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# Hybrid Robust Control Approach to Enhance the Functioning of Semi- Active Suspension System

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**Abstract:** For a car with an intriguing combination of technical characteristics and aesthetic appeal, the suspension system plays a crucial role. The suspension system creates a link between road and the vehicle chassis, providing comfort and managing the dynamics caused due to unsynchronized motion. The goal of this paper is to analyze and model a robust control strategy applicable on Semi-Active Suspension. In this work, optimal H∞ controller is analyzed and an innovative control strategy is proposed by augmenting PID controller with H∞ optimal controller. The proposed controller further enhances the results under uncertain road conditions and gives robust results. Finally the controller is simulated using MATLAB/Simulink for functional evaluation.

**Keywords:** Semi-active Suspension, Optimal Control, Robust Control, H∞ Optimization, PID

## I. INTRODUCTION

Although modern control theory has been widely adapted for solving many engineering problems, but very few efforts are made to collaborate it with the conventional control theory. The modern suspension systems in the vehicles can be controlled on applying these control theories. The suspension systems are provided with dampers that exchange energy in order to provide comfort to the passengers. In general, these dampers are passive, meaning the design parameters cannot vary once they are installed.

The passive suspension systems are still widely used but their response to sudden uncertainties is either sluggish or unexpected. This is why active and semi-active suspension devices came into picture. Therefore, the implementation of these devices is already done in commercial vehicles and constant efforts are made to install them in aircrafts too[1]. Contrary to semi-active devices, active ones have large power requirements and if the controller fails they becomes unstable. But this is not the case with semi-active devices, as on control failure they convert into passive dampers and they have minimal power requirements[2].

Robust control has been trending these days and is being incorporated to solve various control problems. [3] presented the state space approach to evaluate H∞ norm using the solutions of ricatti equations and thus helps in solving H∞ problem for control systems. [4] constructed an actual size test bed using active suspension and proposed a new H-sub-infinity controller design. A nonlinear H∞ controller was designed in [5] to control vehicle vibration optimally. [6] introduced a model following robust control for semi-active suspension. Even autonomous steering of vehicles can be controlled using H∞ algorithm as in [7].

The H∞ control approach has been researched thoroughly in these papers. However, enhancement of suspension control is possible if conventional controller is added to the H∞ robust controller. In this paper, a 2-DOF quarter car model is operated with an innovative controller for improved stability and performance.

The paper is organized as follows. Section II describes the vehicle suspension model for a 2-DOF quarter car model using state space approach. This model is then utilized for Controller Design in Section III. The system thus formed is simulated using MATLAB and the results are analyzed in Section IV. Conclusions are finally made in Section V.

## II. MODELLING OF SUSPENSION SYSTEM

A schematic diagram of quarter car model with a semi-active suspension system is shown in Fig 1[2]. The semi-active suspension contains a spring and a controllable damper. The controllable damper is regulated by a force actuator who receives required feedback from the controller to enhance car ride and handling. The tire has been replaced by its equivalent stiffness and tire damping is neglected.

The model can be represented as a finite dimensional state space model[3]:

$$\dot{x} = Ax + B_1 w + B_2 u \tag{1}$$
$$z = C_1 x + D_{11} w + D_{12} u \tag{2}$$
$$y = C_2 x + D_{21} w + D_{22} u \tag{3}$$

Where x represents state variables, w represents exogenous inputs, u represents control force, y represents the measured variables, z represents performance vector, A represents the state matrix, $B_i$ represents the input matrix, $C_i$ represents the output matrix, and $D_{ij}$ represents the feed through matrix.

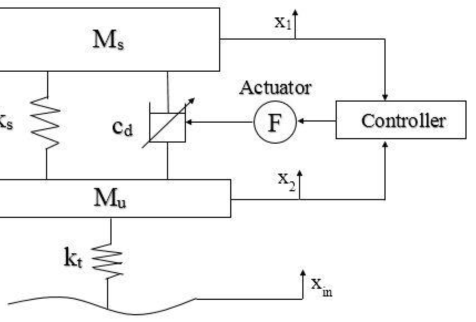


Fig. 1 Quarter car model with semi-active suspension system

Table 1: Vehicle Model Symbols

| Symbols | Physical Meaning |
|---|---|
| $M_s$ | Vehicle sprung mass |
| $M_u$ | Vehicle unsprung mass |
| $k_s$ | Stiffness of suspension spring |
| $k_t$ | Stiffness of tyre |
| $c_{d(initial)}$ | Damping coefficient of suspension |
| $x_1$ | Vertical displacement of sprung mass |
| $x_2$ | Vertical displacement of tyre |
| $x_{in}$ | Vertical road displacement |
| f | Force generated by controller |

