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Estimation of Statistical Energy Analysis Parameters of Gearbox

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Abstract: Transmission error generated in the gears leads to the whine noise which can result in to the undesirable accoustic performance of the gearbox. Determination of vibro-acoustic behavior of such a component at the early stages of design can result in developing an acoustically sound gearbox. The SEA provides a better analytical solution for such a problem over the FEM and other deterministic method. The statistical approach eliminates the possibility of uncertainty in the results. The main focus in the SEA modeling of the complex system is to find out the loss factors. This paper discuss the method for the determination of loss factors which can be used to study the vibro-acoustic behavior of the geared rotor system.

Keywords: Statistical Energy Analysis (SEA), Vibroaccoustic, Gearbox, Loss Factor.

I. INTRODUCTION

Statistical Energy Analysis (SEA) is a method developed to study the transmission of sound and vibration through complex vibro-acoustic structures.[2] SEA is generally applied in high frequency noise analysis which will reduce the computational efforts and cost over other methods like FEM and Lumped parameter method[1][3]. The SEA predicts the behavior of vibro-acoustic structures for a high frequency narrow width frequency band, which are expressed in terms of parameters like coupling loss factor, modal density, reflection factor, radiation loss factor etc.[3] SEA predicts the flow of energy through the subsystem of the main vibro-acoustic system.[4][19] The SEA is applied in numerous areas like interior noise prediction and sound package design in automobiles, aircraft, rotorcraft and trains. The SEA provides prediction of radiated sound in marine, vehicles and spacecraft.[18].

Modern downsizing technology led to a compact transmission devices. The geared rotor system exhibits best characteristics for such applications[1]. As the speed Increases, due to the transmission error in gear meshing leads to higher frequency vibration, this leads to structure borne noise. When a geared rotor system supported by bearings, subjected to transmission error generates vibro-acoustic energy. Part of it is absorbed, transmitted, radiated and part of it is converted in the form of noise. Applying SEA to this problem gives the prediction of SEA parameters which can be used to predict the vibro-acoustic behaviour of the system[26][27][28].

A. Role Of Geared –Rotor System

A simple geared rotor system represents the family of a gearbox so that the gearboxe can be analysed easily and the results obtained from the analysis can be used to study the complex gearboxes[22][4]. Basically the gearbox play's a role of transmission. It may be power or motion transmission. In automotive its work is to transmit the power but in the utility machines these are generally used to transmit motion. The excessive use of the gearbox and the manufacturing errors can lead to the transmission error[17][28]. The transmission error is nothing but the deviation of the tooth profile from its predetermined position. Simply this can be termed as inappropriate changes in the backlash which leads to the error[28]. This error will create the harmonic excitations in the geared rotor system which leads to the whine noise[15].

1.2 Justification Of Using SEA

Vibration analysis of complex structures can be carried out by using the various deterministic methods like FEM and the lumped parameter methods but the difficulty associated with such a methods is those are applicable effectively in lower resonant modes. The FEM becomes less effective at higher resonant modes due to smaller mesh size, as well as the limited DOF in lumped parameter method leads to high computational effort[18][19][17]. It has been observed that complex structures such as gearbox tends to have high resonant modes at high speeds and FEM and LPM fails to provide certainity in results. For such application the statistical or asymptotic methods leads to better results[22][4].

II. SEA FORMULATION

The SEA modelling is relating the modal parameters with the SEA parameters such that the SEA model defined should provide the vibro-acoustic information of the system. Fundamental principles of SEA, is that the average power flow between two coupled groups of dynamical modes is proportional to the difference in the average modal energies. This makes it possible to analyze the dynamical response of a system consisting of many resonant modes in a certain frequency range by dividing the modes into groups (subsystems) and considering a power balance equation for each mode group. The analysis is statistical in that the expected value and variance of the power flow are evaluated assuming a statistical

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distribution in the resonance frequencies of the subsystem modes. The SEA modelling procedure can be broadly specified into three stages as,

