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Fuzzy g**- Closed Sets

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Abstract: In this Paper we introduce the concept of the fuzzy g*-closed set in fuzzy topological spaces. After the introduction of fuzzy sets by Zadeh in 1965 and fuzzy topology by Chang in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1995, Thakur and Malviya extended the concept of g-closed sets in fuzzy topology. Many authors utilized fuzzy g-closed sets for the generalization of various fuzzy topological concepts in fuzzy topology. In 2010 Thakur and Mishra [9] introduced the concept of fuzzy g*-closed sets in fuzzy topology. In 2012 M. and Helen P. Introduced the concept of g**-closed sets in general topology. The present Paper extends the concept of g**-closed set due to Pauline M. and Helen P. In fuzzy topology and explore their study.

Keywords: Fuzzy topology, Fuzzy generalized closed sets, Fuzzy g**-closed set, Fuzzy g** -open set sets

I. INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh in his classical paper [11]. Subsequently several authors have applied various basic concepts from general topology to fuzzy sets and developed the theory of Fuzzy topological spaces.
The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology introduced by Chang [3].

Pu and Liu [5] introduced the concept of quasi-coincidence and q-neighbourhoods by which the extensions of mappings in fuzzy setting can very interestingly and effectively be carried out. The aim of this paper is to introduce the notion of fuzzy g^{**} -closed sets, an alternative generalization of fuzzy g- closed set in fuzzy topological spaces.

II. PRELIMINARIES

A family τ of fuzzy sets of X is called a fuzzy topology [3] on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets.

Throughout this paper, (X , τ) , (Y , σ) and (Z , γ) (or simply X,Y and Z) always mean fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned .

For a fuzzy set A of (X,τ) , Cl(A) and Int(A) denote the closure and the interior of A respectively. By 0_X and 1_X we will mean the fuzzy sets with constant function 0 (Zero function) and 1 (Unit function) respectively.
The following definitions are useful in the sequel.

DEFINITION 2.1: A fuzzy set A of (X,τ) is called:

(1). (Fuzzy semiopen (briefly, Fs-open) if $A \subseteq Cl(Int(A))$ and a fuzzy semiclosed (Briefly, Fs-closed) if $Int(Cl(A)) \subseteq A$ [1];

(2). (Fuzzy preopen (briefly, Fp-open) if $A \subseteq Int(Cl(A))$ and a fuzzy preclosed (briefly , Fp-closed) if $Cl(Int(A)) \subseteq A$ [2];

(3). Fuzzy α-open (briefly, Fα-open) if $A \subseteq Int Cl(Int(A))$ and a fuzzy α-closed (briefly,Fα-closed) if $ClInt(Cl(A)) \subseteq A$ [2] ;

(4). Fuzzy semi-preopen (briefly, Fsp-open) if $A \subseteq ClInt(Cl(A))$ and a fuzzy semi- preclosed (briefly, Fsp-closed) if $IntCl(Int(A)) \subseteq A$ [6] .

DEFINITION 2.2: A fuzzy set A of (X,τ) is called:
(1) Fuzzy generalized closed (briefly, Fg-closed) if $Cl(A) \subseteq H$, whenever $A \subseteq H$ and H is fuzzy open set in X .[8]

(2) Fuzzy g*-closed (briefly, Fg**-closed) if $Cl(A) \subseteq H$, whenever $A \subseteq H$ and H is fuzzy g- open set in X .[9]

(3) Fuzzy regular generalized closed set (briefly, Frg-closed) if $Cl(A) \subseteq H$, whenever $A \subseteq H$ and H is fuzzy regular open set in X . [7]

DEFINITION 2.3: A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_p \, q \, A$ iff $p+A(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by A q B iff there exists $x \in X$ such

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that A(x) +B(x) > 1. If A and B are not quasi-coincident then we write ˥ A q B. Note that A ⊆ B ⇔ ˥ A q B^C [4].

III.FUZZY G**-CLOSED SETS

DEFINITION 3.1: A fuzzy set A of fuzzy topological spaces (X,ℑ) is called a fuzzy g**-closed set if cl(A)) ⊆ U whenever A⊆ U and U is fuzzy g*-open.