The matrix values for entering in the equations (1-3) are as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_s}{M_s} & -\dfrac{c_d}{M_s} & \dfrac{k_s}{M_s} & \dfrac{c_d}{M_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{M_u} & \dfrac{c_d}{M_u} & -\dfrac{(k_s+k_t)}{M_u} & -\dfrac{c_d}{M_u} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k_t}{M_u} \end{bmatrix}; \; B_2 = \begin{bmatrix} 0 \\ -\dfrac{1}{M_s} \\ 0 \\ \dfrac{1}{M_u} \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -\dfrac{k_s}{M_s} & -\dfrac{c_d}{M_s} & \dfrac{k_s}{M_s} & \dfrac{c_d}{M_s} \end{bmatrix};$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

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$$D_{11} = \begin{bmatrix} 0 \end{bmatrix} \; ; \; D_{12} = \begin{bmatrix} -\frac{1}{M_s} \end{bmatrix};$$

$$D_{21} = D_{22} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$\mathrm{x} = [x_1;\ \dot{x}_1;\ x_2;\ \dot{x}_2];$$
$$\mathrm{w} = [x_{in}]\ ;\ \mathrm{u} = [\mathrm{f}];$$
$$\mathrm{z} = [\ddot{x}_1];$$
$$\mathrm{y} = [x_1; x_2; \dot{x}_1;\ \dot{x}_2\ ]$$

## III. INTEGRATED CONTROLLER DESIGN

In this section, the proposed controller is designed integrating H∞ controller with an additional PID controller to enhance the stability and performance of the quarter car semi-active suspension system. Initially the H∞ controller is designed for the required plant and subsequently the PID loop is added to enhance the performance of the system.

### A. Closed loop State-Space Modelling

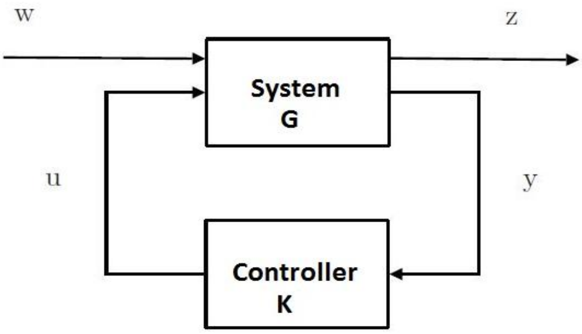


Fig. 2 General H∞ control configuration

In turn, block K represents the controller, an LTI model defined by a finite dimensional state space model[3]

$$\dot{x}_K = A_K x_K + B_K y \tag{4}$$
$$u = C_K x_K + D_K y \tag{5}$$

where the coefficients $A_K$ , $B_K$ , $C_K$ , $D_K$ are to be designed.

On integrating the plant with the designed controller, the block diagram in fig. 2 defines a valid closed loop state space model

$$\dot{x}_{CL} = A_{CL} x_{CL} + B_{CL} w \tag{6}$$
$$z = C_{CL} x_{CL} + D_{CL} w \tag{7}$$

where

$$x_{CL} = \begin{pmatrix} x \\ x_K \end{pmatrix};$$

$$A_{CL} = \begin{pmatrix} A + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{pmatrix}$$

$$B_{CL} = \begin{pmatrix} B_1 + B_2 D_K D_{21} \\ D_K D_{21} \end{pmatrix}$$
$$C_{CL} = (C_1 + D_{12} D_K C_2 \quad D_{12} C_K)$$

$$D_{CL} = (D_{11} + D_{12} D_K D_{21})$$

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Now for designing the coefficients of controller K, we evaluate the closed-loop transfer function from w to z which is given by the linear fractional transformation (lft) is[3]:

$$z = F_l(G, K)w \tag{8}$$

Now, the general H∞ optimal control problem is to find all stabilizing controllers K which minimize

$$\|F_l(G, K)\|_\infty = \sup_{w} \bar{\sigma}(F_l(P, K)(jw)) \tag{9}$$

Practically, it is not necessary to find an optimal controller for H∞ problem, and a suboptimal controller is easier and simpler to design computationally (i.e. approximately closest to the H∞ norm). Let min be the minimum value of $\|F_l(G, K)\|_\infty$ over all stabilizing controllers K. Then the H∞ sub-optimal control problem is: given $\gamma > \gamma_{min}$, find all stabilizing controllers K such that

$$\|F_l(G, K)\|_\infty < \gamma \tag{10}$$

This can be solved efficiently using the solutions of the Ricatti equations, and by reducing $\gamma$ iteratively using bisection algorithm[3], an optimal solution is approached. The algorithm is summarized below with all the simplifying assumptions.

