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Using Mathematical Modelling to Forecast Population Trends in Bangladesh

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Abstract: Bangladesh is among the most populous countries in South Asia. The high population in the country has occasioned serious challenges including pressure on social amenities and for this reason, it is important to devise a reliable population prediction mechanism. This paper will analyze an appropriate model that can be used to forecast the Bangladesh population. Precisely, the paper will be centered on the projected population of the country from 2020 to 2050. An exploration of the projected population using the Malthusian model reveals that the projected population of Bangladesh will be around 267.4 million people by 2050.

Keywords: Resource allocation, Economic policy, Forecast population, Verhulst model, Malthusian growth model

I. INTRODUCTION

Population refers to a collection of people that reside in a particular area. Population is a very fundamental aspect when it comes to the operations of any country since influences resource allocation, economic policy, availability of manpower and other such factors. It is important to limelight that a high population might put pressure on available resources. Key resources such as water, housing and food are finite in nature and a high population might result in the depletion of such resources. To this end, it is very vital to have a reliable population prediction model which can estimate a country's population at any time.

Population prediction relies heavily on biomathematics. Biomathematics also referred to as mathematical biology is an area that deals with the application of mathematical models and analysis techniques to study biological phenomenon. Population growth is regarded to one of the most notable biological phenomena that can be analyzed using biomathematics. Biomathematics thus analyses some of the consequences that come about with respect to population growth. Within a given population, there are individual organisms that interact with one another and this is an important aspect when it comes to analyzing the characteristics of the population.

Mathematical modelling is an analysis technique that lends itself to biomathematics. A precise definition of mathematical modeling is the use of mathematical techniques and computation to represent the real-life scientific phenomenon. Mathematical modelling is integral when it comes to analyzing population growth. Different models can be used in analyzing population growth but the most reliable one is the one that was developed by Thomas R. Malthus, a UK based mathematician in 1798. Malthus underscored the significance of mathematical modeling in analyzing population growth. He further warned that over-population can lead to significant challenges in the society since human population growth is geometric in nature while food supplies increases in an arithmetic fashion.

Precisely, the model of population growth takes into account the constrained nature of resources. This is because; people are expected to live in an area that has adequate resources to support their lives. The aspect of carrying capacity explains the nature of the relationship between population and resources. Under the Verhulst model, population growth should not exceed its carrying capacity.

A less constrained environment is likely to experience higher population growth as compared to a highly constrained environment. In such an environment, there a high level of access to food, water, accommodation and other essential goods. On the other hand, a highly constrained environment is characterized by limited access to some of the aforementioned resources that are crucial to support human life.

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Malthusian growth model

The Malthusian growth model is a reliable approach of analyzing population growth. The model relies on a constant rate. Using the model, t represents time while (t) the number of items present at time t.
Assumption of the model: P(t)is continuously differentiable [1].

Population Growth

Population growth rate represents the rate of change in population [5]. The change might be positive in nature whereby there is an increase in population or, negative which represents a decline in population.
If the population (t) varies at time t ($t + \partial t$) the average rate of per capita growth at the time t is $\frac{P(t+\partial t)-P(t)}{P(t)t}$
Given limit $\partial t \to 0$,instant growth rate at the time *t*
Given,
b = Congenital birth rate.
d =Congenital death rate
The population growth rate can be obtained by calculating the intrinsic growth rate *r*.
$r = b - d$
Now, Take a single population species as defined in the growth model is

$$\frac{P'(t)}{P(t)} = r$$

$$P'(t) = rP(t) \quad \ldots\ldots\ldots (1)$$

With the population at first $P(t_0) = P_0 > 0$. So we've got that
Mathematical model defined the growth of the population of individual species as: -

$$P'(t) = rP(t), P(t_0) = P_0 > 0$$

This equation's general solution (1) is$P(t) = ce^{rt}$. For the initial condition $P(t_0) = P_0$
We have,

$$P(t_0) = ce^{rt_0} = P_0$$

$$c = P_0 e^{-rt_0}$$

$$P(t) = P_0 e^{-rt_0} e^{rt} = P_0 e^{r(t-t_0)}$$

Therefore $P(t) = P_0 e^{r(t-t_0)} \quad \ldots\ldots\ldots (2)$
That is known as the Malthusian growth principle [5]

