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Stability of Hopfield in Higher Dimension

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Abstract: A Hopfield neural network transfers information with feed-back connections. These are similar to magnetic materials where stability of the bit storage plays a crucial role in exchanging strength through the spin(+1 or -1) orientations. More stability helps in storage of multilevel data such as image data. A Complex Valued Hopfield Neural Network (CHNN) with a multi-stable Hopfield model has low stability in two-dimensional phase. Rotor Hopfield Neural Network (RHNN) added to CHNN increases its stability in multidimensional phase. Hyperbolic Hopfield Neural Network (HHNN) is an extension of CHNN by Clifford algebra. In our proposing system, we are extending the theories of stability between HHNN and RHNN by investigating this process through the projection rule. HHNN is independent of the resolution factor and there is a gradual increase in the noise tolerance. Thus, it is comparatively more stable than RHNN.

Keywords: CHNN, RHNN, HHNN, Projection Rule

I. INTRODUCTION

Artificial neural networks is imitation of biological brain. The main motive of neural networks is to recognize patterns in your data. An artificial neural network stores large amount of data and is capable of detecting complex non-linear relationships implicitly. Neural Networks play an important role in image processing with high efficiency, pattern recognition and signal processing [1].

Hopfield network is a form of neural network with feedback connections. It is a uni-layer network that consists of fully connected recurrent neurons. It is generally used to auto-associate and optimize the given task [2]. Hopfield network behaves as a content-addressable memory system with binary threshold nodes[2] (0, -1, +1). These are similar to magnetic materials where stability of the bit storage plays a vital role in exchanging their strength through spin orientations (+1 or -1) [29]. Thus, Stability has the potency to determine the appropriateness of neural network to accomplish a given task [23-26].

II. EXTENSION OF HOPFIELD MODELS FOR STABILITY

Recently, different models of Hopfield neural networks have been introduced. Hopfield neural networks were extended using Clifford algebra. These extensions directly deal with complex numbers and quaternion's of higher algebra. The complex and quaternion fields are 2-D and 4-D Clifford algebra respectively [10].

The neural network which directly deals with the complex numbers are framed as Complex Valued Hopfield Neural Networks (CHNN). Rotor Hopfield Neural Network (RHNN) is an added extension to Complex Valued Hopfield Neural Network (CHNN) in multi-dimensional phase. It is a multistate Hopfield model with excellent storage capacity and noise tolerance but also employ 2D vector form. The storage capacity of a RHNN is twice as that of a CHNN but it also requires double the connection weight parameters as of CHNN. CHNNs have less noise robustness compared to that of RHNN because they store rotated patterns where as RHNNs do not store rotated training patterns [11]. However, conventional learning methods for RHNNs such as Hebbian and gradient descent learning rules present difficulties regarding different parameters [16]. For example, storage capacity, noise robustness etc.

In this paper, we consider a projection rule for RHNN and demonstrate that the noise robustness of RHNN is better than that of CHNN. The proposed algorithm improves the noise robustness of RHNN [11]. As the number of training patterns increases, the noise robustness of CHNN rapidly deteriorates [18]. On the other hand, the noise robustness of RHNN reduces less rapidly for the same case. RHNN can easily recover from rotated patterns, unlike CHNN. We show this ability by computer simulation.

Hyperbolic Hopfield Neural Network (HHNN) is another extension of Complex Valued Hopfield Neural Network (CHNN) by Clifford algebra (higher order algebraic calculations) [12]. Hyperbolic algebra is a 2-D Clifford algebra and also act in higher dimensional phase. We also have analyzed hyperbolic backpropagation learning algorithms [25].

Several multistate models of Hopfield using hyperbolic algebra have been proposed. Hyperbolic algebra is does not act as a field but a ring since it has zero divisors. It is a commutative algebra. HHNNs and CHNNs need the equal number of connection weights. An HHNN with a directional activation function has been proposed to improve the noise tolerance [13].

Observations of computer simulation using the projection rule that we consider are as follows:
1) HHNN's noise tolerance, is independent of the resolution factor. It decreases rapidly as the number of training patterns increase [12].
2) CHNN's noise tolerance, rapidly decreases as the resolution factor increase and gradually decreases as the number of training patterns increase.

