IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Vol. 6, Issue 3, March 2019

Algorithm for Doubly Even Magic square

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Abstract: An magic square is N*N matrix containing integers and addition result of each row, column and diagonally get the same value. The commonly used methods of constructing magic squares are the cross diagonals method for doubly even. But these method cannot be used to find higher order magic square. In this article we will find the method to solve higher order doubly even magic squares.

Keywords: Magic Square, Bhavya squares, 4nX4n Magic square, Doubly Even Magic Square

I. INTRODUCTION

An n × n matrix A consisting of nonnegative integers is a general magic square of order n if the sum of elements in each row, column, and main diagonal is the same. The sum is the magic number. A general magic square A of order n is a magic square, denoted by MS(n), if the entries of A are distinct. A magic square A of order n is normal if the entries of A are n_2 consecutive integers. Usually, the entry in position (i, j) of a matrix A is denoted by $a_{i,j}$.

Magic squares have been studied for 4000 years. The Loh-Shu magic square is the oldest known magic square; its invention is attributed to Fuh-Hic, the mythical founder of Chinese civilization [4]. A lot of work has been done on construction of magic squares; for more details, the interested reader may refer to [1,3–6,8,11], and the references therein.

The magic square are of three order:
1. Odd order magic square
2. Doubly even magic square : which are divisible by 4.
3. Singly even magic square : which are divisible by 2.

II. THEORUMS AND PROOFS

Lemma 1.1: if any 4 no. put in this formula then it will be the magic square of order 4nx4n for n≥1 for all n belongs to Positive integer.

Square of order

A	B	C	D
D+3	C-3	B-1	A+1
B-2	A+2	D+2	C-2
C-1	D+1	A-1	B+1

A_1	B_1	C_1	D_1
D_1+3	C_1-3	B_1-1	A_1+1
B_1-2	A_1+2	D_1+2	C_1-2
C_1-1	D_1+1	A_1-1	B_1+1

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A_2	B_2	C_2	D_2
D_2+3	C_2-3	B_2-1	A_2+1
B_2-2	A_2+2	D_2+2	C_2-2
C_2-1	D_2+1	A_2-1	B_2+1

A_3	B_3	C_3	D_3
D_3+3	C_3-3	B_3-1	A_3+1
B_3-2	A_3+2	D_3+2	C_3-2
C_3-1	D_3+1	A_3-1	B_3+1

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Vol. 6, Issue 3, March 2019

.A to A_1 are in arithmetic progression similarly, A To A_2/A_3 in arithmetic progression and similarly, B to $B_1/B_2/B_3$,C to $C_1/C_2/C_3$ and D to $D_1/D_2/D_3$.

Where A_1 , B_1 , C_1 , D_1, D_2 ,C_2 ,B_2, A_2 , D_3 ,C_3 ,B_3, A_3

Are related to A,B,C,D As

A_1 = A- 4 X (n-1)

B_1 = B+ 4 X (n-1)

C_1 = C+ 4 X (n-1)

D_1 = D- 4 X (n-1)

A_2 = A- 4n X (n-1)

B_2 = B+ 4n X (n-1)

C_2 = C+ 4n X (n-1)

D_2 = D- 4n X (n-1)

A_3 = A- 4 X (n^2-1)

B_3 = B+ 4 X (n^2-1)

C_3 = C+ 4 X (n^2-1)

D_3 = D- 4 X (n^2-1)

A to A_1 are in arithmetic progression similarly, A To A_2/A_3 in arithmetic progression and similarly, B to $B_1/B_2/B_3$,C to $C_1/C_2/C_3$ and D to $D_1/D_2/D_3$. And A_1 is n^{th} term.

Lemma 1.2 : For n≥1 A,B,C,D should follow these conditions:
(i) B-A ≥5
(ii) C-B ≥(4 X n^2 +1)
(iii) D-C ≥((n^2-1) X 8 +1)
(iv) A ≥4n^2-3 + 1

Proof
Lemma 1.1 proof
Considering n=3

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{110}	a_{111}	a_{112}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{210}	a_{211}	a_{212}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{310}	a_{311}	a_{312}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{410}	a_{411}	a_{412}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{510}	a_{511}	a_{512}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{610}	a_{611}	a_{612}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{710}	a_{711}	a_{712}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{810}	a_{811}	a_{812}
a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{910}	a_{911}	a_{912}
a_{101}	a_{102}	a_{103}	a_{104}	a_{105}	a_{106}	a_{107}	a_{108}	a_{109}	a_{1010}	a_{1011}	a_{1012}
a_{111}	a_{112}	a_{113}	a_{114}	a_{115}	a_{116}	a_{117}	a_{118}	a_{119}	a_{1110}	a_{1111}	a_{1112}
a_{121}	a_{122}	a_{123}	a_{124}	a_{125}	a_{126}	a_{127}	a_{128}	a_{129}	a_{1210}	a_{1211}	a_{1212}