1. System model definition
2. SEA parameters evaluation
3. Evaluation of response variables.

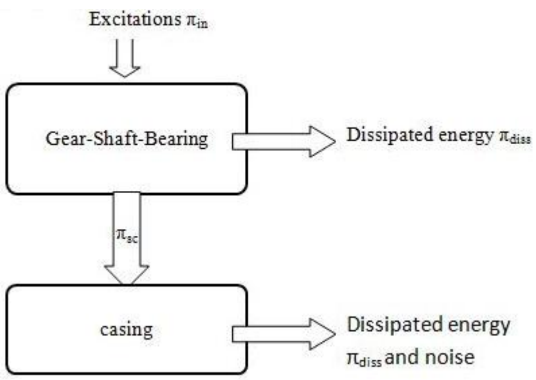

Figure 1 SEA Model

A. Assumptions

1. The dynamics of the gearbox are considered to be steady state and time invariant.
2. The error gear mesh frequency is the function of static transmission error.
3. The most dominant path of excitation is considered to be the structure borne path.
4. The energy transmitted through the bearing is assumed to be identical.
5. The nature of vibration mode is assumed to be bending.
6. The shafts will have equal amount of energy transmission.
7. The gearbox is mounted on rigid support
8. The effect of combination of torsional vibrations is neglected.
9. The effect of prime mover and load is also neglected.

B.System model definition

The SEA model is based on a balance of dynamical energy and power flow among groups of natural modes in a system. A complex system is modelled as a set of coupled mode groups (or subsystems) that are associated with the physical components of the system. The most dominant mode of vibration is considered to be bending mode. The bearings are single groove ball bearings having identical energy transmission between them. The energy transmission path in the above described geared-rotor system can be observed as follow

Fiureg. 1 Vibration Transmission Path

Since the gear shaft and bearing of the system are dominated by the similar mode of excitation hence grouping them into a single subsystem and the remaining casing and mount into another. Hence the whole system is divided into two subsystem as

1. Gear-shaft-bearing
2. Casing

The energy interactions in the subsystems are

π_{in} = input power to the system

π_{diss} = dissipated power into the system due to its inherent damping

π_{sc} = vibratory energy flow between the two subsystems

Suffix s and c represents the shaft and casing.

The power balance equation can be written as,

For subsystem 1

$$\pi_{in} = \pi_{sdiss} + \pi_{sc}$$

For subsystem 2

$$0 = \pi_{cdiss} + \pi_{cs}$$

The excitations induced into the system are generated due to the transmission error which is generated due to the manufacturing errors, wear of the gear tooth, deviation of the gear tooth profile from its predetermined position etc.The excitations are considered to be harmonic in nature.

C. SEA Parameters

The modal characteristics of interest in the work are modal density and coupling loss factor which can evaluated as

1. Modal density of shaft : The bending motion equation for the shaft is given by,

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = 0 \text{............(1)}$$

where, EI= Flexural rigidity if the shaft, m= mass per unit length and y=transeverse displacement of particle. Solution of the above equation can be given as,

$$y(x,t) = e^{j(\omega t - k_b x)} \text{......... (2)}$$

On further solving the final expression can be given as,

$$n(\omega) = L_s \times \sqrt[4]{\frac{\rho_s \pi {d_s}^2}{4EI_s \omega^2}} \text{..........(3)}$$

Here, L_s & d_s denotes length and diameter of the shaft.

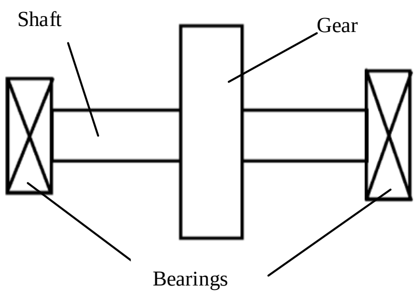

Figure 3 Shaft-Rotor system (subsystem 1)

2. Modal density of casing: Consider a two-dimensional, simply supported, homogenous and isotropic rectangular plate as shown in figure,

The governing equation of motion for the plate excited to bending mode of vibration is given by,

$$D\left\{\frac{\partial^4 y}{\partial x_1^4} + 2\frac{\partial^4 y}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 y}{\partial x_2^4}\right\} + \rho h\frac{\partial^2 y}{\partial t^2} = 0 \text{.......(4)}$$

Where, $D = \frac{E h_c^3}{12\times(1-\mu^2)}$ is the plate stiffness factor.