### B. Design of central controller

Before applying the H∞ algorithm the following assumptions are to be followed:

1. $D_{12}$ and $D_{21}$ have full rank,
2. $\begin{pmatrix} A - jwI & B_2 \\ C_1 & D_{12} \end{pmatrix}$ has full column rank for all w,
3. $\begin{pmatrix} A - jwI & B_1 \\ C_2 & D_{21} \end{pmatrix}$ has full row rank for all w,
4. $(A, B_1)$ is stabilizable and $(A, C_1)$ is detectable.

For the general control configuration of fig. 2 described by equations (1)-(7), with assumptions, there exists a stabilizing controller K such that $\|F_l(G, K)\|_\infty < \gamma$ if and only if[5]

i. $X_\infty \geq 0$ is a solution to the algebraic Riccati equation
$A^T X_\infty + X_\infty A + C_1^T C_1 + X_\infty(\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X_\infty = 0$
such that Re $\lambda_i[A + (\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X_\infty] < 0, \forall i$; and

ii. $Y_\infty \geq 0$ is a solution to the algebraic Riccati equation
$A Y_\infty + Y_\infty A^T + B_1 B_1^T + Y_\infty(\gamma^{-2} C_1^T C_1 - C_2^T C_2) Y_\infty = 0$
such that Re $\lambda_i[A + Y_\infty(\gamma^{-2} C_1^T C_1 - C_2^T C_2)] < 0, \forall i$; and

iii. $\rho(X_\infty Y_\infty) < \gamma^2$.[where $\rho(.)$ is spectral radius]

All such controllers are then given by K = $F_l(K_c, Q)$
where

$$K_c = \begin{bmatrix} A_\infty & -Z_\infty L_\infty & Z_\infty B_2 \\ F_\infty & 0 & I \\ -C_2 & I & 0 \end{bmatrix}$$

$$F_\infty = -B_2^T X_\infty\,, \; L_\infty = -Y_\infty C_2^T\,, Z_\infty = (I - \gamma^{-2} Y_\infty X_\infty)^{-1}$$

$$A_\infty = A + \gamma^{-2} B_1 B_1^T X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2$$

and Q(s) is any stable proper transfer function such that $\|Q\|_\infty < \gamma$. For Q(s) = 0, we get

$$K = K_{C_{11}} = -Z_\infty L_\infty (sI - A_\infty)^{-1} F_\infty \tag{11}$$

This is called the "central" controller and has the same number of states as the generalized plant G. The central controller can be separated into a state estimator (observer) of the form

$$\dot{\hat{x}} = A\hat{x} + B_1 \underbrace{\gamma^{-2} B_1^T X_\infty \hat{x}}_{\hat{w}_{worst}} + B_2 u + Z_\infty L_\infty (C_2 \hat{x} - y) \tag{12}$$

and a state feedback

$$u = F_\infty \hat{x} \tag{13}$$

Now u is used as the control force given as a feedback to the plant G. This controller can now be used in SIMULINK for the purpose of simulation. These mathematical equations can be solved using MATLAB Robust Toolbox.

### C. The Proposed Controller

To further enhance the stability and performance of the system in comparison to the H∞ controller, the system is augmented using a PID controller with the H∞ controller. This PID – H∞ Controller (fig. 3) is thus the proposed controller.

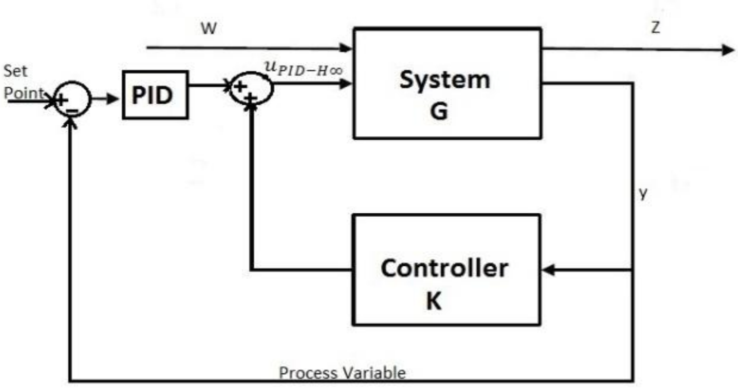


Fig. 3: PID - H∞ configuration

The PID tuning is performed using Ziegler-Nichols method. This method tunes the values of the PID controller and gives optimal values of the PID constants. This proposed controller helps in disturbance rejection and further stabilizes the system.