DOI 10.17148/IARJSET.2019.6319

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
Vol. 6, Issue 3, March 2019

From the formula we can put the values of a_{ij} where $1 \le i, j \le 12$

$A_1 = A- 4 X (n-1) = A-8$

$B_1 = B+ 4 X (n-1) = B+8$

$C_1 = C+ 4 X (n-1) = C+8$

$D_1 = D- 4 X (n-1) = D-8$

$A_2 = A- 4n X (n-1) = A-24$

$B_2 = B+ 4n X (n-1) = B+24$

$C_2 = C+ 4n X (n-1) = C+24$

$D_2 = D- 4n X (n-1) = D-24$

$A_3 = A- 4 X (n^2-1) = A-32$

$B_3 = B+ 4 X (n^2-1) = B+32$

$C_3 = C+ 4 X (n^2-1) = C+32 D_3 = D- 4 X (n^2-1) = D-32$

By putting the values of a_{ij} we get this square,

A	B	C	D	A-12	B+12	C+12	D-12	A-24	B+24	C+24	D-24
D+3	C-3	B-1	A+1	D-9	C+9	B+11	A-11	D-21	C+21	B+23	A-23
B-2	A+2	D+2	C-2	B+10	A-10	D-10	C+10	B+22	A-22	D-22	C+22
C-1	D+1	A-1	B+1	C+11	D-11	A-13	B+13	C+23	D-23	A-25	B+25
A-4	B+4	C+4	D-4	A-16	B+16	C+16	D-16	A-28	B+28	C+28	D-28
D-1	C+1	B+3	A-3	D-13	C+13	B+15	A-15	D-25	C+25	B+27	A-27
B+2	A-2	D-2	C+2	B+14	A-14	D-14	C+14	B+26	A-26	D-26	C+26
C+3	D-3	A-5	B+5	C+15	D-15	A-17	B+17	C+27	D-27	A-29	B+29
A-8	B+8	C+8	D-8	A-20	B+20	C+20	D-20	A-32	B+32	C+32	D-32
D-5	C+5	B+7	A-7	D-17	C+17	B+19	A-19	D-29	C+29	B+31	A-31
B+6	A-6	D-6	C+6	B+18	A-18	D-18	C+18	B+30	A-30	D-30	C+30
C+7	D-7	A-9	B+9	C+19	D-19	A-21	B+21	C+31	D-31	A-33	B+33

Adding 1st row

$\sum a_{1j}=3(A+B+C+D)$ where $1 \le j \le 12$

Similarly, adding 2nd row

$\sum a_{2j}=3(A+B+C+D)$ where $1 \le j \le 12$

Similarly 3rd to 12th row

$\sum a_{3j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{4j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{5j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{6j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{7j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{8j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{9j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{10j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{11j}=3(A+B+C+D)$ where $1 \le j \le 12$

$\sum a_{12j}=3(A+B+C+D)$ where $1 \le j \le 12$

DOI 10.17148/IARJSET.2019.6319

So we can say ,

Rows are giving the same sum which we can call magic sum .

For it to become magic square columns and diagonals should have to give the same sum.

1st Column

$\sum a_{i1} = 3(A+B+C+D)$ where $1 \le i \le 12$

Similarly other columns are also giving the same sum.

Diagonal

$\sum a_{ii} = 3(A+B+C+D)$ where $1 \le i \le 12$

So we can say the formula is giving the magic square of sum n times the sum of 4X4 square.

Hence Proved

Lemma 1.2 proof

(i) $A \ge 4n^2 - 3 + 1$ as n=3

(ii) $B - A \ge 5$

(iii) $C - B \ge (4 \times n^2 + 1)$

(iv) $D - C \ge ((n^2 - 1) \times 8 + 1)$

(i) As in this case consider term $a_{1211} = A - 33$

Its minimum term for A series $34 = 4X3^2 - 3 + 1$

(ii) It is clearly seen that maximum value of A series is =A +2

And minimum value of B series is = B-2

So minimum number of elements between A and B is 4 so minimum difference is 5 to distinguish all terms.