II.VERHULST MODEL

Another model for predicting population growth is the Verhulst model. The Verhulst model was put forward by Pierre Verhulst, a Belgian based mathematical biologist [6]. The model draws a correlation between population growth and it's carrying capacity [10]. In developing the Verhulst model [13], Pierre took into account the impact of the previous population on population growth with terms a and b representing integral coefficients of the population.

$$\frac{a - bP(t)}{a} \ldots\ldots\ldots\ldots (3)$$

As the value of the population gets near to$\frac{a}{b}$, the term gets reduces significantly and eventually tends to zero [15]. Ultimately, Verhulst model takes into consideration the impact of competition for the limited resources that are required to support the population.
Under the Verhulst model, sustained increase in population can be represented by the equation:

$$\frac{dP}{dt} = rP$$

P in this equation is the population at time t and r represents the proportionality constant.

$$\frac{dp}{dt} = \frac{ap(t)[a - bp(t)]}{a} \ldots\ldots\ldots\ldots (4)$$

Solving (4) and applying the initial condition, Verhulst law of population growth P (t0) = P0, where P0 is the population at time t =0.

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The above equation (4) P (t0) = P0 is the expected Bangladesh population using the Verhulst model.

$$\frac{dp}{dt} = ap - bb^2$$

$$\frac{dp}{p(a-bp)} = dt \qquad \ldots\ldots\ldots (5)$$

Through applying variables separation

$$\frac{1}{a}\left(\frac{1}{p} + \frac{b}{a-bp}\right) = dt \qquad \ldots\ldots\ldots\ldots (6)$$

Now integrating (6), we get

$$\frac{1}{a}[lnp - \ln(a-bp)] = t + c \qquad \ldots\ldots\ldots\ldots (7)$$

At $t = 0$ we get $P(t_0) = P_0$

$$c = \frac{1}{a}[lnp - \ln(a-bp_0)] \qquad \ldots\ldots\ldots\ldots (8)$$

Equation (7) transforms into

$$\frac{1}{a}[lnp - \ln(a-bp)] = t + \frac{1}{a}[lnp_0 - \ln(a-bp_0)]$$

$$P = \frac{a/b}{1 + (\frac{a/b}{P_0} - 1)e^{-at}} \qquad \ldots\ldots\ldots\ldots (9)$$

Using the limit of equation (9) as $t \to \infty$, we get

$$P_\infty = \lim_{t\to\infty} P = \frac{a}{b}$$

Using t=0, t=1, t=2, the values of P_0 , P_1 , P_2 correspondingly.
Then we have equation (9)

$$\frac{b}{a}(1 - e^{-a}) = \frac{1}{P_1} - \frac{e^{-a}}{P_0}$$

$$\frac{b}{a}(1 - e^{-2a}) = \frac{1}{P_2} - \frac{e^{-2a}}{P_0} \qquad \ldots\ldots\ldots\ldots (11)$$

Remove $\frac{b}{a}$ we get $e^{-a} = \frac{P_0(P_2 - P_1)}{P_2(P_1 - P_0)}$ $\ldots\ldots\ldots\ldots (12)$

Replacing the values of e^{-a} in to the equation (9), we obtain

$$\frac{b}{a} = \frac{{P_1}^2 - P_0P_2}{P_1(P_0P_1 - 2P_0P_2 + P_1P_2)} \qquad \ldots\ldots\ldots\ldots (13)$$

Thus, limiting the value of P, we get

$$P_{max} = \lim_{t\to\infty} P = \frac{a}{b} = \frac{P_1(P_0P_1 - 2P_0P_2 + P_1P_2)}{{P_1}^2 - P_0P_2} \qquad \ldots\ldots (14)$$

III.RESULTS AND DISCUSSIONS

Malthusian Model:
The equation of Malthusian Model is

$$P(t) = P_0e^{rt}$$

By calculating we find the growth rate $r = 0.01385$
The general solution, therefore, is $P(t) = 143.4e^{0.01385t}$ $\ldots\ldots\ldots\ldots (15)$
This suggest that population growth is projected at 2% in Bangladesh with this we project the population of Bangladesh from 2005 to 2050.