CHNNs are impractical for high-resolution like image data while HHNNs are practical only for a small number of training patterns.
The projection rule is done in a single learning algorithm that realizes fast training. Self-loops are usually removed in this projection rule. Meanwhile in the case of HHNNs, they can't be removed and so they can cause many pseudo memories and deteriorate noise tolerance. Behavior of the real parts of self-loops are investigated by computer simulation. The number of self-loops increase with the number of training patterns. Thus, the noise tolerance of HHNNs deteriorates as the number of training there is an increase in patterns [12].

The condition for stability of HHNNs is extended. The extended stability condition is applied to the projection rule and its noise tolerance is improved. We show this improvement in noise tolerance in the form of computer simulations.

III. STABILITY OF ROTOR HOPFIELD NEURAL NETWORKS WITH PROJECTION RULE

We will prove a theorem for the projection rule of RHNNs for its stability. Let's prove by using projection rule.
The r^{th} training vector is denoted as follows:

$$x^r = \begin{bmatrix} x_1^r \\ x_2^r \\ \vdots \\ x_N^r \end{bmatrix} \quad (1)$$

Where$x_n^r$is a vector. We define the training matrix as given below:

$$X = (x^1, x^2, x^3 \ldots\ldots x^R) \quad (2)$$

Where R is the number of training vectors, and is less than 2N (number of considerations). X is a 2N×Rmatrix. The training vectors $\{x^r\}$ are necessary to be linearly independent. Then, Lemma 1 is said to be true.
Lemma 1: $X^T X$is positive definite.
Proof: $X^T X$is obviously a symmetric matrix of order R. Let us consider a vector $m = (m_1, m_2, m_3 \ldots \ldots, m_R)^T$
Then, we obtain the following inequality equation:

$$^T(X^T X) = (Xm)^T (Xm) \quad (3)$$

$$= |Xm|^2 \geq 0 \quad (4)$$

Therefore, $X^T X$is nonnegative. Suppose $Xm = 0$Then, the following equality is true:

$$\sum_{r=1}^{R} m_r x^r = 0 \quad (5)$$

Since$m \neq 0$ the training vectors $\{x^r\}$ are linear dependent. This contradicts our assumption of the Training vectors.
From Lemma 1, there exists $(X^T X)^{-1}$We can consider the connectionweight matrix W=$X(X^T X)^{-1} X^T$Then, W X =X hold s for all r from Therefore, all the training vectors are fixed. This training algorithm(learning) is referred to as our Projection rule. Let's consider the use of the projection rule. In CHNNs, the diagonal components of W are often replaced with 0. This substitution is not allowed in RHNNs [19]. In CHNNs, the training vectors are fixed even after the diagonal components disappear. But, in the case of RHNNs, the training vectors are not fixed without the diagonal components when K (resolution factor[4]) is large. We prove the following significant theorem on the stability of RHNNs with the projection rule in synchronous mode.

Theorem 1: An RHNN with a projection rule W=$X(X^T X)^{-1} X^T$and synchronous mode converges to a fixedpoint.
Proof: From Lemma 1, S= $X^T X$is positive definite. For an eigenvector **v** corresponding to an eigen value λ, $S^{-1} v = \lambda^{-1} v$
Therefore, **v** is an eigenvector of S^{-1}corresponding to λ^{-1}
This implies that $S^{-1}$is positive definite.
We consider any vector $m = (m_1, m_2, m_3 \ldots \ldots, m_R)^T \neq 0$

$$m^T W m = m^T X (X^T X)^{-1} X^T m \quad (6)$$

$$= (X^T m)^T S^{-1} (X^T m) \geq 0 \quad (7)$$

Since $S^{-1}$is positive definite, the last inequality is true. Therefore,is nonnegative definite. Theorem 1 is not true in the case of CHNNs, and this represents oneof the great advantages of RHNNs.

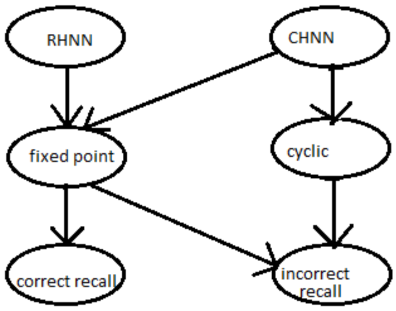


Fig.1.Although CHNNs may be trapped at a cycle, RHNNs with projection rule will not be. The patterns in cycles are not the training patterns. To recall a training pattern, both CHNNs and RHNNs must converge to a fixed point.