(iii) As in this case minimum of C series is = C-3

And maximum value of B series is = B+33

So minimum no. of elements between B and C is 36 so minimum difference is 37 which can be written as $=4X3^2 + 1$

(iv) Minimum value of D series = D-32

Maximum value of C series = C+32

So minimum no. of elements between D and C is 64 so minimum difference is 65 which can be written as $=(3^2 - 1)X8 + 1$

Hence Proved

Example 1: Consider n=3

Solution: 12X12 magic square we have to make

$A = 4n^2 - 3 + 1 = 34$

$B = A + 5 = 39$

$C = B + 4 \times n^2 + 1 = 76$

$D = C + ((n^2 - 1) \times 8 + 1) = 141$

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
Vol. 6, Issue 3, March 2019

A	B	C	D	A-12	B+12	C+12	D-12	A-24	B+24	C+24	D-24
D+3	C-3	B-1	A+1	D-9	C+9	B+11	A-11	D-21	C+21	B+23	A-23
B-2	A+2	D+2	C-2	B+10	A-10	D-10	C+10	B+22	A-22	D-22	C+22
C-1	D+1	A-1	B+1	C+11	D-11	A-13	B+13	C+23	D-23	A-25	B+25
A-4	B+4	C+4	D-4	A-16	B+16	C+16	D-16	A-28	B+28	C+28	D-28
D-1	C+1	B+3	A-3	D-13	C+13	B+15	A-15	D-25	C+25	B+27	A-27
B+2	A-2	D-2	C+2	B+14	A-14	D-14	C+14	B+26	A-26	D-26	C+26
C+3	D-3	A-5	B+5	C+15	D-15	A-17	B+17	C+27	D-27	A-29	B+29
A-8	B+8	C+8	D-8	A-20	B+20	C+20	D-20	A-32	B+32	C+32	D-32
D-5	C+5	B+7	A-7	D-17	C+17	B+19	A-19	D-29	C+29	B+31	A-31
B+6	A-6	D-6	C+6	B+18	A-18	D-18	C+18	B+30	A-30	D-30	C+30
C+7	D-7	A-9	B+9	C+19	D-19	A-21	B+21	C+31	D-31	A-33	B+33

Putting the values of A,B,C and D in the above magic square.

34	39	76	141	22	51	88	129	10	63	100	117
144	73	38	35	132	85	50	23	120	97	62	11
37	36	143	74	49	24	131	86	61	12	119	98
75	142	33	40	87	130	21	52	99	118	9	64
30	43	80	137	18	55	92	125	6	67	104	113
140	77	42	31	128	89	54	19	116	101	66	7
41	32	139	78	53	20	127	90	65	8	115	102
79	138	29	44	91	126	17	56	103	114	5	68
26	47	84	133	14	59	96	121	2	71	108	109
136	81	46	27	124	93	58	15	112	105	70	3
45	28	135	82	57	16	123	94	69	4	111	106
83	134	25	48	95	122	13	60	107	110	1	72

And their Magic sum is **870.**

Example 2: Consider n=2

Solution: 8X8 magic square we have to make

$A=4n^2-3+1=14$

$B=A+5=19$

$C=B+4 \times n^2+1=36$

$D=C+((n^2-1) \times 8+1)=61$

DOI 10.17148/IARJSET.2019.6319

A	B	C	D	A-8	B+8	C+8	D-8
D+3	C-3	B-1	A+1	D-5	C+5	B+7	A-7
B-2	A+2	D+2	C-2	B+6	A-6	D-6	C+6
C-1	D+1	A-1	B+1	C+7	D-7	A-9	B+9
A-4	B+4	C+4	D-4	A-12	B+12	C+12	D-12
D-1	C+1	B+3	A-3	D-9	C+9	B+11	A-11
B+2	A-2	D-2	C+2	B+10	A-10	D-10	C+10
C+3	D-3	A-5	B+5	C+11	D-11	A-13	B+13

Putting the values of A,B,C and D

14	19	36	61	6	27	44	53
64	33	18	15	56	41	26	7
17	16	63	34	25	8	55	42
35	62	13	20	43	54	5	28
10	23	40	57	2	31	48	49
60	37	22	11	52	45	30	3
21	12	59	38	29	4	51	46
39	58	9	24	47	50	1	32

Their magic sum is **260**.

Remarks: All 4X4 magic square in the bigger square can be shuffled to make new magic square of same order . Previously there is no way to find these square but now we can solve these square with above method.

If n is even it will always give even sum and if n is odd it can give both odd and even sum. By selecting any 4X4 square which are used in the formation of bigger order square can change shuffle its positions with other square it will not affect the overall sum and new magic square can be made.

ACKNOWLEDGEMENT

I would like to thanks **Prof. A.k. Lal** and **Prof. S.S. Bhatia** of Thapar University for their helpful discussions and suggestions.

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