Verhulst Model: Again from the verhulst model we have

$$P(t) = \frac{KP_0}{P_0 + (K - P_0)e^{-rt}} \qquad \ldots\ldots\ldots\ldots (16)$$

By calculating we find the growth rate $r = 0.024$ and carrying capacity $K = 318.6$

Hence the general solution of verhulst model becomes

$$P(t) = \frac{45687.24}{143.4 + 175.2e^{-0.024t}} \qquad \ldots\ldots\ldots (17)$$

This formula is used to determine the population's expected values [17].

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Bangladesh's projected population using both models is provided with actual data in the Table from 2005 to 2050.[25] Bangladesh's predicted population was estimated from the equations (15) and (17),

Year	Time Notation	Actual population (in millions)	Projected population (in millions)			
			Malthusian model	Absolute Percentage Error	Verhulst model	Absolute Percentage Error
2005	0	143.4	143.4	0.0%	143.4	0.0%
2006	1	145.4	145.4	0.0%	145.3	-0.1%
2007	2	147.1	147.4	0.2%	147.2	0.1%
2008	3	148.8	149.5	0.5%	149.1	0.2%
2009	4	150.5	151.6	0.7%	151.0	0.3%
2010	5	152.2	153.7	1.0%	152.9	0.5%
2011	6	153.9	155.8	1.2%	154.8	0.6%
2012	7	155.7	158.0	1.5%	156.7	0.6%
2013	8	157.6	160.2	1.6%	158.6	0.6%
2014	9	159.4	162.4	1.9%	160.6	0.8%
2015	10	161.2	164.7	2.2%	162.5	0.8%
2016	11	163.0	167.0	2.5%	164.4	0.9%
2017	12	164.7	169.3	2.8%	166.3	1.0%
2018	13	166.4	171.7	3.2%	168.2	1.1%
2019	14	168.1	174.1	3.6%	170.1	1.2%
2020	15		176.5		172.0	
2021	16		179.0		173.9	
2022	17		181.5		175.8	
2023	18		184.0		177.7	
2024	19		186.6		179.6	
2025	20		189.2		181.4	
2026	21		191.8		183.3	
2027	22		194.5		185.2	
2028	23		197.2		187.0	
2029	24		199.9		188.9	
2030	25		202.7		190.7	
2031	26		205.6		192.6	
2032	27		208.4		194.4	
2033	28		211.3		196.2	
2034	29		214.3		198.0	
2035	30		217.3		199.8	
2036	31		220.3		201.6	
2037	32		223.4		203.3	
2038	33		226.5		205.1	
2039	34		229.6		206.8	
2040	35		232.8		208.6	
2041	36		236.1		210.3	
2042	37		239.4		212.0	
2043	38		242.7		213.7	
2044	39		246.1		215.4	
2045	40		249.5		217.1	
2046	41		253.0		218.7	
2047	42		256.6		220.4	
2048	43		260.1		222.0	
2049	44		263.8		223.6	
2050	45		267.4		225.2	
Mean Absolute				1.5%		0.6%

Let us now present, using the *Malthusian model* (15) and the *Verhulst model* (17), the simulation results of the real and projected Bangladesh population from *2005 to 2019* in Figures 1 and 2, respectively.

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Graph:

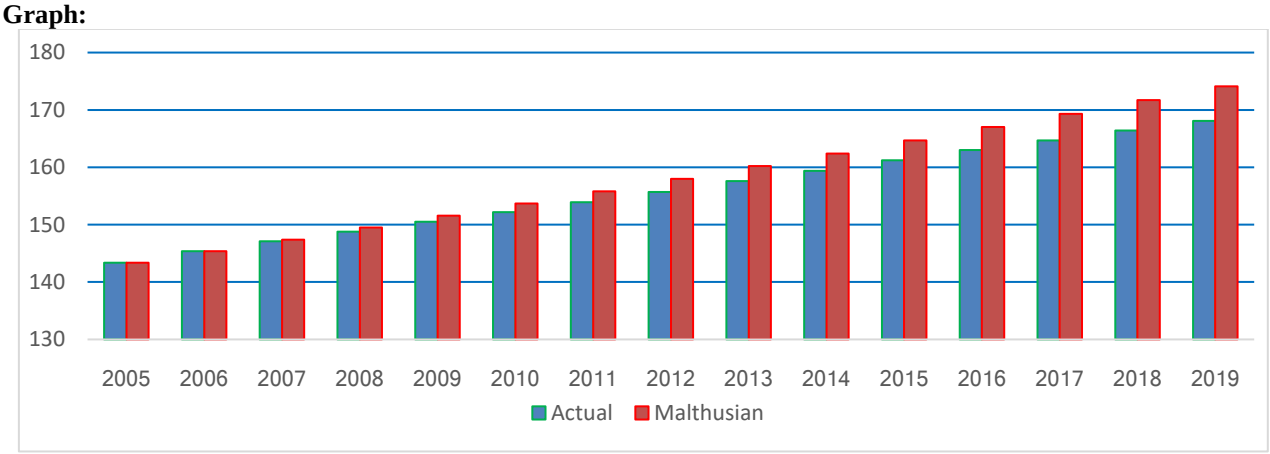

Figure 1: Bangladesh's real(green) & expected(red) populations using the Malthusian model(15) between 2005 & 2019

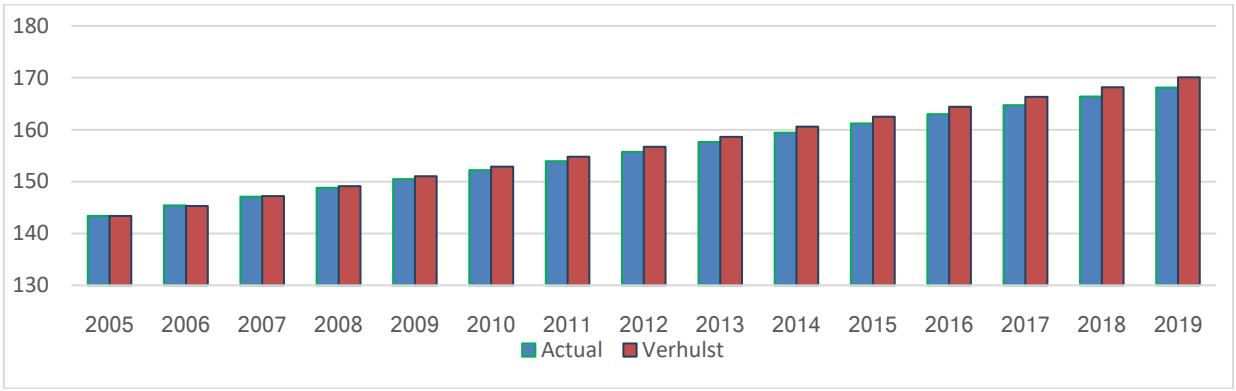

Figure 2: Bangladesh's real (blue) and expected (red) populations from 2005 to 2019 using the Verhulst model (17)

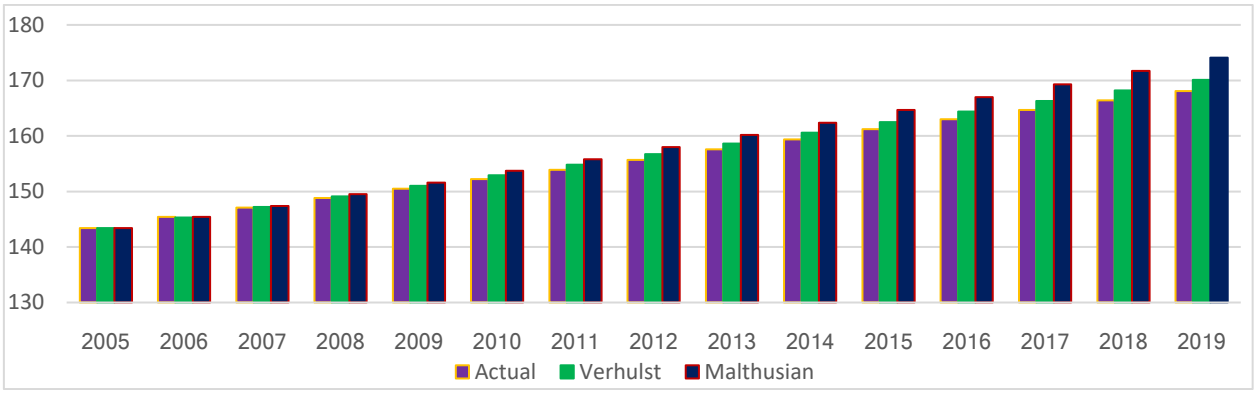

Figure 3: Comparison of Malthusian-Verhulst population projections with real data from 2005 to 2019