## IV. SIMULATION PARAMETERS AND RESULTS

The simulation parameters and results of the H∞ and PID - H∞ controllers are presented in this section. The 2-DOF quarter car model has been subjected to a random road input and required control damping force is gained. The quarter car model is simulated at car velocity of 60 km/hr. The results were obtained in terms of body acceleration, body displacement and Fourier transform of acceleration

Table 2: Model Parameter for 2-DOF quarter vehicle model

| Parameter | Value | Unit |
|---|---|---|
| $M_s$ | 365 | kg |
| $M_u$ | 40 | kg |
| $k_s$ | 19960 | N/m |
| $k_t$ | 175500 | N/m |
| $c_{d(initial)}$ | 1290 | Ns/m |

A random road profile is generated according to the International Organization for Standardization (ISO 8608)[8]. It gives a depiction of the road profile through estimation of the PSD of the vertical displacements $G_d$, as a function of spatial frequency $n$ $(n = \Omega/2\pi\ cycles/m)$. The ISO 8608 introduces a classification which is evaluated in accordance with conventional values of spatial frequency $n_0 = 0.1\ cycles/m$. Eight classes of roads are identified; from class A to class H according to the values of $G_d(n_0)$ established in ISO 8608.

In simulations, the ISO 8608 gives that the roughness of the road surface profile can be defined using the equations[8];

$$G_d(n) = G_d(n_0) \cdot \left(\frac{n}{n_0}\right)^{-w} \quad (14)$$

where, $w$ is the waviness and taken to be 2, the PSD of vertical displacement $G_d$ as a function of spatial frequency $n$. This random road profile is thus utilized for the purpose of simulation as an input to the system.

Fig.4 and fig. 5 demonstrates the body acceleration vs time plot for the H∞ control and PID-H∞ control of semi-active suspension system.

It can be observed from fig. 4 that the maximum value of acceleration achieved by passive system is about 4m/s$^2$, whereas, the same for H∞ is found to be about 2.97m/s$^2$. The pattern also shows significant reduction in the values of acceleration.

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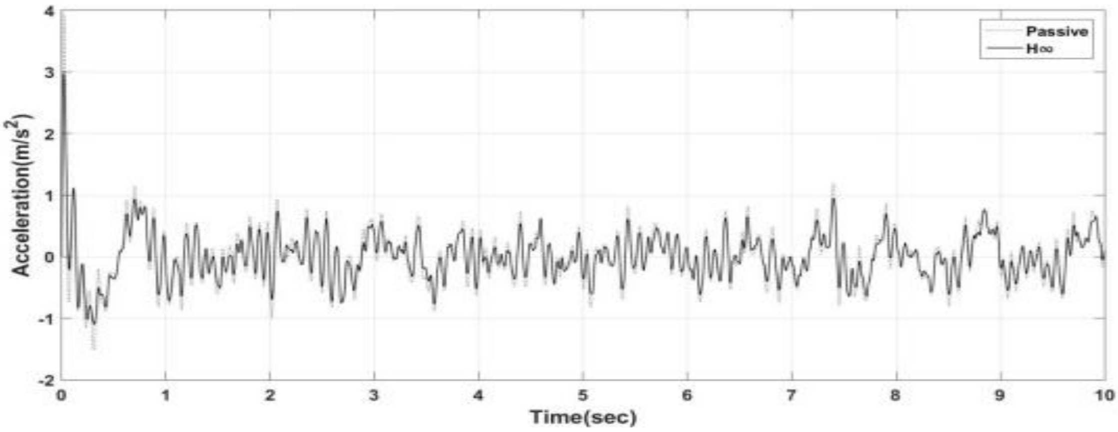


Fig 4 : Body acceleration response of 2-DOF quarter car H∞ controller for random road input

Fig. 5 shows use of PID – H∞ control. It is observed that the algorithm has much more diminished values of acceleration. The initial magnitude of acceleration is found to be less than 1.8m/s$^2$ which is almost 55% less than the passive system.

