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Spectrum and Energy of the Transformation Graph K_n^{++-}

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Abstract: Let G be a graph. The eigen values of G are the eigen values obtained from the adjacency matrix of G. The energy of a graph G denoted by E(G), is the sum of absolute values of its eigen values. One of the questions that arises here is, Is it possible to find the energy of graph by graph transformation? This paper answers this question. Here a category of graph called complete graph is taken and is operated to obtain transformation graph K_n^{++-}. It has been proved that the spectrum and energy of this transformation graph is -1 with multiplicity (n+2)(n-1)/2 and (n+2)(n-1)/2 and its energy is (n+2)(n-1).

Keywords: Energy, complete graph, transformation graph, K_n^{++-}

I. INTRODUCTION

Graph theory is a branch of mathematics which deals with vertices and edges. Edges connecting the vertices. Graphs are ever-present miniature of both from nature and man-made structures. Graphs are the miniature of physical problems it can handle changes in the real world which is dynamic. The subject Graph Theory had its beginnings in recreational problem, but now it has evolved into a major area of mathematical research work with application in various fields like chemistry, operation research, social science, computer science, logistics etc. The concept of graph was first mentioned by Euler. The actual problem that he was considering can be regarded as flippant puzzle.; which arose from the physical world. After a century of a paper by Euler on the famous Konigsberg bridge problem. Topology was introduced by listing, Cayley was studying a specific analytical form that arose from differential calculus to learn a specific type of graphs, which is called the trees. From the above almost frivolous beginning has grown modern graph theory, which now embraces topics such as domination, neighbourhood, algebraic graph theory, fractional graph theory, switching, design theory, to name but a few. All topics deal essentially with abstract concepts, although many have potential application in practical. Application of graph theory can also be seen in chemistry. As we know that adjacency, matrix, represents the non-pictorial form of corresponding graph and also many researchers concluded that from adjacency matrix one can calculate the Eigen values of the corresponding graph. An application of graph theory in chemistry was started by Erich Huckel in 1930's in his study of molecular orbital theory. A frame of non-saturated hydrocarbon in quantum chemistry, was structured by graphs and was converted to a graph theory problem. The level of energy of electrons in molecules of such non-saturated hydrocarbon were surprisingly Eigen values of the corresponding graphs and hence the name was coined relating to it. In our present work, we study energy of graph, equitable energy of graphs and try obtain equitable spectrum.

1.1 Objectives: Objectives of the work is to obtain the spectrum and energy of a graph obtained by graph transformation. Here I have considered the total graph and one additional condition is laid on the total graph. With that it transforms into the K_n^{++-} transformation graph of the K_n which is the complete graph and used the spectra of the transformation graph K_n^{++-} to find the energy of the transformation graph K_n^{++-} obtained from a complete graph. It is purely in the interest of mathematical aspect.

1.2 Scope and Limitation: Energy of graphs is essentially an abstract concept, although it has many potential applications in practical. The results derived can be used in chemistry and other fields. The significance of transformation graphs its spectrum and energy lies when it could be transformed to a real-time problem and the parameters and related results answers the real-time problems.

II. LITERATURE REVIEW

In 1936, D. Konig [14], the Hungarian Mathematician wrote the first book on graph theory. Later book on graph theory also written by C. Berge [2], O. Ore[15] and F. Harary[7]. Application of graph theory with different branches like engineering technology, biological science, archeology, ecology, planning etc can be found in the book of F.S Robestx. The connection between graph theory with other branches of mathematics were discussed in the book of L.U Beincles and R.J Wilson.

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The topic of domination was the fastest expanding area within the graph theory because of its wide variety of application in fields like networks, algorithm, designs etc. In 1958, C. Berge [2] introduced the concept of external stability and co-efficient of external stability in his book, "Theory of graphs and its application ". In 1962 O. Ore[15] published a famous book "Theory of Graphs", in which he introduced the word domination. The domination concept was in inhibition till 1975 as there was no much work done till 1977. By the end of the year T.W Haynes, S.T Hedetniemi and P.J Slater [8,9] brought out a comprehensive two volume text books "Fundamentals of domination in graphs" and "Domination in graphs: Advanced topics", which contains more detailed survey on domination theory see[10,11,12,16]