CHNNs and RHNNs are often applied to associative memories to store multilevel data (image data). These are necessary to remove noise from training patterns with noise. The training patterns are fixed points. Patterns in cycles are not training patterns. Convergence to a fixed point is effective for correct recall (Fig. 1).

IV. STABILITY OF HYPERBOLIC HOPFIELD NEURAL NETWORKS USING PROJECTION RULE

The projection rule is done in a single learning algorithm that realizes fast training. The storage capacity is $N - 1$. Let $x^z = (x_1^z, x_2^z, x_3^z \dots . x_N^z)^T$be the r^{th} training pattern, R is the number of training patterns, then the training matrix is given as
$X = (x^1 x^2 x^3 \dots \dots . . x^R)$(8)
Projection rule for CHNNs is described as
$B = P\,(P^T P)^{-1} P^T$ (9)

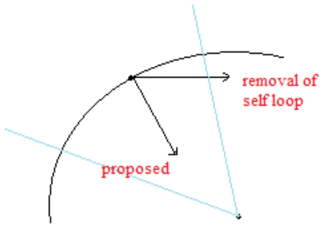
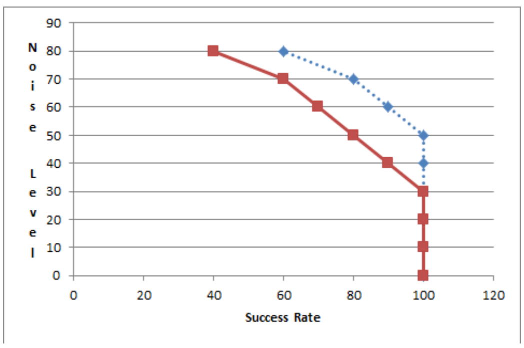


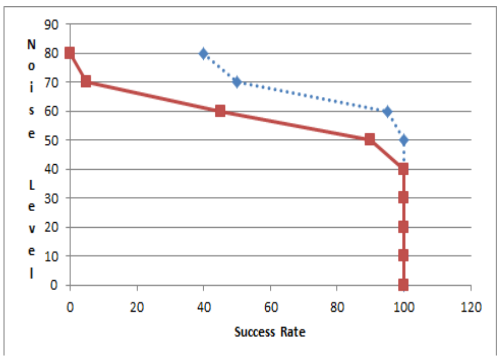
Fig. 2. The direction of weighted sum input is differing from that of the training pattern, if the self-loops were removed. If only the real part of self-loop is removed, the direction is kept. However, the HHNN does not converge to affixed point and can be trapped at a cycle. Based on Theorems, the real part of self-loop is removed.

V. RESULTS

a)Rhnn And Chnn:



K=4 P=10



K=4 P=20

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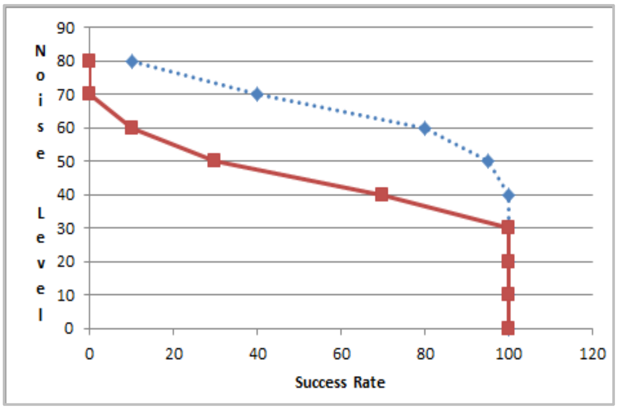
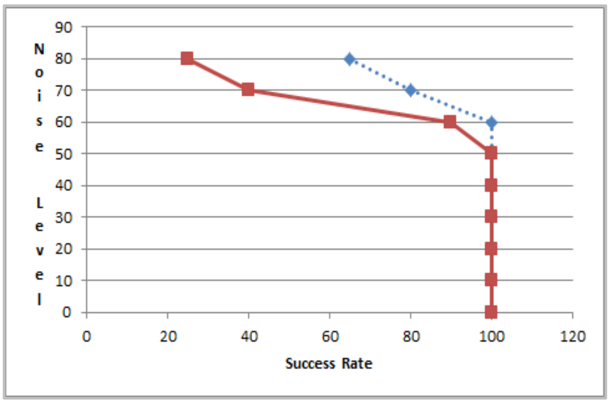
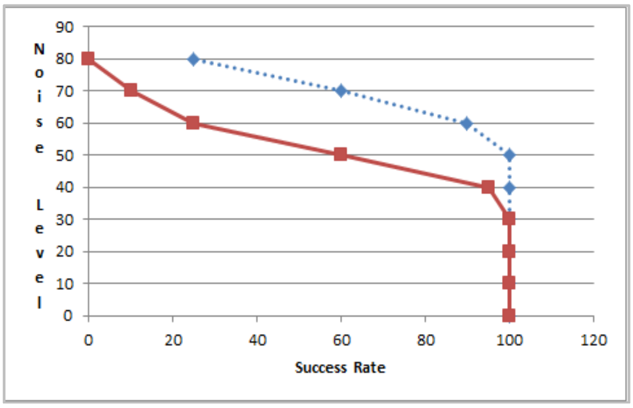


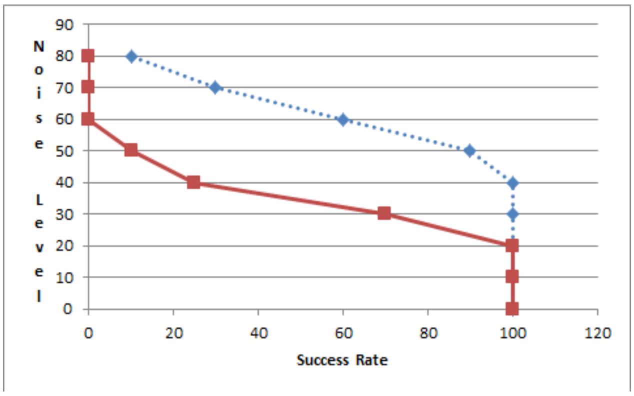
K=4 P=30



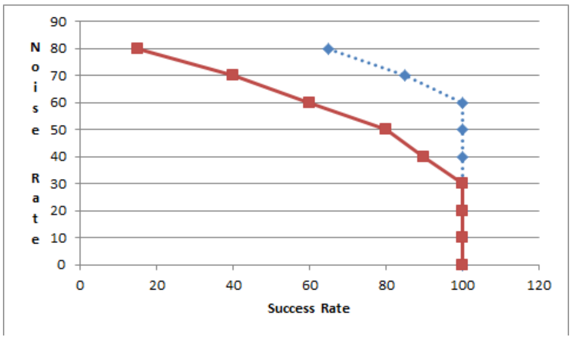
K=8 P=10



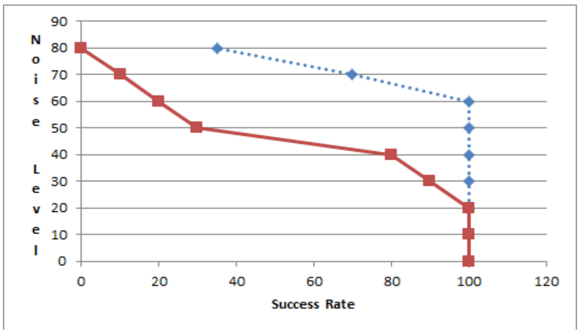
K=8 P=20



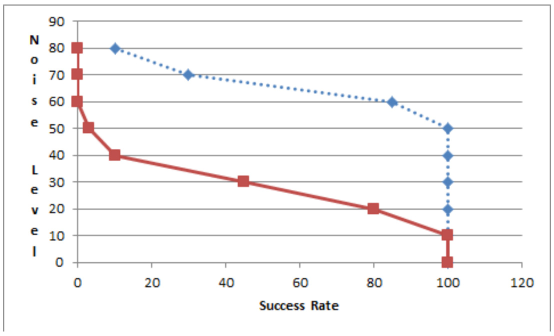
K=8 P=30



K=16 P=10



K=16 P=20



K=16 P=30

Fig.3.Computer simulations results different data with impulse noise.