THEOREM 3.1: Every fuzzy closed set is fuzzy g**-closed set.
Proof: Let A is fuzzy closed set of topological space (X,ℑ) then A= cl(A). Now we have to prove that A is fuzzy g**- closed set. Let A⊆ U and U is fuzzy g-open which implies that cl(A) ⊆ U Since A=cl(A) Therefore cl(A) ⊆ U whenever A⊆ U and U is fuzzy g-open Hence A is fuzzy g**-closed set.

REMARK 3.1: The converse of above theorem need not be true. For,

EXAMPLE 3.1: Let X= {a, b} and ℑ = {0, U, 1}. Where U(a) = 0.5 and U(b)= 0.4 . Then fuzzy set A defined by
A(a) = 0.3, A(b)= 0.3 is fuzzy g**-closed set but not a fuzzy closed set.

THEOREM 3.2: If a fuzzy set A of fuzzy topological space (X,ℑ) is both fuzzy open and fuzzy g**- closed then it is fuzzy closed set.

Proof: Suppose a fuzzy set A of fuzzy topological space (X, ℑ) is both fuzzy open and fuzzy g** closed set . Since every fuzzy open set is g-open set, therefore A is fuzzy g-open set of fuzzy topological space (X,ℑ) such that A ⊆ A Now A is fuzzy g**-closed set of topological space (X,ℑ) then by definition of fuzzy g** -closed set we have cl(A) ⊆ A. Since A ⊆ cl(A) for every fuzzy set A. Therefore cl(A) = A . Thus A is fuzzy closed set in fuzzy topological space (X, ℑ).

THEOREM 3.3: Every fuzzy g*- closed set is fuzzy g**-closed set.
Proof: Let A is fuzzy g*- closed set of fuzzy topological space (X,ℑ). We have to prove that A is fuzzy g**- closed set .Let A⊆ U and U is fuzzy g*-open. Since every g*-open set is g-open. Since U is fuzzy g-open Such that A⊆ U. Then by def of fuzzy g*-closed set cl(A) ⊆ U .Hence we have cl(A) ⊆ U whenever A⊆ U and U is g*-open.
Hence A is fuzzy g**-closed set.

REMARK 3.2: The converse of above theorem need not be true. For,

EXAMPLE 3.2: Let X= {a, b}, ℑ = {0, A, B, D, 1} and fuzzy sets A, B, D and H are defined as follows.
A(a) = 0.2 , A(b)=0.4
B(a) = 0.6 , B(b)=0.7;
D(a)=0.4 , D(b)=0.6 D
H(a)=0.4, H(b)=0.5.
Then H is fuzzy g** -closed set but it not fuzzy g*-closed.

THEOREM 3.4: If a subset A of topological space (X, ℑ) is both fuzzy open and fuzzy g**- closed then it is g*- closed set.
Proof: Suppose fuzzy set A of topological space (X, ℑ) is both fuzzy open and fuzzy g**- closed set . Since every fuzzy open set is fuzzy g-open set , therefore A is fuzzy g-open subset of topological space (X, ℑ) such that A ⊆ A .Now A is s g**-closed set of fuzzy topological space (X, ℑ) then cl(A) ⊆ A . Therefore cl(A) ⊆ A whenever A⊆ A and A is fuzzy g-open in (X, ℑ). Thus A is fuzzy g*- closed set in fuzzy topological space (X, ℑ).

THEOREM 3.5: Every Fuzzy g**-closed set A of topological space (X, ℑ) is Fuzzy g- closed set.
Proof: Let A is fuzzy g**-closed set of topological space. We have to prove that A is fuzzy g-closed sets in (X, ℑ)
Let A ⊆ U and U is open subset of X.

Since every open set is g-open, U is g-open set of X. Now A is g**-closed set of topological space (X, ℑ) then by definition of g** -closed set we have cl(A)⊆ U we have cl(A) ⊆ U whenever A⊆ U and U is -open in (X, ℑ).
Thus A is g- closed set in topological space (X, ℑ).

