mechanical performance. In the last few decades many literatures have been attracted by the analysis of FGM plates.

A comprehensive review of various analytical and numerical models for predicting the bending, buckling and vibration responses of FG plates under mechanical and thermal loadings was recently carried out by Swaminathan et al. [1]. Roque et al. [2] employed the bending behavior of FG plates using a meshless collocation method with multiquadric RBFs. Pandya and Kant [3] used higher order shear deformation theory with 7 unknowns and accounting for a cubic variation of the in-plane displacements. For analyzing purpose many researchers show their interest in meshless method due to their simplicity requirements.

Meshless methods use a set of nodes scattered within the problem domain and sets of nodes scattered on the boundaries of the domain to represent the problem domain and its boundaries. Meshless methods not required mesh and information on the relationship between the nodes is required. Free vibration analysis of laminated composite plates by a meshless local collocation method based spline RBF was carried out by Xiang and Kang [4] . Liu and Chen [5] introduced free vibration analysis of thin plates of complicated shapes using mesh free method.

II. MATHEMATICAL FORMULATION

A rectangular shape plate of edge length a, b along x, y axes respectively and thickness h is the thickness along z axis whose mid plane is coinciding with x-y plane of the coordinate system is considered. The diagram of rectangular shaped functionally graded material (FGM) plate in rectangular coordinate system is shown in Figure 1.

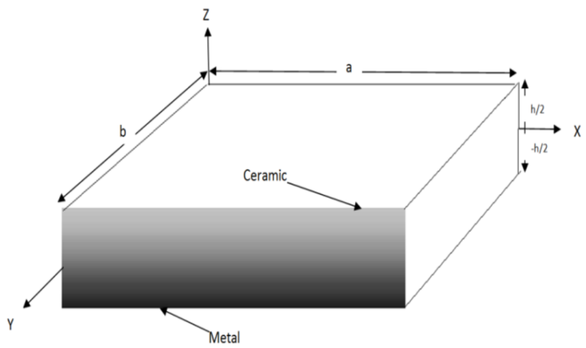

Fig 1. Geometry of rectangular FGM plate in rectangular coordinate system

The homogenization technique considered in this work is the law of mixtures, which provides the following elastic properties at each material layer. The top surface of the plate is ceramic rich and the bottom surface is metal rich.

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^n \quad (1)$$

Where 'n' is exponent governing the material properties along the thickness direction known as volume fraction exponent or grading index,

The volume fraction of the metal phase is obtained by

$$V_m(z) = 1 - V_c(z) \quad (2)$$

The material property gradation through the thickness of the plate is assumed to have the following form

 DOI 10.17148/IARJSET.2016.3331

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
Vol. 3, Issue 3, March 2016

$$E(z)=\left[E_c-E_m\right]\left(\frac{2z+h}{2h}\right)^n+E_m \qquad (3)$$

Here E denote the modulus of elasticity of FGM structure, while these parameters come with subscript m or c represent the material properties for pure metal and pure ceramic plate respectively., h is the thickness of the plate, E_m and E_c are the corresponding Young's modulus of elasticity of metal and ceramic and z is the thickness coordinate.

The displacement field at any point in the plate made up of uniform thickness is expressed as:

$$\begin{Bmatrix} U_x \\ U_y \\ U_z \end{Bmatrix} = \begin{Bmatrix} u_x(x,y) - z\frac{\partial u_z(x,y)}{\partial x} + \left[\frac{h}{\pi}\sin(\frac{\pi z}{h}) - z\right]\psi_x(x,y) \\ u_y(x,y) - z\frac{\partial u_z(x,y)}{\partial y} + \left[\frac{h}{\pi}\sin(\frac{\pi z}{h}) - z\right]\psi_y(x,y) \\ u_z(x,y) \end{Bmatrix} \qquad (4)$$

The constitutive stress-strain relations for any FGM plate are expressed as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