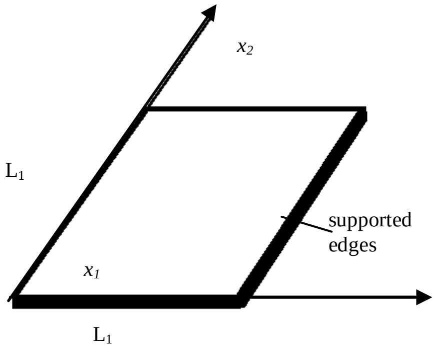

Figure 4 Simply supported plate(subsystem 2)

Harmonic response of the plate can be assumed as,

$$y(x_1, x_2, t) = Y(x_1, x_2)e^{j\omega t} \text{......................(5)}$$

On further solving the final expression for the natural frequency's in both direction

$$\omega^2{}_{n_1 n_2} = \left\{\left(\frac{n_1\pi}{L_1}\right)^2 + \left(\frac{n_2\pi}{L_2}\right)^2\right\}^2 \kappa_l{}^2 c_l{}^2 \text{......(6)}$$

After simplifying the final expression for the modal density of casing can be given as

$$n_c(f) = \frac{A_c}{h_c} \times \sqrt{\frac{3\rho_c(1-\mu^2)}{E_c}} \text{.......................(7)}$$

3. Coupling loss factor: The coupling loss factor can be evaluate by using two approach,

- Wave approach
- Modal approach

The appropriate approach can be selected on the basis whether the mode can be represented as a superposition of waves or the wave can be represented as sum of modes.
The coupling loss factor simplified in terms of point moment impedances and can be given as[4],

$$\eta_{sc} = \left(\frac{4EI}{\omega L_s}\right) R_e \left[\left(\frac{1}{Z_c}\right)\left|\frac{Z_c}{Z_c + Z_s}\right|^2\right] \text{...............(8)}$$

Where, R_e represents the real part.
Here the task is to determine the value of point moment impedances Z_c and Z_s.
When the time integration in the equation (1) is replaced by jω and solving for velocity response we get[21],

$$v = v_+ e^{-jkx} + v_- e^{jkx} + v_{+j} e^{-kx} + v_{-j} e^{kx} \text{ .... (9)}$$

Where, $k = \sqrt[4]{\frac{\omega^2 \rho A}{EI}}$ = bending wave no.
Assuming all the excitations are moving away from the excitation point the points from +jkx and +kx are vanished.

$$v = v_+ e^{-jkx} + v_{+j} e^{-kx} \text{ ....(10)}$$

The force vector can be given as,

$$\frac{F}{2} = \frac{Bk^3}{j\omega}(-jv_+ - v_{+j}) \text{ ....(11)}$$

The point moment impedance can be given by,

$$Z_s = 2m' c_B (1+i) + jm_g \omega \text{ ....(12)}$$

The term $jm_g\omega$ represents the excitations coming from the gear.
Similarly the point moment impedance for the casing can also be formulated which can be given as[22],

$$Z_c = \frac{4Eh_c^3}{3\omega(1-\mu^2) \times \left[1 - \left[\frac{4i}{\pi}\right] \ln\left(\frac{9}{20} k_c d_s\right)\right]} \text{....(13)}$$

Where k_c= wave no. of casing.
The value of point moment impedances obtained from equation (12) and (13) are substituted in equation (2) and the value of coupling loss factor can be determined.

4. Radiation loss factor
The radiation loss factor comes into the picture when the enrgy transmission from the casing plate to the atmosphere is considered. When the intereaction between the gearbox and the atmosphere is considered the radiation loss factor itself can be treated as coupling loss factor of the gearbox. The radiation loss factor can be estimated by following expression as,

$$\eta_{CT} = \frac{z_0 A_c \sigma(\omega)}{\omega_h m} \text{....................... (14)}$$

where z_0 is the characterictic impedance of the atmoshere, σ(ω) accoustic radiation constant and for gearbox σ(ω) = 1. Application of this equation to the gearbox can provide results of the coupling loss factor of the gearbox casing to the atmosphere.