REMARK 3.3: The converse of above theorem need not be true. For,

EXAMPLE 3.3: Let X= {a, b} and ℑ = {0, U, 1}. Where U(a) = 0.4 and U(b)= 0.6 . Then fuzzy set A defined by
A(a) = 0.5, A(b)= 0.3 is fuzzy g -closed set but not a fuzzy g**- closed set.

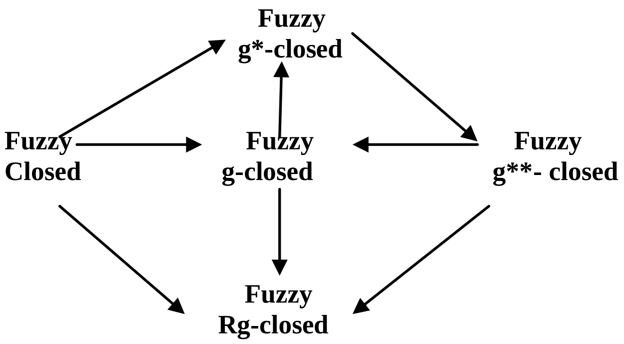
THEOREM 3.6: Every fuzzy g**-closed set A of topological space (X,ℑ) is Fuzzy rg- closed set.
Proof: Let A is fuzzy g**-closed set of topological space. We have to prove that A is fuzzy rg-closed sets in (X,ℑ)

Let A ⊆ U and U is regular open subset of X.
Since every fuzzy regular open set is fuzzy open set and every fuzzy open set is fuzzy g-open. U is fuzzy g-open set of X. Now A is fuzzy g**-closed set of topological space (X,ℑ) then by definition of fuzzy g** -closed set we have cl(A)⊆ U. Therefore cl(A) ⊆ U whenever A⊆ U and U is fuzzy regular open in (X, ℑ).
Thus A is fuzzy rg- closed set in topological space (X, ℑ).

REMARK 3.4: The converse of above theorem need not be true. For,

EXAMPLE 3.4: Let X= {a, b}, $\Im$ = {0, A, B, 1} and fuzzy sets A, B and H are defined as follows.
A(a) = 0.3 , A(b)=0.4
B(a) = 0.5 , B(b)=0.6;
H(a)=0.4, H(b)=0.5.
Then H is fuzzy rg -closed set but it not fuzzy g**-closed

REMARK 3.5: Theorem 3.1, 3.3, 3.5 and 3.6 reveals the following diagram of implication.



THEOREM 3.7: A fuzzy set A of fuzzy topological space (X,$\Im$) is fuzzy g** -closed if and only if ˥A q B ⇒ ˥ Cl (A) qB for every fuzzy g* -closed set B of X

Proof: Suppose that A is a fuzzy g** - closed set of X such that ˥A q B Then A⊆ B^c and B^c is a fuzzy g*-open set X. which implies that Cl (A) ⊆ B^c since A is fuzzy g** -closed. Hence ˥ Cl (A) qB
Conversely, Let U be a fuzzy g*- open set in X such that A⊆ U. Then ˥A q U^c and U^c is fuzzy g*-closed set in X. Then by hypothesis ˥ Cl (A) qU^c which implies that cl(A)) ⊆ U . Hence A is fuzzy g**-closed set.

THEOREM 3.8: If A is a fuzzy g** - closed set in fuzzy topological space (X,$\Im$) and A⊆ B⊆ cl (A), then B is fuzzy g **- closed set in fuzzy topological space (X,$\Im$)

Proof: Let A be a fuzzy g** - closed set in fuzzy topological space (X, $\Im$) Let B ⊆ U where U is a fuzzy g*- open se in X. Then A ⊆ U. Since A is fuzzy g** - closed set, it follows that Cl (A) ⊆ U. Now B ⊆ cl(A) which implies that cl(B) ⊆ cl(A) ⊆ U
Hence B is fuzzy g**-closed set in X.