Where, the parameters Q_{ij} are the stiffness coefficients and are expressed in terms of elastics constants as:

$$\bar{Q}_{11} = \bar{Q}_{22} = \frac{E}{(1-\nu^2)}, \quad \bar{Q}_{12} = \frac{\nu E}{(1-\nu^2)}$$

$$\bar{Q}_{44} = \bar{Q}_{55} = \bar{Q}_{66} = G = \frac{E}{2(1+\nu)}$$

The governing differential equations of plate are obtained using energy equation, in mathematical form it is expressed as:

$$\int_{t_1}^{t_2} \delta(U+V)dt = 0 \qquad (5)$$

Where, U = Strain energy
V = work done due to transverse load

The strain energy of the plate due to internal stress resultants is expressed as:

$$U = \frac{1}{2}\int_{Volume} \left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{xy}\gamma_{xy} + \sigma_{yz}\gamma_{yz} + \sigma_{xz}\gamma_{xz}\right)dxdydz \qquad (6)$$

$$V = \int_{Area} u_z q_z dxdy \qquad (7)$$

The governing differential equations of plate are obtained using Hamilton's principle and expressed as :

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + q_z = 0$$

$$\frac{\partial M^f_{xx}}{\partial x} + \frac{\partial M^f_{xy}}{\partial y} - Q^f_x = 0 \qquad (8)$$

$$\frac{\partial M^f_{xy}}{\partial x} + \frac{\partial M^f_{yy}}{\partial y} - Q^f_y = 0$$

The force and moment resultants in the plate and plate stiffness coefficients are expressed as:

$$N_{ij}, M_{ij}, M^f_{ij} = \int_{-h/2}^{+h/2} \left(\sigma_{ij}, z\sigma_{ij}, z.\left[\frac{h}{\pi}\sin(\frac{\pi z}{h}) - z\right]\sigma_{ij}\right)dz \qquad (9)$$

$$Q^f_x, Q^f_y = \int_{-h/2}^{+h/2} \left(\sigma_{xz}, \sigma_{yz}\right)\left(\frac{\partial\left[\frac{h}{\pi}\sin(\frac{\pi z}{h}) - z\right]}{\partial z}\right)dz \qquad (10)$$

$$A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \int_{-h/2}^{h/2}\left\{Q(z)\times\left(1, z, z^2, z.\left[\frac{h}{\pi}\sin(\frac{\pi z}{h}) - z\right], z\times z.\left[\frac{h}{\pi}\sin(\frac{\pi z}{h}) - z\right], z.\left[\frac{h}{\pi}\sin(\frac{\pi z}{h}) - z\right]^2\right)\right\}dz$$

i, j = 1, 2, 6

$$A_{ij} = \int_{-h/2}^{h/2}\left\{Q(z)\times\left(\frac{\partial\left[\frac{h}{\pi}\sin(\frac{\pi z}{h}) - z\right]}{\partial z}\right)^2\right\}dz$$

i, j = 4, 5

$$where, \quad Q(z) = \left(\left(\left[Q^c_{ij} - Q^m_{ij}\right]\right)\left(\frac{2z+h}{2h}\right)^n + Q^m_{ij}\right)$$

The boundary conditions for an arbitrary edge with simply supported conditions are as follows:

$$x = 0, a : u_y = 0; \psi_y = 0; u_z = 0; M_{xx} = 0; N_{xx} = 0$$

$$y = 0, b : u_x = 0; \psi_x = 0; u_z = 0; M_{yy} = 0; N_{yy} = 0$$

1. SOLUTION METHODOLOGY

The governing differential equations (8) are expressed in terms of displacement functions. Radial basis function based formulation works on the principle of interpolation of scattered data over entire domain. A 2D rectangular domain having NB boundary nodes and ND interior nodes is shown in Figure-2.