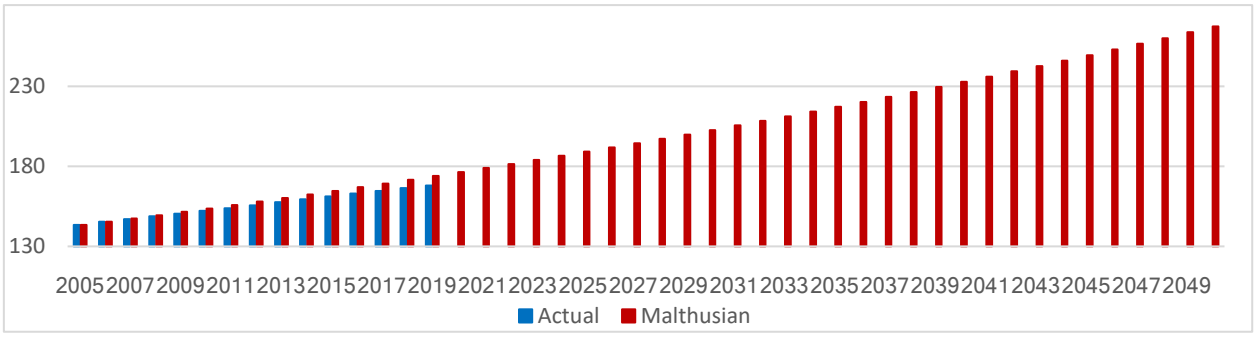

Fig 4: Bangladesh's real(blue) & expected(brown) populations using the Malthusian model(15) between 2005 to 2050

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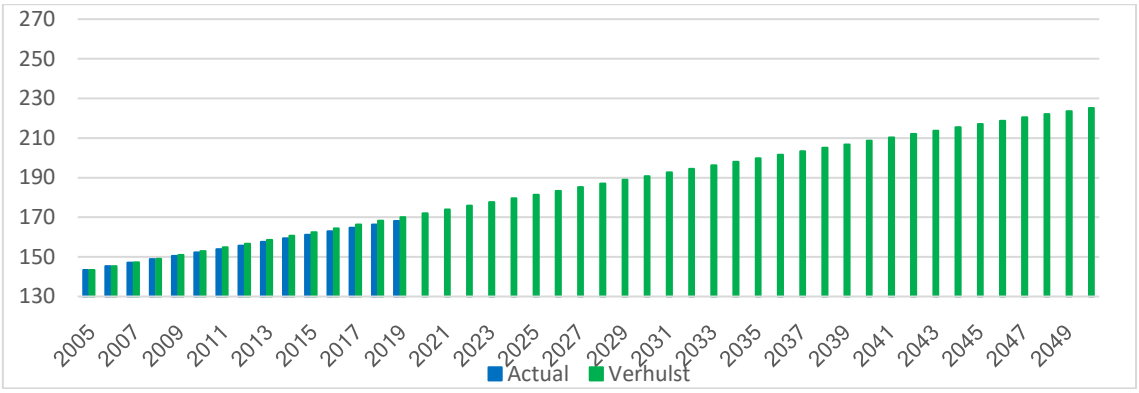

Fig 5: Bangladesh's real(blue) & expected(green) populations using the Verhulst template (17) between 2005 and 2050.

IV.DISCUSSION

Here is a composite chart of the 2005 to 2019 Malthusian and Verhulst models showing Bangladesh's comparative population size (figure 3)

Figures 1 and 2 contain data representing the population of Bangladesh. The figures contain data reflecting both the expected population as well as the country's real population. The third figure contains data of Bangladesh's comparative population calculated using the two models.