III. GEARBOX DESCRIPTION

The gearbox considered for the application is a single stage simple gear train. The gear selected are 20^o full depth involute tooth profiled having equal module. The bearings used are single groove deep ball bearing. The casing is considered to be made of the plates welded together. The specifications of the gear manufactured with high precision are as follow

TABLE 1 GEAR SPECIFICATIONS

Module	2.5mm
Addendum	2.5mm
Dedundum	3.125mm
Clearance	0.625mm
Working depth	5mm
Tooth thickness	3.927mm

TABLE 2 BEARING SPECIFICATIONS

d	15 mm
D	32mm
d_1	20.5mm
D_1	28.2mm
B	9mm

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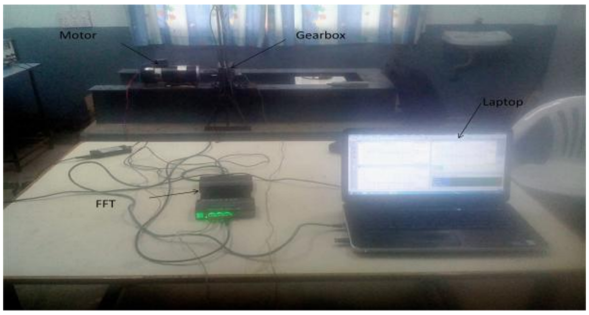
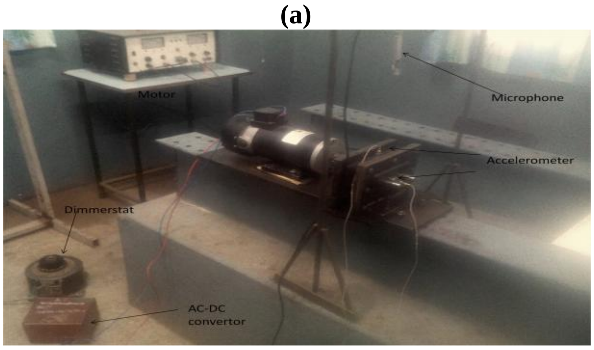
IV. EXPERIMENTAL SETUP

The experimentation involves determination of modal density and acoustic radiation loss factor of the gearbox experimentally. The experimental setup incorporates gearbox manufactured with high precision manufacuring as the specification mentioned in the Table1 and Table 2 is driven by the prime mover at various speed. For our application we have used the prime mover as variable speed control permanant magnet DC motor which is of 1HP and having speed range of 1000-3000 RPM. The accelerometers are used to measure the casing response and the microphone probe is used to measure the pressure alterations in atmosphere as shown in Figure 5 (b). The signals coming from the accelerometers and the microphone are recorded and analysed using B n K data aquisition system with 4-Channel FFT which is shown in Figure 5 (a). For key interest the gearbox has ran at only two speeds at 1500 and 2000 RPM.



(a)

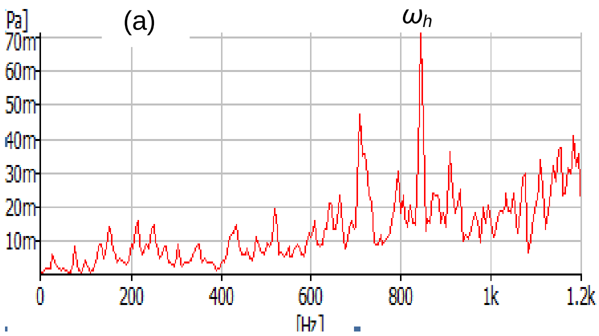


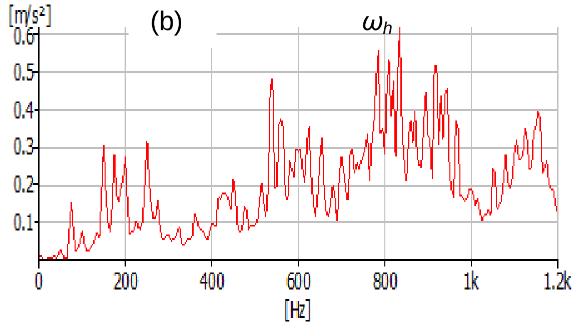
(b)

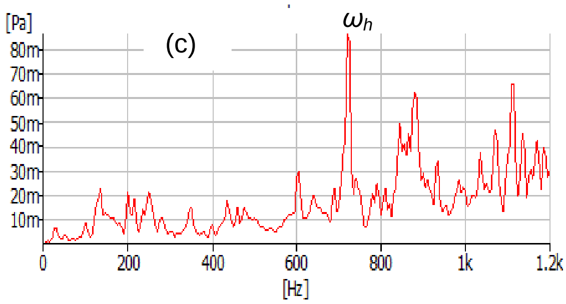
Figure 5. Experimental Setup

V. RESULTS AND DISCUSSION

The gearbox is tested for the speed of 1500 RPM and 2000 RPM and the results are recorded. The recorded results are shown in figure,







