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Central Automorphisms Group of Metabelian Lie Algebras Generated by Three Sandwich Elements

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Abstract: Let L be a metabelian Lie algebra generated by three sandwich elements. In this paper, we investigate the group of central automorphisms of L.

Keywords: Central automorphism, metabelian Lie algebra, sandwich elements.

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INTRODUCTİON

Let L be a metabelian Lie algebra over a field of characteristic zero generated by three sandwich elements x,y and z. We denote by C(L) and Aut(L), respectively, the center and the group of all automorphisms of L. An automorphism θ of L is called a central automorphism if it induces the identity mapping on the algebra $L/C(L)$. That is, if θ is a central automorphism then for every u∈L we have θ(u)-u∈C(L). The set of all central automorphisms of L forms a subgroup of Aut(L). We shall denote the group of all central automorphisms of L by $Aut_C(L)$. For free nilpotent Lie algebras, the characterization of these automorphisms group was given by O.Oztekin and N. Ekici [8] and the form of central automorphisms of free center-by- metabelian Lie algebras was given by Z. Esmerligil [7].

A non-zero element x of a Lie algebra L over F is called an extremal element if [[L,x],x]⊆Fx. Sandwich, that is, elements x∈L with [[L,x],x]=0 are extremal elements of a special kind. In [10], Zel'manov and Kostrikin proved that, the universal Lie algebra generated by a finite number of sandwich elements is nilpotent and finite dimension. Cohen,Steinbach, Ushirobira, and Wales [4] generalized this result and proved that a Lie algebra generated by a finite number of extremal elements is finite dimensional. In [9], Roozemond showed that if Lie algebra generated by three extremal elements over a field of characteristic distinct from 2 then the dimension of this Lie algebra is 8.

In this paper we use the commutator notation for the Lie multiplication. Suppose that L is generated by three sandwich elements x,y and z. For each u ∈L the derivation adu:L→L is nilpotent. Hence the linear mapping

$$e^{adu}=1+((adu)/(1!))+((ad^2u)/(2!))$$

is well defined and it is an inner automorphism of L. The set of all inner automorphisms L forms a subgroup of Aut(L).We shall denote the group of all inner automorphisms of L by Inn(L).

In this study we give the form of central automorphisms of metabelian Lie algebras generated by three sandwich elements and we show that, these central automorphisms are also inner automorphisms.

MAİN RESULTS

Theorem 1. Let L is metabelian Lie algebra generated by three sandwich elements x,y and z. Then

x,y,z,[x,y],[x,z],[y,z],[[z,x],y],[[z,y],x]

span L.

Proof. It is obtained from [4,9,10].

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Theorem 2. Let L is metabelian Lie algebra generated by three sandwich elements x,y and z. Then the center of L is

$$C(L)=\langle[[z,x],y],[[z,y],x]\rangle$$

Proof.

$$[[[z,x],y],x] = -[[y,x],[z,x]]-[x,[z,x],y]=[[[z,x],x],y]=0$$

$$[[[z,x],y],y] = [[L,y],y]=0$$

$$[[[z,x],y],z] = -[[y,z],[z,x]]-[z,[z,x],y]=[[[z,x],z],y]=-[[[x,z],z],y]=0$$

Since L is metabelian Lie algebra we have

$$[[[z,x],y],[x,z]]=[[[z,x],y],[x,y]]=[[[z,x],y],[y,z]]=[[[z,x],y],[[z,y],x]]=0$$

therefore $[[z,x],y]\in C(L)$. Similarly,

$$[[[z,y],x],y] = -[[x,y],[z,y]]-[y,[z,x],x]=[[[z,x],y],x]=0$$

$$[[[z,y],x],x] = [[L,x],x]=0$$

$$[[[z,y],x],z] = -[[x,z],[z,y]]-[z,[z,y],y]=[[[z,y],z],y]=-[[[y,z],z],y]=0$$

Since L is metabelian Lie algebra we have

$$[[[z,y],x],[x,z]]=[[[z,y],x],[x,y]]=[[[z,y],x],[y,z]]=[[[z,y],x],[[z,y],x]]=0$$

therefore $[[z,y],x]\in C(L)$

Theorem 3. Let L is metabelian Lie algebra generated by three sandwich elements x,y and z. Then $Aut_C(L)<\theta_1,\theta_2,\theta_3>$where $\theta_1,\theta_2,\theta_3$ are

$$\theta_1 : \{x\to x+[x,[y,z]],\ y\to y,\ z\to z\}$$

$$\theta_2 : \{x\to x,\ y\to y+[y,[x,z]],\ z\to z\}$$

$$\theta_3 : \{x\to x,\ y\to y,\ z\to z+[z,[x,y]]\}$$

Proof.θ_1 is inner automorphisms of L determined by [y,z],

$$e^{ad[y,z](x)} = x+[x,[y,z]]+(([x,[y,z],[y,z]])/2)+...$$

$$= x+[x,[y,z]]$$

$$e^{ad[y,z](y)} = y+[y,[y,z]]+(([y,[y,z],[y,z]])/2)+...$$

$$= y+[y,[y,z]]$$

$$= y$$

$$e^{ad[y,z](z)}= z+[z,[y,z]]+(([z,[y,z],[y,z]])/2)+...$$

$$= z+[z,[y,z]]$$

$$= z$$

Similarly it is shown that θ_2 and θ_3 are inner automorphisms of L determined by [x,y] and [x,z], respectively.

Now we prove that these automorphisms are central,

$$\theta_1(x)-x = [x,[y,z]]\in C(L)$$

$$\theta_1(y)-y = 0\in C(L)$$

$$\theta_1(z)-z = 0\in C(L)$$

so θ_1 is central automorphism of L. Similarly,

$$\theta_2(x)-x = 0\in C(L)$$

$$\theta_2(y)-y = [y,[x,z]]\in C(L)$$

$$\theta_2(z)-z = 0\in C(L)$$

θ_2 is central automorphism of L. And

$$\theta_3(x)-x = 0\in C(L)$$

$$\theta_3(y)-y = 0\in C(L)$$

$$\theta_3(z)-z = [z,[x,y]]\in C(L)$$

θ_3 is central automorphism of L. $<\theta_1,\theta_2,\theta_3>$ is the subgroup of Aut(L), so we get

$Aut_C(L)=<\theta_1,\theta_2,\theta_3>$.

Corollary. Let L is generated by three sandwich elements x,y and z then$Aut_C(L)<Inn(L)$.

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