Copyright to IARJSET

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
Vol. 3, Issue 3, March 2016

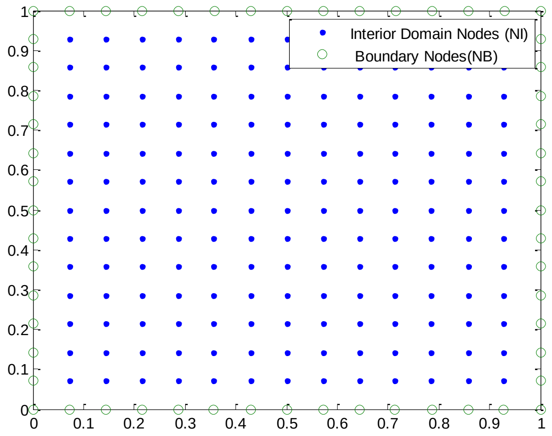

Fig 2. An arbitrary two dimensional domains

The variable $u_x, u_y, u_z, \psi_x, \psi_y$ can be interpolated in form of radial distance between nodes. The solution of the linear governing differential equations (8) is assumed in terms of multiquadric radial basis function for nodes 1:N, as;

$$u_x, u_y, u_z, \psi_x, \psi_y = \sum_{j=1}^{N} (\alpha_j^{u_x}, \alpha_j^{u_y}, \alpha_j^{u_z}, \alpha_j^{\psi_x}, \alpha_j^{\psi_y})\ g\left(\left\|X - X_j\right\|, m, c\right)$$

Where, N is total numbers of nodes which is equal to summation of boundary nodes NB and domain interior nodes ND $g\left(\left\|X - X_j\right\|, m, c\right)$. is multiquadric radial basis function expressed as $g = \left(r^2 + c^2\right)^m$,

$(\alpha_j^{u_x}, \alpha_j^{u_y}, \alpha_j^{u_z}, \alpha_j^{\psi_x}, \alpha_j^{\psi_y})$ are unknown coefficients. $\left\|X - X_j\right\|$ is the radial distance between two nodes.

Where, $r = \left\|X - X_j\right\| = \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2}$ and m, c are shape parameter. The value of 'm' and 'c' taken here is 0.5 and 1.3/(N)0.25.

2. COMPUTATION AND DISCUSSION OF RESULTS:

The study here has been focused on the flexural response of simply supported square functionally graded plates under line transverse loads. A RBF based meshless code in MATLAB 2013 is developed. Several examples have been analysed and the computed results are compared. Based on convergence study, a 15×15 node is used throughout the study. The material properties of FGMs have been taken as follows:

$$\text{Ceramic } E_c = 151 GPa, \quad \nu_c = 0.3$$

$$\text{Aluminium (Al) } E_m = 70 GPa, \quad \nu_m = 0.3$$

In order to show the accuracy and efficiency of the present solution methodology, detailed convergence studies for simply supported FGM plate (a/h=20) is carried out.

The convergences of the deflection are shown in Fig. 3. It can be seen that convergence achieved is within 1 % at 15×15 nodes.

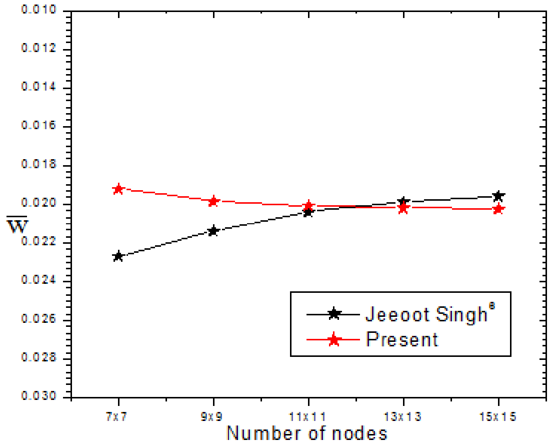

Fig. 3 Convergence study for deflection $\overline{w}$ of a simply supported FGM plate (a/h = 20, 'n'=2)

Table1 Effect of gradation index 'n' on deflection, stresses and Moments of a simply supported FGM Plate(a/h=5)