The total pie-electron energy E(G), created lots of curiosity among the researchers after C.A Coulson [4] initiated his work on it. I. Gutman [15] in 1978, for the first time brought forth the concepts of energy of graph. He defined it as addition of absolute value of Eigen values of a graph. Due to the great success of graph energy concept few analogous quantities such as Laplacian energy, incidence energy, distance energy, skew energy and soon were also conceived. For more detailed survey, see [1,6,13]

III. RESEARCH METHODOLOGY

3.1 Formation of transformation graph

Two kinds of graph discussed here are Complete graph K_n which is transformed into the K_n^{++-} transformation graph. We know that a complete graph is a graph where every pair of unique vertices is joined by a unique edge. It is written as K_n. The transformation graph written as K_n^{++-} whose vertex set is $V(K_n) \cup E(K_n)$ and there is an edge uv in K_n^{++-} if the vertices u and v are connected under the following conditions : (i) u, v are both vertices and are adjacent in K_n, (ii) u, v are both edges and are adjacent in K_n (iii) one of u is a vertex and the other v is an edge and they are not incident in K_n. The matrix formed by the adjacency of transformation graph K_n^{++-} the general notation is $A(K_n^{++-})$. The n vertices and n(n-1)/2 edges of K_n will now be the vertices of transformation graph K_n^{++-} . Hence there will be n(n+1)/2 vertices in the transformation graph K_n^{++-}. Using the definition of the transformation graph K_n^{++-} , we can form the adjacency matrix for K_n^{++-}.

3.2 Adjacency matrix of the transformation graph $\mathbf{K_n^{++-}}$

The adjacency matrix for $\mathbf{K_n^{++-}}$will have dimension n(n+1)/2 x n(n+1)/2. The set of eigen values obtained by means of the adjacency matrix of graph is called the spectra or spectrum of the graph. The general representation is

$$\mathrm{Spec}(G) = \begin{pmatrix} \lambda_1 & \lambda_2 \dots\dots\dots \\ n_1 & n_2 \dots\dots\dots \end{pmatrix}$$

A matrix is constructed depending on the vertices and their adjacencies in the graph. As there are no self-loops the diagonal elements will be 0. The transformation graph K_n^{++-}, has all the vertices adjacent to each other, we obtain $J_n - I$ matrix.

$$\begin{array}{c} V_1\ V_2\ V_3 \dots\dots\dots\dots\dots\dots\dots\dots V_{n(n+1)/2} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1\dots\dots. & 1 \\ 1 & 0 & 1 & 1 & 1\dots\dots. & 1 \\ 1 & 1 & 0 & 1 & 1\dots\dots. & 1 \\ . & . & . & . & .\dots\dots.. & . \\ . & . & . & . & .\dots\dots.. & . \\ 1 & 1 & 1 & 1 & 1\dots\dots\dots & 0 \end{pmatrix}_{n(n+1)/2 \times n(n+1)/2} \end{array}$$

3.3 Statement: If K_n is a complete graph with $n \geq 2$ vertices. Let K_n^{++-} be the transformation graph of K_n, and $A(K_n^{++-})$ be the adjacency matrix then,

a) The eigen values of K_n^{++-} are '-1' repeated $\frac{(n+2)(n-1)}{2}$ times and $\frac{(n+2)(n-1)}{2}$ repeated once.

b) The spectrum of K_n^{++-} is given by $\begin{pmatrix} -1 & \frac{(n+2)(n-1)}{2} \\ \frac{(n+2)(n-1)}{2} & 1 \end{pmatrix}$

c) The energy of K_n^{++-} denoted by $E(K_n^{++-})$ is (n+2)(n-1).

The above statement will be proved by Mathematical Induction. Firstly, the statement will be shown is true for n = 2. ie, for complete graph K_2. The above result will be assumed to be true for some number of vertices say 'm'. Then the result will be proved for 'm + 1' vertices. Which would complete the process of Mathematical Induction. The results obtained will be verified on few graphs.