The table shows Bangladesh's projected population based on both the Malthusian and Verhulst models. The calculations are based on actual data [25] and on equations (15) and (17).

The population expected in Bangladesh has been calculated from the years 2000 to 2050. In order to calculate this population, we have applied both equations (15) and (17). From our calculations, the predicted population of Bangladesh in 2000 is approximately 340.3 million while the predicted population of the country in 2050 is approximately 236.3 million.

The population growth of Bangladesh can be described as unsustainable. This is due to the fact that the growth might exceed the carrying capacity of the country on account of constrained resources. The problem of unsustainable population growth is one that is quite prevalent in many developing countries and Bangladesh is one such country. In order to counter this problem, many countries are taking measures to avert the situation. On its part, Bangladesh has implemented measures such as increasing literacy rates, improving access to education and creating awareness on the need to control overall population levels.

Finally, numerical simulation of the results of the data relating to predicted population from during the period is contained in figure 4 and 5. Figure 4 contains data that was obtained using Malthusian model while figure 5 contains data that was obtained using Verhulst model. The third figure represents a graphical representation of the country's comparative population using both models. The information relates to the period 2005 to 2019.

V.CONCLUSION

The paper has applied both the Malthusian and Verhulst models to estimate Bangladesh's population. The prediction is over a 45-year period and from the analysis, the population carrying capacity is 318.6 million. Using the two aforementioned models, the Bangladesh population forecast is 225.2 million in year 2050. Based on actual statistics from the country's authorities, the population of Bangladesh stood at 166.4 million in 2018 and 168.1 million the following year. Both figures are fairly close to the predicted population thus underscoring the reliability of the Malthusian and Verhulst models in predicting population. Verhulst model is a model that recognizes the constrained nature of resources and how resources should match to population. For Bangladesh and other developing countries, it is important for them to try and address the problem of high population growth since such countries have lowest access to useful resources such as medical care, housing, and food.