	'n'					
	0	0.5	1	2	5	Metal
$\overline{w}$	0.013	0.017	0.019	0.021	0.023	0.029
$\overline{\sigma}_{xx}$	2.692	3.137	3.374	3.612	4.005	5.805
$\overline{\sigma}_{yy}$	2.692	3.137	3.374	3.612	4.005	5.805
$\overline{\sigma}_{xy}$	1.428	1.663	1.789	1.914	2.118	3.078
$\overline{\sigma}_{xz}$	0.406	0.505	0.554	0.601	0.661	0.875
M_{xx}	0.033	0.042	0.046	0.049	0.054	0.072
M_{yy}	0.033	0.042	0.046	0.049	0.054	0.072
M_{xy}	0.018	0.022	0.024	0.026	0.029	0.038
M^f_{xx}	0.033	0.042	0.046	0.049	0.054	0.072
M^f_{yy}	0.033	0.042	0.046	0.049	0.054	0.072
M^f_{xy}	0.018	0.022	0.024	0.026	0.029	0.038

Table1 Effect of span to thickness ratio on deflection, stresses and Moments of a simply supported FGM Plate (n=2)

	a/h				
	5	10	20	50	100
$\overline{w}$	0.0248	0.0214	0.0205	0.0203	0.0202
$\overline{\sigma}_{xx}$	3.8887	1.8792	0.9314	0.3716	0.1858
$\overline{\sigma}_{yy}$	3.8887	1.8792	0.9314	0.3716	0.1858
$\overline{\sigma}_{xy}$	2.0052	0.9717	0.4818	0.1922	0.0961
$\overline{\sigma}_{xz}$	0.6668	0.1672	0.0754	0.1066	0.1110
M_{xx}	0.0491	0.0246	0.0123	0.0049	0.0025
M_{yy}	0.0491	0.0246	0.0123	0.0049	0.0025
M_{xy}	0.0288	0.0132	0.0065	0.0026	0.0013
M^f_{xx}	0.0491	0.0246	0.0123	0.0049	0.0025
M^f_{yy}	0.0491	0.0246	0.0123	0.0049	0.0025
M^f_{xy}	0.0288	0.0132	0.0065	0.0026	0.0013

Copyright to IARJSET

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
Vol. 3, Issue 3, March 2016

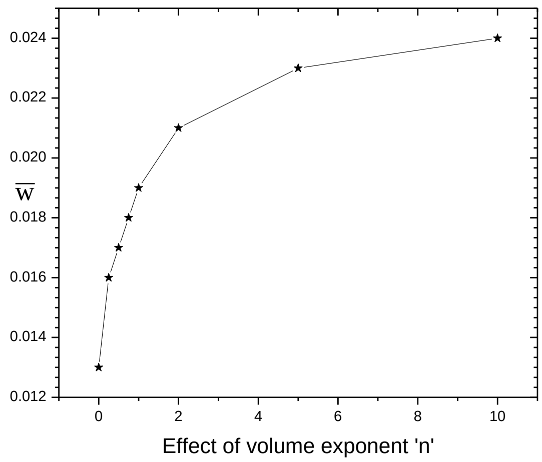

Fig 4. Effect of grading index 'n' on deflection of a square FGM plate (a/h=5)

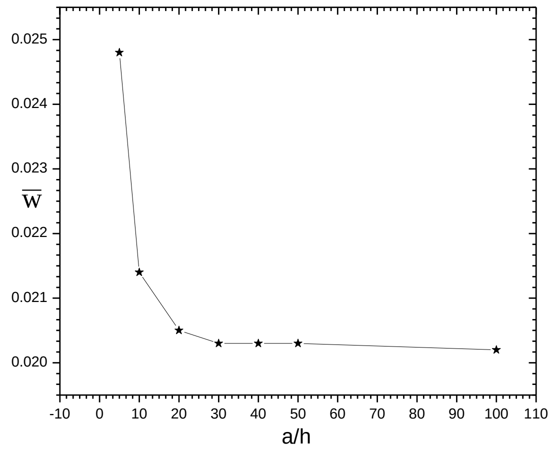

Fig 7 Effect of span to thickness ratio on deflection of a square FGM plate (‘n’=2)