IV. RESULTS AND DISCUSSIONS

4.1 Theorem: If K_n is a complete graph with $n \geq 2$ vertices. Let K_n^{++-} be the transformation graph of K_n, and $A(K_n^{++-})$ be the adjacency matrix, its dimension is $\frac{n(n+1)}{2}$ X $\frac{n(n+1)}{2}$ then,

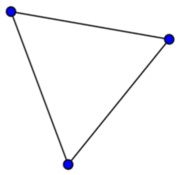
a) The eigen values of K_n^{++-} are '-1' repeated $\frac{(n+2)(n-1)}{2}$ times and $\frac{(n+2)(n-1)}{2}$ repeated once.

b) The spectrum of K_n^{++-} is given by $\begin{pmatrix} -1 & \frac{(n+2)(n-1)}{2} \\ \frac{(n+2)(n-1)}{2} & 1 \end{pmatrix}$

c) The graph energy of K_n^{++-} denoted by $E(K_n^{++-})$ is (n+2)(n-1).

Proof: The above theorem is proved by Mathematical Induction.
Step1: n = 2
We obtain the graph K_2 :

K_2^{++-} is



Hence $A(K_2^{++-}) = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

The eigen values are obtained by $|A(K_2^{++-}) - \lambda I| = 0$
That is,

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = -\lambda^3 + 3\lambda + 2$$

$$-\lambda^3 + 3\lambda + 2 = 0$$

$$-(\lambda + 1)^2(\lambda - 2) = 0$$

$$\lambda = 2 \text{ and } \lambda = -1$$

Hence the spectrum of K_2^{++-} is given by $\begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$

The energy of K_2^{++-} is $E(K_2^{++-}) = \sum_{i=1}^{2} |\lambda_i|$

$= 2 + |-1| + |-1| = 4$

When n = 2 is substituted in the subdivision of theorem a) , b) and c) , the result obtained is same as done by conventional methods.

Step2: Assume the statement is true for n = m,

Which means for the graph K_m^{++-} the adjacency matrix has dimension $\frac{m(m+1)}{2}$ X $\frac{m(m+1)}{2}$

And the eigen values are assumed to be (-1) repeated $\frac{(m+2)(m-1)}{2}$ times and $\frac{(m+2)(m-1)}{2}$ repeated once.

Similarly the statement,

b) The spectrum of K_m^{++-} is given by $\begin{pmatrix} -1 & \frac{(m+2)(m-1)}{2} \\ \frac{(m+2)(m-1)}{2} & 1 \end{pmatrix}$

c) The energy of K_m^{++-} denoted by $E(K_m^{++-})$) is (m+2)(m-1).
Are assumed to be true.

Step3: To prove the above for n = m+1

From step2 we have K_m^{++-} with $\frac{m(m+1)}{2}$ vertices. Add one vertex to the graph K_m so as to obtain K_{m+1} graph. The 'm' edges connecting this vertex. Hence the K_{m+1}^{++-} graph will have ($\frac{m(m+1)}{2} + m + 1$) vertices. On simplifying it becomes $\frac{(m+1)(m+2)}{2}$. This now becomes the dimension of the adjacency matrix.

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$$\text{ie, } A(K_{m+1}^{++-}) = \begin{bmatrix} \dots\dots \\ \dots\dots. \\ \dots\dots.. \end{bmatrix}_{\frac{(m+1)(m+2)}{2} X \frac{(m+1)(m+2)}{2}}$$

From the assumption of step2, the eigen values are (-1) repeated (dim of A(K_m^{++-}) – 1)
And (dim of A(K_m^{++-}) – 1) repeated once.

ie, (-1) repeated $\frac{(m-1)(m+2)}{2}$ times and $\frac{(m-1)(m+2)}{2}$ repeated once. ________________(1)

Therefore, the eigenvalues of A(K_{m+1}^{++-}) are (-1) repeated (dim of A(K_{m+1}^{++-}) – 1)
And (dim of A(K_{m+1}^{++-}) – 1) repeated once.

ie, (-1) repeated $\left(\frac{(m+1)(m+2)}{2} - 1\right)$ times and $\left(\frac{(m+1)(m+2)}{2} - 1\right)$ repeated once.
ie, (-1) repeated $\left(\frac{m(m+3)}{2}\right)$ times and $\left(\frac{m(m+3)}{2}\right)$ repeated once.
ie, (-1) repeated $\left(\frac{((m+1)-1)((m+1)+2)}{2}\right)$ times and $\left(\frac{((m+1)-1)((m+1)+2)}{2}\right)$ repeated once. ________________(2)