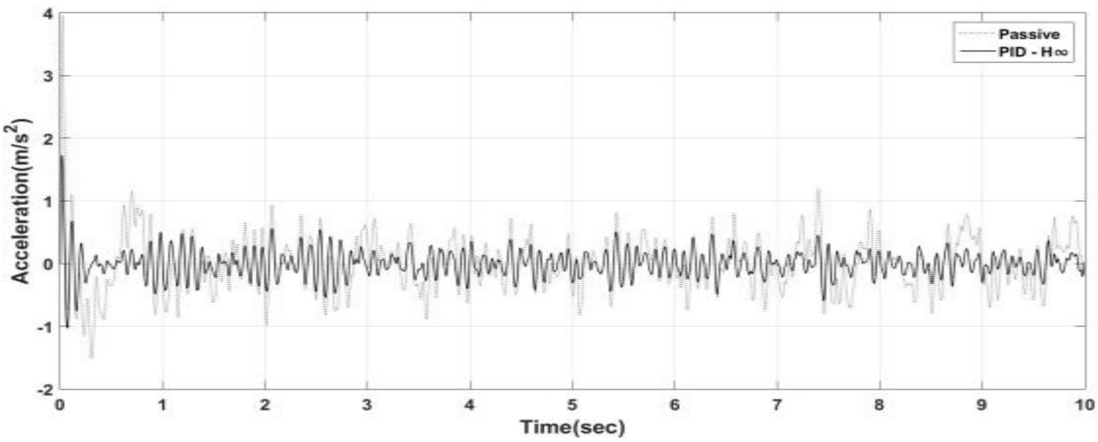


Fig. 5: Body acceleration response of 2-DOF quarter car PID – H∞ controller for random road input.

Fig. 6 displays body displacement vs time plot for PID-H∞ control for semi- active suspension system.

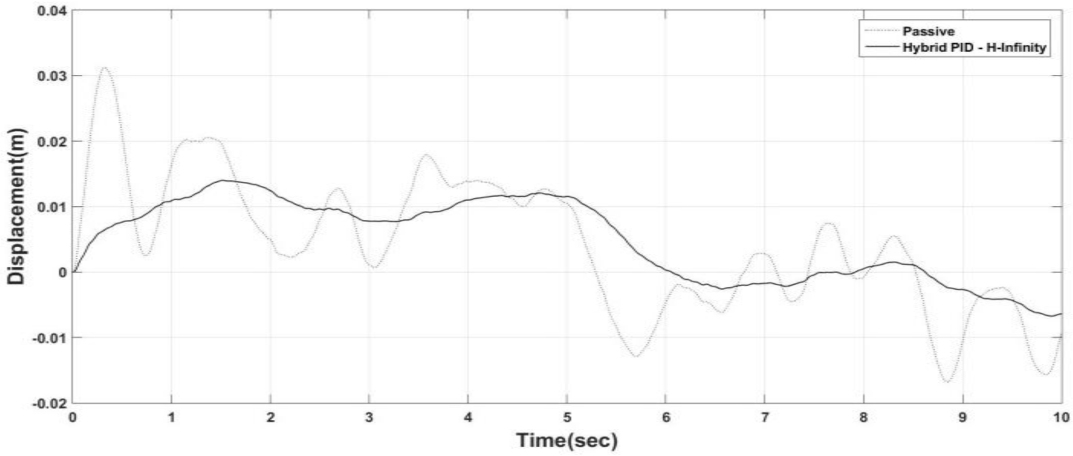


Fig. 6: Body displacement response of 2-DOF quarter car PID – H∞ controller for random road input.

It is perceived from the fig. 6 that body displacement reduces to much greater extent and sprung mass do not make much vertical displacement in comparison to the passive suspension system.
Fig. 7 and fig 8 represents the frequency domain response of the H∞ controller and the PID-H∞ controller respectively.