X-Success Rate Y- Noise Level

From B P = P, we have B x ^{r}= x ^{r}Let b_{ij}be the (i, j)component Of B. From B^T = B is a real number, and $b_{ii}$is true for$b_{ij} = \overleftrightarrow{b_{ij}}$Then, its connection weight matrix is given by $i \neq j$(14)From$W = B - \text{diag}\, B, W^T = W$and $\text{diag}W = O$. For the sth training pattern, from$B\, x^s = x^s$the sum of inputs weights to neuron I is

$$I_i^s = \sum_{j=1}^{N} b_{ij}\, x_j^s - b_{ii}\, x_i^s \qquad (10)$$

$$= (1 - b_{ii}) x_i^s \qquad (11)$$

If b_{ii}, then we have $f(I_i^s) = x_i^s$ Therefore, if all the diagonalcomponents of Aare smaller than 1, all the training patterns are fixpoints' define the matrix

$$C = P\,(P^T P)^{-1} P^T \qquad (12)$$

The projection rule of the HHNNs is given W = C from W P = P, we find that all the training patterns are fixed points. The projection rule for the HHNNs satisfies condition. The self-loops, which are the diagonal components of W, that

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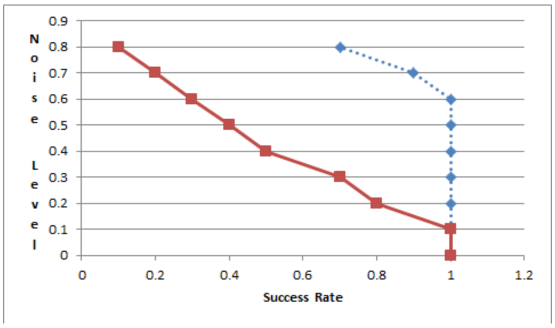
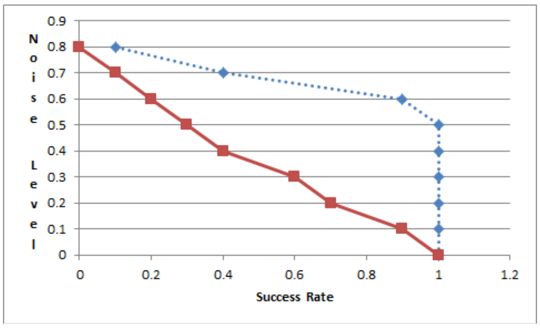
cannot be removed. If the self-loops are removed, the training patterns will not be fixed points. HHNN with the self-loops does not converge to a fixed point. However, the practically it does happen and did converge to a fixed point. Noise tolerance rapidly decreased whenever number of training patterns increased. Thus, we can say that stability of HHNN is better compared to that of CHNN.

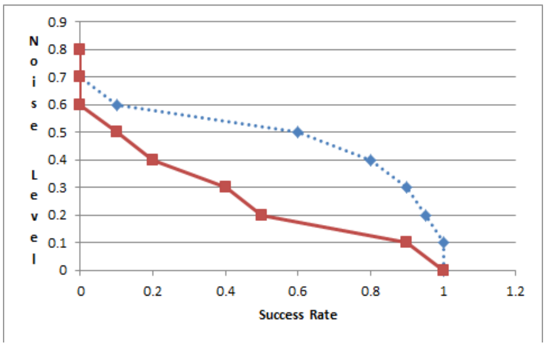
b)Hhnn and Chnn:



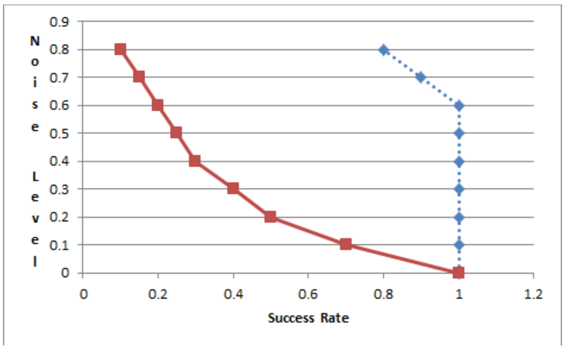
K=4 P=10



K=4 P=30



K=4 P=50



K=8 P=10

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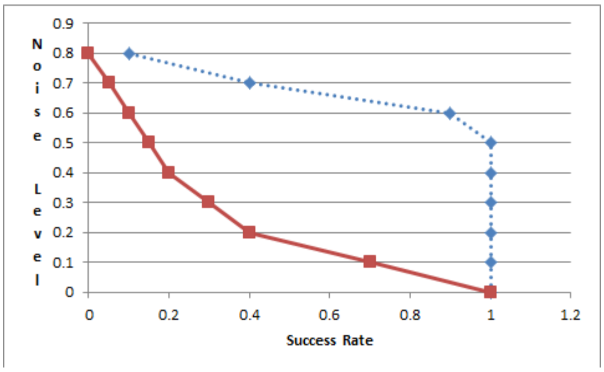
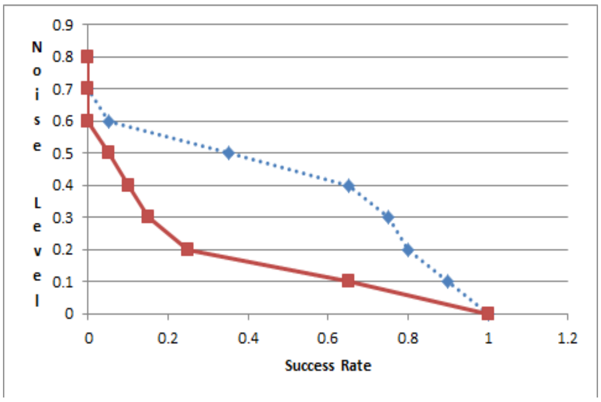
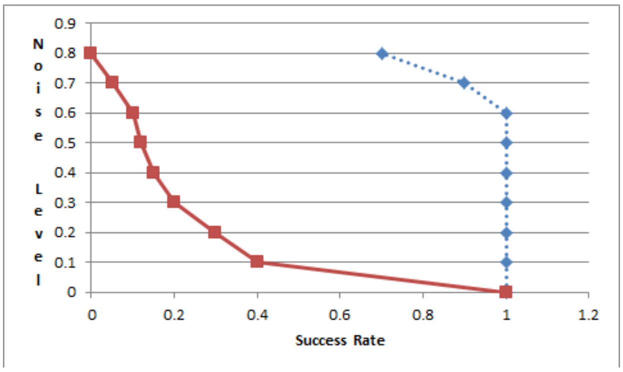


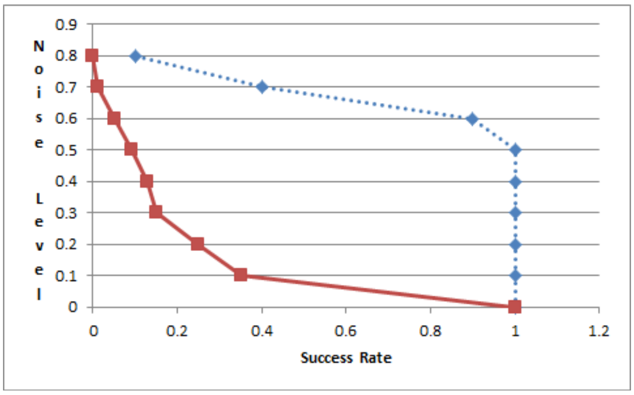
K=8 P=30



K=8 P=50



K=16 P=10



K=16 P=30

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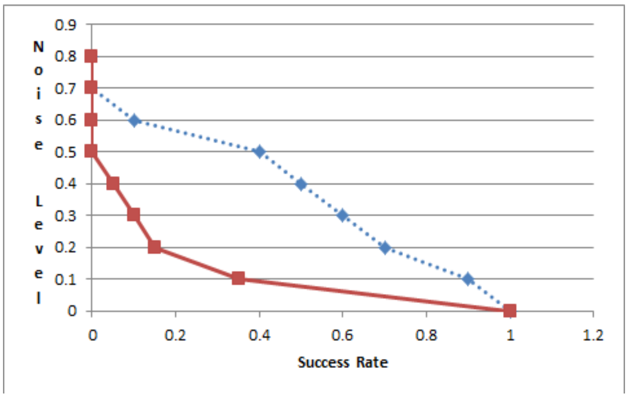
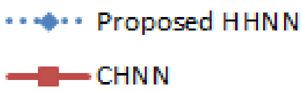


K=16 P=50

Fig.4.Computer simulations results using various different data with impulse noise.
The noise tolerance of CHNN is gradually decreases as K increases and gradually decreases as P increases.
Proposed HHNN is independent of K and decreases as P increases.