From (1) and (2) the statement is proved for n = m+1.

b) The set of graph eigen values called the spectrum of the graph. The general representation of the is

$$\text{Spec(G)} = \begin{pmatrix} \lambda_1 & \lambda_2 \dots\dots\dots \\ n_1 & n_2 \dots\dots\dots \end{pmatrix}$$

Since we know the eigen values of K_m^{++-} we can write the its spectrum, which is

$$\text{Spec()} = \begin{pmatrix} -1 & \frac{(m+2)(m-1)}{2} \\ \frac{(m+2)(m-1)}{2} & 1 \end{pmatrix}$$

C) The energy is given by E(G) = $\sum_{i=1}^{n} |\lambda_i|$

$$E(K_m^{++-}) = \sum_{i=1}^{n} |\lambda_i|$$
$$= |-1| + |-1| + |-1| \dots.. \frac{(m+2)(m-1)}{2}\ times + \left|\frac{(m+2)(m-1)}{2}\right|$$
$$= (m+2)(m-1)$$

Hence the theorem

4.2 Illustrations

4.2.1 Consider the transformation graph of K_3^{++-},

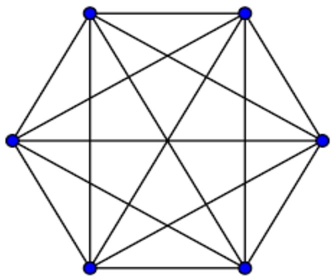

Fig. 1 K_3^{++-},

The adjacency matrix is

$$A(K_3^{++-}) = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

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To find the energy,

$$\begin{vmatrix} -\lambda & 1 & 1 & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 & 1 & 1 \\ 1 & 1 & -\lambda & 1 & 1 & 1 \\ 1 & 1 & 1 & -\lambda & 1 & 1 \\ 1 & 1 & 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 & 1 & 1 & 1 \\ 0 & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{-\lambda^2+1}{\lambda} \\ 0 & 0 & \lambda+1 & \lambda+1 & \lambda+1 & -\lambda^2+\lambda+2 \\ 0 & 0 & 0 & (\lambda+1) & (\lambda+1) & -\lambda^2+2\lambda+3 \\ 0 & 0 & 0 & 0 & (\lambda+1) & -\lambda^2+3\lambda+4 \\ 0 & 0 & 0 & 0 & 0 & -\lambda^2+4\lambda+5 \end{vmatrix} = 0$$

$$(\boldsymbol{\lambda}+\mathbf{1})^5(\boldsymbol{\lambda}-\mathbf{5}) = \mathbf{0}$$
$$\boldsymbol{\lambda} = 5 \ and \ \boldsymbol{\lambda} = -1$$

The spectrum is Spec(K_3^{++-}) = $\begin{pmatrix} -1 & 5 \\ 5 & 1 \end{pmatrix}$

And the energy is given by

$$E(K_3^{++-}) = \sum_{i=1}^{n} |\lambda i|$$
$$E(K_3^{++-}) = \sum_{i=1}^{n} |\lambda_i|$$
$$= 5 + |-1| + |-1| + |-1| + |-1| + |-1| = 10$$

The above is conventional method
Verification using the above theorem,

The spectrum of

$$K_n^{++-} = \begin{pmatrix} -1 & \frac{(n+2)(n-1)}{2} \\ \frac{(n+2)(n-1)}{2} & 1 \end{pmatrix}$$