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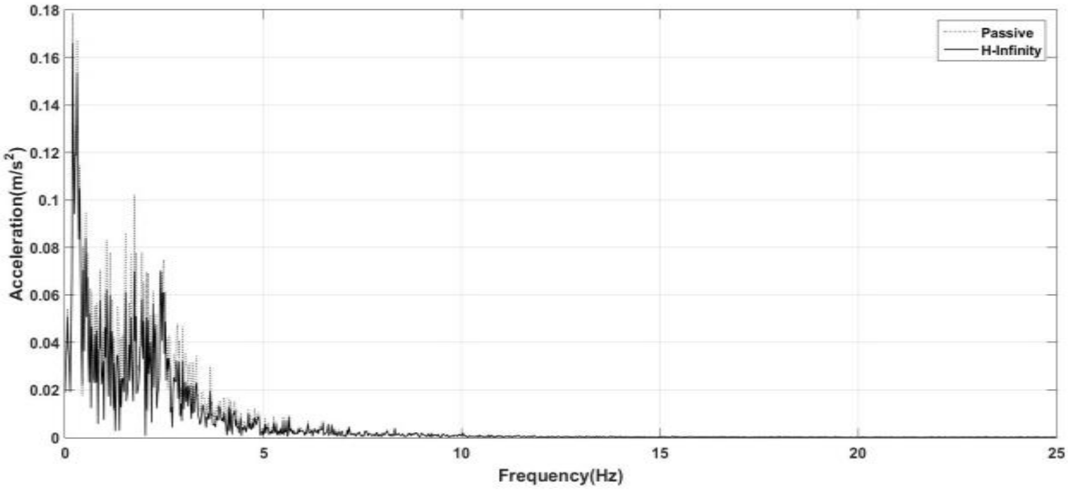


Fig. 7: FFT of body acceleration of 2-DOF quarter car H∞ controller for random road input.

It can be observed in fig. 7 that during frequencies from 1 Hz to 10 Hz the H∞ controller based semi active suspension gives better results than passive suspension. Humans are sensitive to vibrations dependent on frequency. Humans are more sensitive to vertical acceleration at 4 Hz to 8 Hz, based on ISO 2631-1[9].
Similar observations can be made in fig. 8 that PID-H∞ controller reduces magnitude of frequency response to a great extent in comparison to passive system. Thus less effort is experienced by the person travelling in the vehicle. The force generated by the controllers are also reduced to a much extent.

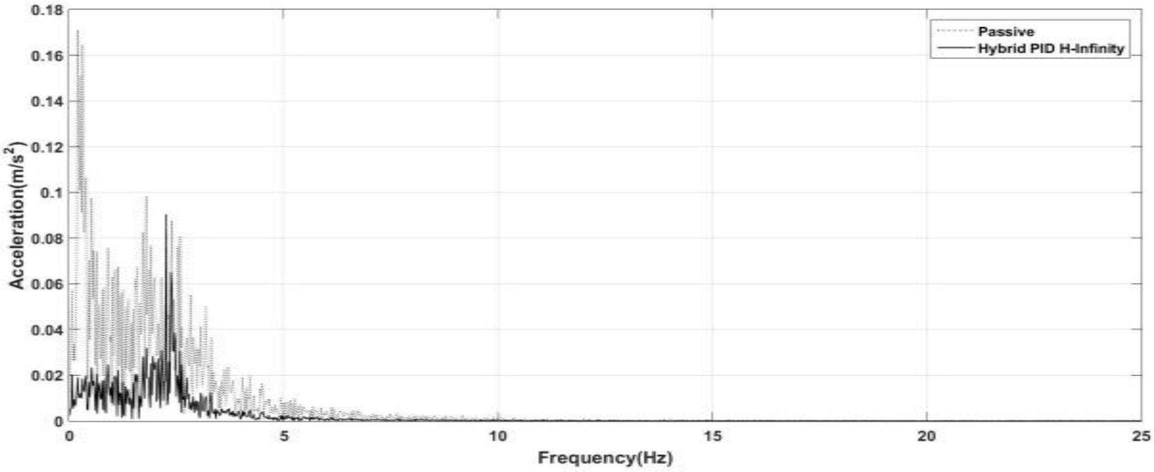


Fig. 8: FFT of body acceleration of 2-DOF quarter car PID-H∞ controller for random road input.

In summary, the PID-H∞ controller simultaneously improves the body acceleration as well as frequency response of the system when compared with conventional H∞ controller. Thus the proposed controller is better in terms of performance and stability.

## V. CONCLUSION

This paper shows a design for a PID-H∞ controller for 2-DOF quarter car semi-active suspension system to improve stability as well as performance of the system. The dynamic model of road vehicle had been simulated through MATLAB/Simulink® environment. Vertical dynamics has been carried out for the road vehicle model. A 2-DOF quarter car model is used for the analysis. Velocity input at the tire is given by considering random road irregularity based on ISO standard.
The robust H∞ controller gives optimal results for body acceleration and body displacement. Uncertain road conditions can be handled safely by the H∞ controller. PID Controller when augmented with H∞ controller creates optimal changes in sprung mass acceleration but no change is observed in sprung mass displacement. Smoothness during the ride is enhanced using PID controller. The control strategy presented in this paper can improve ride comfort without sacrificing much of the road handling conditions. In future, full car structure will be considered as well as lateral dynamics will be inspected.

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