DEFINITION 3.2: A fuzzy set A of fuzzy topological space (X, $\Im$) is said to be fuzzy g**-open set in A if its complement A^c is fuzzy g**-closed.

THEOREM 3.9: Every fuzzy open set is fuzzy g**-open.

Proof – Let A is fuzzy open set in fuzzy topological space (X, $\Im$).
Then A^C is fuzzy closed set in fuzzy topological space(X, $\Im$).
Now by theorem 3.1. Every fuzzy closed set is fuzzy g**-closed.
Therefore A^c is fuzzy g**-closed set.
Hence By definition 4.1 A is fuzzy g**-open set in (X, $\Im$).

REMARK 3.6: The converse of above theorem need not be true. For,

EXAMPLE 3.5: Let X= {a, b} and $\Im$ = {0, U, 1}. Where
U(a) = 0.5 and
U(b)= 0.4
Then fuzzy set A defined by
A(a) = 0.7 , A(b)= 0.6 is fuzzy g^{**}-open set but not a fuzzy open set.

THEOREM 3.10: Every fuzzy g*-open set is fuzzy g**-open.
Proof: Let A is fuzzy g*-open set in topological space (X, $\Im$).
Then A^C is fuzzy g*-closed set in fuzzy topological space (X, $\Im$).
Now by theorem 3.3 every fuzzy g*-closed set is fuzzy g**-closed. Therefore A^c is fuzzy g**-closed set.
Hence By definition 4.1 A is fuzzy g**-open set in (X, $\Im$).

REMARK 3.7: The converse of above theorem need not be true. For,

EXAMPLE 3.6: Let X= {a, b, c} and $\Im$ = {∅, X, {a}, {a, b}} . where
U(a) = 0.3 and
U(b)= 0.6
Let A= {a, c} then A is fuzzy g^{**}-open set but not a fuzzy g*-open set.

THEOREM 3.11: A set A of fuzzy topological space (X, $\Im$) is fuzzy g**-open if and only if F ⊆ cl(A) whenever F is fuzzy g* -closed and F⊆ A

Proof: **Necessity:** Let A is fuzzy g^{**}-open set. F is fuzzy g*-closed set such that F⊆ A. Then F^c is g*-open set such that A^c ⊆ F^c Now by hypothesis A^c is fuzzy g**-closed set .
We have cl((A^c) ⊆ F^c which implies that F ⊆ cl((A))
Sufficiency: Let F ⊆cl(A) whenever F is g*-closed and F⊆ A

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We have to prove that A is fuzzy g**-open. Let U is a fuzzy g*-open set of X such that $A^c \subseteq U$. Then U^c is fuzzy g*-closed set such that $U^c \subseteq A$. Therefore by hypothesis $U^c \subseteq cl(A)$ which implies that $cl(A)^c \subseteq U$ we have $cl((A^c)) \subseteq U$ where $A^c \subseteq U$ and U is fuzzy g*-open.
Hence A^c is fuzzy g**-closed. Thus A is fuzzy g**-open.

THEOREM 3.12: Let A is fuzzy g**-closed set of X and int(A)) ⊆ B ⊆ A. Then B is fuzzy g**-open set.

Proof: Let A is fuzzy g**-open set in X. such that int(A) ⊆ B ⊆ A which implies that $A^c \subseteq B^c \subseteq (int(A)^c = cl(A^c)$ Now A^c is s fuzzy g**-closed set such that $A^c \subseteq B^c \subseteq cl(A^c)$) Then By theorem 3.7 B^c is fuzzy g**-closed set Hence B is fuzzy g**-open set.

IV. CONCLUSION

The theory of g-closed sets plays an important role in general topology. Since its inception many weak and strong forms of g-closed sets have been introduced in general topology as well as fuzzy topology. The present paper investigated a new weak form of fuzzy g-closed sets called fuzzy g**-closed sets which contain the classes of fuzzy closed sets and fuzzy g*- closed sets and contained in the classes of fuzzy g-closed sets and fuzzy rg-closets. Several properties and application of fuzzy g**-closed sets are studied. Many examples are given to justify the result.

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