The energy of K_n^{++-} = (n+2)(n-1).
Substituting n = 3

$$\text{Spec}(K_3^{++-}) = \begin{pmatrix} -1 & 5 \\ 5 & 1 \end{pmatrix}$$

And energy of $K_3^{++-} = 10$

4.2.2 Consider the transformation graph K_4^{++-}

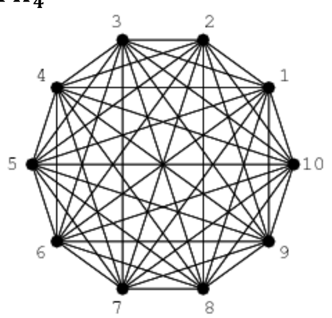

Fig. 2 K_4^{++-},

The adjacency matrix is A(K_4^{++-}) =

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

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To find the energy,

$$\begin{vmatrix} -\lambda & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -\lambda & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -\lambda & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -\lambda & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -\lambda & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -\lambda & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -\lambda & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{\lambda+1}{\lambda} & \frac{-\lambda^2+1}{\lambda} \\ 0 & 0 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & -\lambda^2+\lambda+2 \\ 0 & 0 & 0 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & -\lambda^2+2\lambda+3 \\ 0 & 0 & 0 & 0 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & -\lambda^2+3\lambda+4 \\ 0 & 0 & 0 & 0 & 0 & \lambda+1 & \lambda+1 & \lambda+1 & \lambda+1 & -\lambda^2+4\lambda+5 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda+1 & \lambda+1 & \lambda+1 & -\lambda^2+5\lambda+6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda+1 & \lambda+1 & -\lambda^2+6\lambda+7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda+1 & -\lambda^2+7\lambda+8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda^2+8\lambda+9 \end{vmatrix} = 0$$

$\boldsymbol{(\lambda+1)^9(\lambda-9)=0}$

$\lambda = 9 \text{ and } \lambda = -1$

The spectrum is Spec(K_4^{++-}) = $\begin{pmatrix} -1 & 9 \\ 9 & 1 \end{pmatrix}$

And the energy is given by E(K_4^{++-}) = $\sum_{i=1}^{n} |\lambda i|$

E(K_4^{++-}) = $\sum_{i=1}^{n} |\lambda_i|$

= 9 + $|-1| + |-1| + |-1| + |-1| + |-1| + |-1| + |-1| + |-1| + |-1| = 18$

The above is conventional method

Verification using the above theorem,

The spectrum of K_n^{++-} = $\begin{pmatrix} -1 & \frac{(n+2)(n-1)}{2} \\ \frac{(n+2)(n-1)}{2} & 1 \end{pmatrix}$

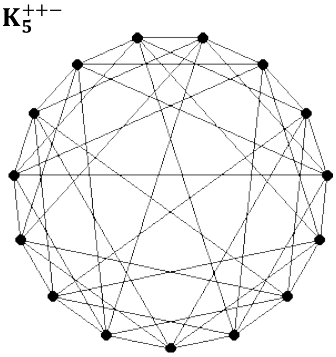
The energy of K_n^{++-} = (n+2)(n-1).

Substituting n = 4

Spec(K_4^{++-}) = $\begin{pmatrix} -1 & 9 \\ 9 & 1 \end{pmatrix}$

And energy of $K_4^{++-} = 18$

4.2.3 Consider the transformation graph K_5^{++-}



The adjacency matrix for K_5^{++-} has dimension 15. Due to large dimensions the eigen values are calculated using MATLAB.

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>> A = [0 1 1 1 1 1 1 1 1 1 1 1 1 1 1; 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1; 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1; 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1;1
e = eig(A)

A =

     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     0     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     0     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     0     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     0     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     0     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     0     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     0     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     0     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     0     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     0     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     0     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     0     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     0     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     0
```

```
e =

   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   -1.0000
   14.0000
```

This is the output.

Substituting n = 4 in the above theorem,

Spec(K_4^{++-}) = $\begin{pmatrix} -1 & 14 \\ 14 & 1 \end{pmatrix}$

And energy of $K_5^{++-} = 28$

Hence the results obtained can be used in calculating the energy and spectrum of any total graph with additional condition with no restriction on the number of vertices. Whereas the conventional methods become tedious as the number of vertices of graph increases. The four illustrations shown here explains this

CONCLUSIONS

The Theorem can be understood as a general formula for finding the eigenvalues, graph spectrum and energy of graph got by transformation of graph. The graph energy is calculated for most of the graphs, there are many papers published on it. But this paper is on energy of graph obtained by graph transformation. The study revealed that the eigen values of K_n^{++-} which is the transformation graph of K_n are

'-1' repeated $\frac{(n+2)(n-1)}{2}$ times and $\frac{(n+2)(n-1)}{2}$ repeated once.

The spectrum of K_n^{++-} is given by $\begin{pmatrix} -1 & \frac{(n+2)(n-1)}{2} \\ \frac{(n+2)(n-1)}{2} & 1 \end{pmatrix}$

The energy of K_n^{++-} denoted by E(K_n^{++-}) is (n+2)(n-1).

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