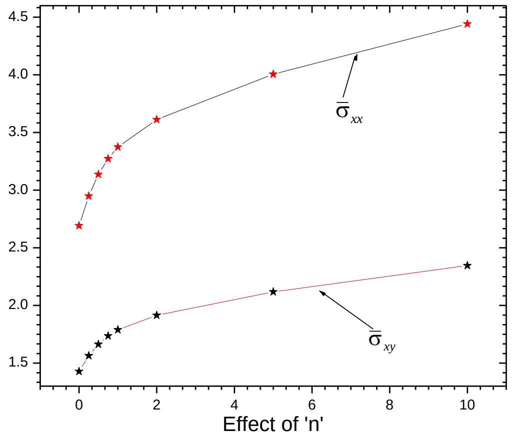

Fig 5. Effect of grading index 'n' on stresses of a square FGM plate (a/h=5)

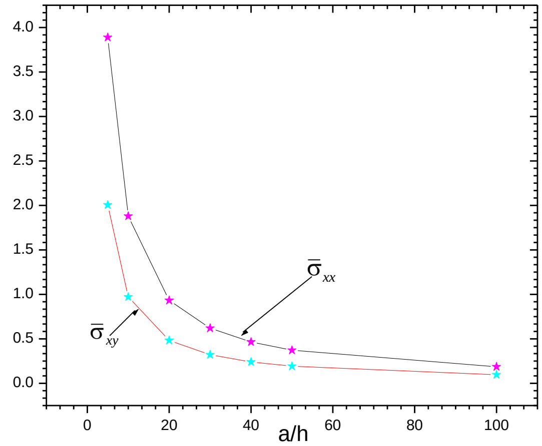

Fig 8 Effect of span to thickness ratio on stresses of a square FGM plate (‘n’=2)

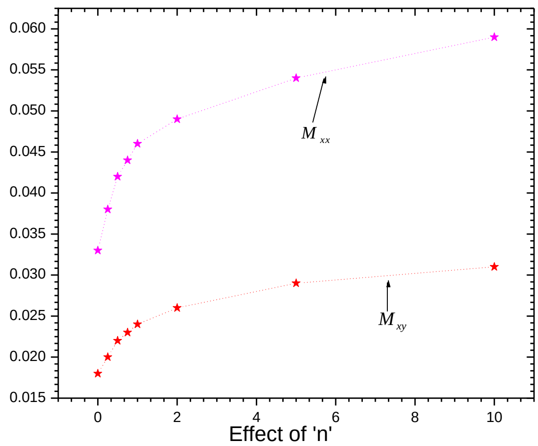

Fig 6 Effect of grading index 'n' on M_{xx} and M_{xy} of a square FGM plate (a/h=5)

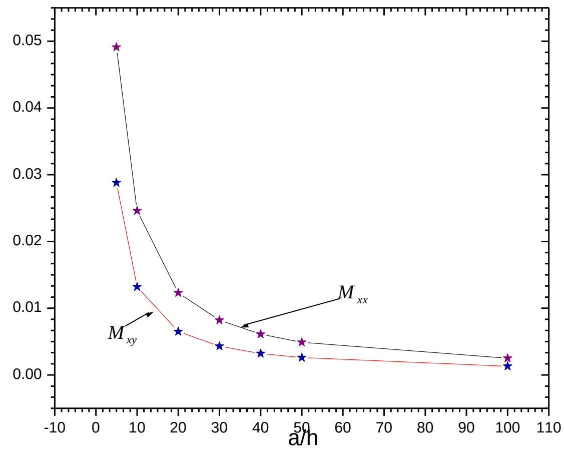

Fig 9 Effect of span to thickness ratio on moments of a square FGM plate (‘n’=2)

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
Vol. 3, Issue 3, March 2016