REFERENCES

[1]. Akçakaya HR, Gulve PS. (2000). Population viability analysis in conservation planning: An overview. Ecological Bulletins 48: 9–21.
[2]. Biswas MHA, Ara M, Haque MN, Rahman MA. (2011). Application of control theory in the efficient and sustainable forest management. International Journal of Scientific & Engineering Research 2(3): 26–33. [3]Biswas MHA, Paiva LT, Pinho MD. (2014). A SEIR model for control of infectious diseases with constraints. Mathematical Biosciences and Engineering 11(4): 761– 784. https://doi.org/10.3934/mbe.2014.11.761 [4]Biswas MHA. (2014). Optimal control of nipah virus (niv) infections: a bangladesh scenario. Journal of Pure and Applied Mathematics: Advances and Applications 12(1): 77–104.
[3]. Biswas MHA. (2012). Model and control strategy of the deadly nipah virus (NiV) infections in Bangladesh. Research & Reviews in BioSciences 6(12): 370–377.
[4]. Cohen JE. (1995). Population growth and earth's human carrying capacity. American Association for the Advancement of Science 269(5222): 341–346.
[5]. Deshotel D. (2013). Modeling World Population.
[6]. Avail at http://home2.fvcc.edu/~dhicketh/DiffEqns/spring13projects/Population%20Model%20Project%202013/PopulationModels2013.pdf
[7]. EdwardsCH, PenneyDE. (2004). Differential equations & boundary value problems computing & modeling. 3rd Edition, Pearson Education Inc
[8]. Farid KS, Ahamed JU, Sharma PK, Begum S. (2011). Population dynamics in bangladesh: data sources, current facts and past trends. J Bangladesh Agriculture University.
[9]. Haque MM, Ahamed F, Anam S, Kabir MR. (2012). Future population projection of bangladesh by growthrate modeling using Verhulst population model. Annals of Pure and Applied Mathematics 1(2): 192–202. [10]Islam MR. (2011). Modeling and predicting cumulative population of Bangladesh. American Journal of Computational and Applied Mathematics 1(2): 98–100.
[10]. Islam T, Fiebig DG, Meade N. (2002). Modelling multinational telecommunications demand with limited data. International Journal of Forecasting 18: 605–624. https://doi.org/10.1016/S0169-2070(02)00073-0
[11]. Koya PR, Goshu AT. (2013). Generalized mathematical model for biological growths. Open Journal of Modelling and simulation.
[12]. Malthus TR. (1893). An Essay on the Principle of Population) 1st edition, plus excepts1893 2nd edition (Introduction by Philip Appeman, and assorted commentary on Malthus edited by Appleman, Norton Critical Edition.
[13]. Mahsin M, Hossain SS. (2012). Population forecasts for Bangladesh, using a bayesian methodology. Journal of Health, Population and Nutrition 30(4): 456–463. http://dx.doi.org/10.3329/jhpn.v30i4.13331
[14]. Murray JD. (1989). Mathematical Biology. 2nd edition, Springer–Verlag Berlin. http://dx.doi.org/10.1007/978- 3-662-08542-4
[15]. Ofori T, Ephraim L, Nyarko F. (2013). Mathematical modeling of Ghana's population growth. International Journal of Modern Management Sciences 2(2): 57–66.
[16]. Omale D, Gochhait S. (2018). Analytical solution to the mathematical models of HIV/AIDS with control in a heterogeneous population using Homotopy Perturbation Method (HPM). AMSE journals-AMSE IIETA-Series: Advances A 55(1): 20-34.
[17]. Pozzi F, Small C, Yetman G. (2002). Modeling the distribution of human population with night-time satellite imagery and gridded population of the world. Proceedings of Pecora 15/Land Satellite Information IV/ISPRS Commission I/FIEOS Conference.
[18]. Roy B, Roy SK. (2015). Analysis of prey-predator three species models with vertebral and invertebral predators. International Journal Dynamics and Control 3: 306–312. http://dx.doi.org/10.1007/s40435-015-0153-6
[19]. Roy SK, Roy B. (2016). Analysis of prey-predator three species fishery model with harvesting including prey refuge and migration. International Journal of Bifurcation and Chaos 26(1650022). http://dx.doi.org/10.1142/S021812741650022X
[20]. Sardar AK, Hanif M, Asaduzzaman M, Biswas MHA. (2016). Mathematical analysis of the two species lotkavolterra predator-prey inter-specific game theoretic competition model. Advanced Modeling and Optimization 18(2): 231 -242.
[21]. Shen J, Tang S, Xu C. (2017). Analysis and research on home-based care for the aged based on insurance policy under government leading. AMSE journals-AMSE IIETA-Series: Advances A 54(1): 106-126.
[22]. TsoularisA.(2001). Analysis of Verhulst growth model. Res. Lett. Inf. Math. Sci. 2: 23–46. http://dx.doi.org/10.1016/S0025-5564(02)00096-2
[23]. The World Bank Population Report. Available at http://data.worldbank.org/indicator/SP.POP.TOTL.
[24]. Wali A, Ntubabare D, Mboniragira V. (2011). Mathematical modeling of rwanda's population growth. Applied Mathematical Science 5(53): 2617–2628.
[25]. Wali A, Kagoyire E, Icyingeneye P. (2012). Mathematical modeling of Uganda's population growth. Applied Mathematical Science 6(84): 4155–4168.
[26]. Bangladesh Bureau of statistics http://203.112.218.65:8008/

Graph References

Here a combined graph of the Malthusian and Verhulst models showing a comparative population size in Bangladesh from 2005 to 2019
Here is a composite graph of the Malthusian and Verhulst models showing a comparable population size between 2005 and 2019 in Bangladesh
Here is a composite graph of the Malthusian and Verhulst models showing a comparable population size from 2005 to 2019 in Bangladesh
Here is a composite chart of the Malthusian and Verhulst models showing Bangladesh's comparative population size from 2005 to 2019
Here is a composite map of the Malthusian and Verhulst models from 2005 to 2019 showing a comparable population size in Bangladesh
Here is a composite graph of the Malthusian and Verhulst models showing a comparable population size between 2005 and 2019 in Bangladesh
Here is a composite chart of the 2005 to 2019 Malthusian and Verhulst models showing a comparable population size in Bangladesh