X-Success Rate Y-Noise Level



So far, computer simulations on CHNNs, RHNNs, proposed HHNNs have been conducted using randomly generated data and impulsive noise to investigate their noise tolerance.

Our assessed information through these computer simulations:
1) After randomly selecting training pattern, noise has been added. Random state is selected from S at the rate r, each neuron state is replaced with the selected one.
2) The trial is regarded as success if original pattern is completely recovered, else, it is regarded as failure.

Different K and P values are considered. K (resolution factor) = [4,8,16], P (noise tolerance) = [10, 30, 50]
For each (K, P) pair, we generated 100 training pattern sets - The noise rate r varied from 0.0 to 0.8 in steps of 0.05. For each r, we conducted 100 trials. A total of 10,000 trials were conducted for each triplet(K, P, r).
For the range of values of P varied from 10 to 50 in steps of 10, we conducted 100 trials and the behavioral changes in all the extensions of neural networks are noted. As P increased, the noise tolerance of proposed HHNNs decreased gradually. When we compare it to RHNN its noise is almost similar to that of CHNN. While, noise tolerance was almost independent of K. But in case of CHNNs its noise tolerance is rapidly deteriorated as K increased, and gradually decreased as P increased.

For P = 10, the CHNNs underperformed HHNNs noise tolerance. As P increased, the noise tolerance of CHNNs rapidly deteriorated. For P = 50, the noise tolerance of proposed HHNNs deteriorated faster than that of CHNNs. There is a gradual difference between CHNN and RHNN than CHNN and RHNN.

The following conclusions are derived from the simulation results:
1) The noise tolerance of CHNNs gradually deteriorated as P increased, and rapidly as K increased.
2) The noise tolerance of proposed HHNNs was independent of K, and gradually deteriorated as P increased.
3) Connection weights parameters used in HHNN are half of that of RHNN.

CHNNs showed low noise tolerance for large K as a result of forming pseudo memories. This is because of the inherent property Rotational variance of CHNN which forms pseudo memories for large K values. On the other hand, HHNNs settle the rotational in-variance. However, when the proposed projection rule is employed, it is not possible to remove the self-loops. All neuron states are stabilized by the positive real parts of self-loops. The computer simulations showed that the real parts of self-loops became larger as P increased. Thus, when P was small, the proposed HHNNs had better noise tolerance than the RHNNs, but when P was large, proposed HHNNs underperformed the others and reduced the real parts of self-loops and thus improved noise tolerance even in cases where P was large [12].

VI. CONCLUSION

An RHNN is an extension of a CHNN that improves storage capacity and noise tolerance. If the connection weight matrix is nonnegative, then the CHNN in synchronous mode converges to a fixed point. We extended this theorem to RHNNs. We also investigated the stability of RHNNs in the case of the projection rule. We proved that RHNNs converges to a fixed point. This is the one of the biggest advantages of RHNNs. In real time, for data such as image data, K (resolution factor) tends to be large. Since the noise tolerance of CHNNs is low under such conditions, they are not practical for the storage of high-resolution data [15]. Whereas in the case of HHNNs, noise tolerance is robust to K [6]. The noise tolerance is majorly affected by self-loops. Hence, we consider a proposed HHNNs, we modified the self-loops based on extended stability conditions to improve noise tolerance under large P conditions. Thus, the noise tolerance is improved with proposed HHNNs. But when we compared both RHNNs and HHNNs through CHNNs, HHNNs show best results than RHNNs as taking resolution factors and noise tolerances into consideration. Quaternionic Hopfield Neural Networks (QHNNs) is a twin-multistate activation function. QHNNs require half connection parameters as that of number of CHNNs[14]. We plan to extend the stability concepts in QHNNs using projection rule.

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