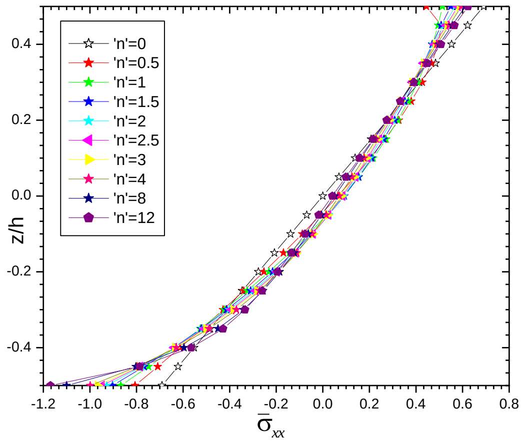

Fig 10 Effect of grading index 'n' on normalized stress $\bar{\sigma}_{xx}$ of simply supported square FGM plate along the thickness

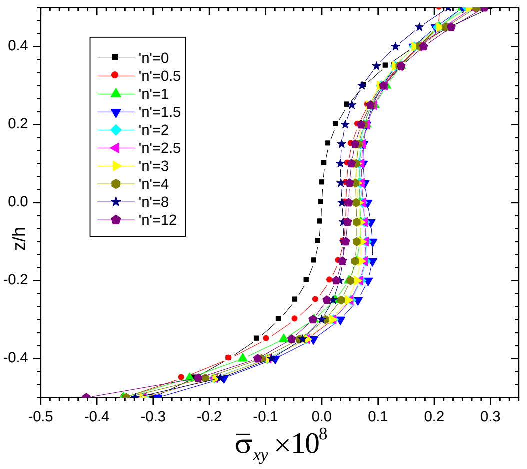

Fig 11 Effect of grading index 'n' on normalized stress $\bar{\sigma}_{xy}$ of simply supported square FGM plate along the thickness

The result obtained for deflection, stresses and moments due to different span ratio for simply supported FGM plates with gradation index 2 is shown in table 1 and table 2 shows the effect of gradation index 'n' for a thick simply supported FGM plates.

It is observed from Fig 4, Fig 5 and Fig 6 that the effect of grading index is more prominent when the value of n is less than 2 for deflection stresses and moments respectively. Fig 7 shows the variation in deflection become almost negligible as plates become thinner (i.e a/h > 30). Fig 8 and Fig 9 shows the effect of span to thickness ratio on stresses and moments of a square FGM plate with 'n'=2. However it is more prominent for thick plate.

Fig 10and11 represent the through thickness variation of stresses for different values of gradation index 'n'.

III.CONCLUSION

Bending response of functionally graded material plate (FGM) is presented using shear deformation theory. The effect of span to thickness ratio decreases for $a/h \geq 30$. The effect of gradation index 'n' is prominent for lesser values of 'n' and decreases as 'n' increases. The present results can be used for validation purpose. Present solution mythology is good for obtaining the result and the concentrated load. The same can be extended for other types of concentrated load like sinusoidal varying line load, point load, patch load etc.

REFERENCES

[1] Swaminathan K, Naveenkumar DT, Zenkour AM, Carrera E. Stress, vibration and buckling analyses of FGM plates-A state-of-the-art review. Compos Struct., 31, 120-10–, 2015
[2] Roque CMC, Ferreira AJM, Jorge RMN. A radial basis function approach for the free vibration analysis of functionally graded plates using a refined theory. J Sound Vib,300(3),1048–70, 2007
[3] Pandya BN, Kant T. Higher-order shear deformable theories for flexure of sandwich plates-finite element evaluations. Int J Solids Struct., 24(12), 1267–86, 1988
[4] Xiang S, Kang GW. Local thin plate spline collocation for free vibration analysis of laminated composite plates. Eur J Mech-A/Solids,33, 24–30, 2012
[5] Liu GR, Chen XL. A mesh-free method for static and free vibration analysis of thin plates of complicated shape. J Sound Vib, 241, 839–55, 2001.
[6] Jeeoot Singh "Some studies on Linear and Nonlinear analysis of rectangular plates using RBF based meshfree method," PHD thesis, page 82, 2012.

Copyright to IARJSET
DOI 10.17148/IARJSET.2016.3331