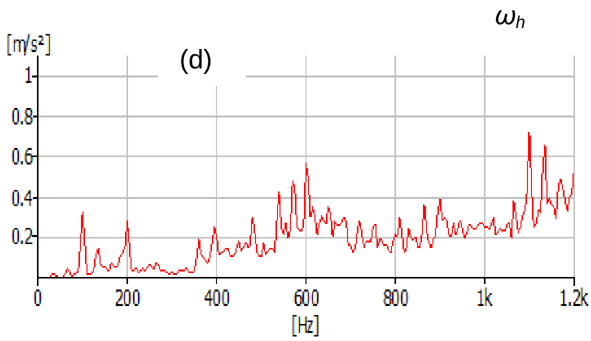


Figure 6 (a)&(c) Microphone spectrum of 1500 & 2000 RPM, (b)&(d) Accelerometer spectrum at 1500&2000 RPM

The SEA model described above can be used to predict the vibro-acoustic behaviour of the gearbox explained above in section 2. The methodology used here is to calculate the modal density's first then finding the point moment impedances of the shaft and the casing. The modal density of the casing does not depend on the speed of the rotor hence it is assumed to be constant. After finding the point moment impedances the coupling loss factor can be determined.

As shown in figure 7(a) it has been observed that the modal density of the shaft at lower speed is higher and as the speed increases the modal density goes decreasing at certain limit and then becomes constant for higher speeds of the shaft. This phenomena is generally due to the mode overlapping. As shown in figure 7(b) the predicted values for the coupling loss factor are considered to be large at lower speeds of the shaft but as speed increases the coupling loss factor goes on reducing and becomes constant for higher speeds. The modal density of casing is independent of the shaft speed hence it is constant irrespective to the speed of the shaft. The modal density of casing is found to be 0.00386mode/Hz.

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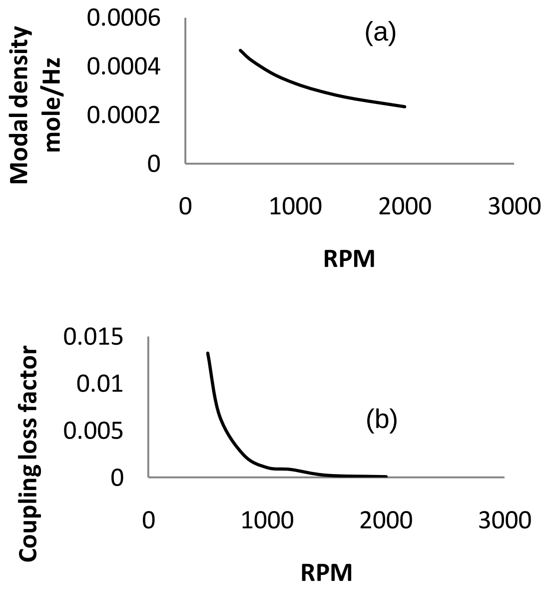

Figure 7 (a) Modal density of shaft Vs RPM (b)Shaft to Casing Coupling Loss Factor vs RPM

The accoustic radiation loss factor is estimated from the SEA model developed and the results are compared with the experimental as,

TABLE 3 ACOUSTIC RADIATION LOSS FACTOR

Sr. No.	ω_h	η_c	η_{rad}	
			By SEA model	Experiment
1	825	0.010382	$5.7654x10^{-4}$	$5.0637x10^{-4}$
2	1100	0.00665	$4.376x10^{-4}$	$4.1396x10^{-4}$

From the above table is clear that the SEA model developed can be applied to the gearbox with certain assumptions.

VI. CONCLUSION

The SEA model developed can be easily applied to the gearbox with including certain assumption. The modal density of casing deduced analytically is supposed to remain constant but the observations show variation in it due to changes in gear mesh frequency. The SEA model developed can be used in the early stages of the product design so that the vibro-acoustic behaviour of the gearboxes can predicted earlier and acoustically sound product can be manufactured. The SEA model explained here can be used with further improvements for the multi-mesh gearboxes.

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