

Response of Network Circuits Connected to Exponential Excitation Sources

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Abstract: In science and engineering, network circuit's analysis is a principal course. In this paper, we discuss the application of Mohand Transform for obtaining the Response of electric network circuits with an exponential excitation Source which is generally determined by the application of calculus method or different integral Transforms or convolution method or Matrix method. The response of electric network circuits with an exponential excitation source provides an expression for the current or voltage. This paper presents demonstrated the use of the Mohand Transform for determining the response of electric network circuits with an exponential excitation source.

Keywords: Mohand Transform; Network Circuits; Exponential Excitation Source; Response

I. INTRODUCTION

The electric network circuits are generally analyzed by adopting calculus method or different integral Transforms [1-6] or convolution method [7, 8] or Matrix method [9, 10] and their response depends on the property of elements- inductor L, capacitor C, and resistor R. Such series and parallel network circuits are widely used as a tuning or resonant circuit and are also widely used in oscillatory circuits [7-11]. Mohand Transform has been applied in solving boundary value problems in most of the science and engineering disciplines [12]. This paper presents the application of Mohand Transform for the analysis of network circuits connected to an exponential excitation source and reveals that Mohand Transform is an effective tool for analysis of such network circuits.

II. BASIC DEFINITION

2.1 Mohand Transform

If the function $h(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Mohand transform [6, 12] of $h(y)$ is given by $M\{h(y)\} = \bar{h}(q) = q^2 \int_0^{\infty} e^{-qy} h(y) dy$.

The Mohand Transform [6, 12] of some of the functions are given by

- $M\{y^n\} = n!/q^{n-1}$, where $n = 0, 1, 2, \dots$
- $M\{e^{ay}\} = \frac{q^2}{q-a}$,
- $M\{\sin ay\} = \frac{aq^2}{q^2+a^2}$,
- $M\{\cos ay\} = \frac{q^3}{q^2+a^2}$,

2.2 Mohand Transform of Derivatives

The Mohand Transform [6, 12] of some of the Derivatives of $h(y)$ are given by

$$M\{h'(y)\} = qM\{h(y)\} - q^2h(0)$$

$$\text{or } M\{h'(y)\} = q\bar{h}(p) - q^2h(0)$$

$$M\{h''(y)\} = q^2\bar{h}(p) - q^3h(0) - q^2h'(0), \text{ and so on.}$$

III. MATERIAL AND METHOD

Analysis a series R-L-C network connected to a source of exponential potential

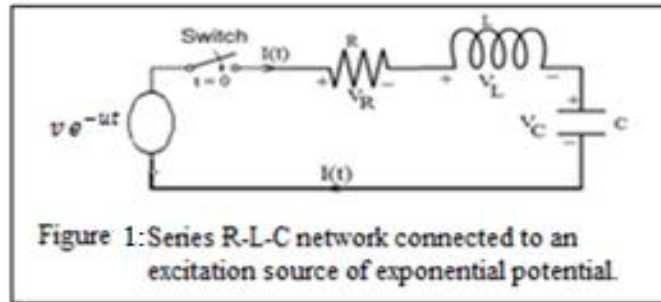
The differential equation for a series R-L-C network connected to an exponential excitation potential source as shown in figure (1) is given by

$$I(t)R + L \dot{I}(t) + \frac{Q(t)}{C} = v e^{-ut} \quad \dots (1)$$

Differentiating (1) w.r.t. t and simplifying we get,

$$\ddot{I}(t) + \frac{R}{L} \dot{I}(t) + \frac{1}{LC} I(t) = \frac{-vu}{L} e^{-ut} \quad \dots (2)$$

Here, I(t) is the instantaneous current through the series R – L – C network circuit.



The initial conditions are:

(i) At t = 0, I(0) = 0..... (3)

(ii) Since at t = 0, I(0) = 0, therefore, equation (1) gives $\dot{I}(0) = \frac{v}{L}$ (4)

Taking Mohand transformation of equation (2), we get

$$q^2 \bar{I}(q) - q^3 I(0) - q^2 \dot{I}(0) + \frac{R}{L} \{q \bar{I}(q) - q^2 I(0)\} + \frac{1}{LC} \bar{I}(q) = \frac{-vuq^2}{L(q+u)} \quad \dots (5)$$

Applying conditions: I(0) = 0 and $\dot{I}(0) = \frac{v}{L}$ and simplifying (5), we get

$$\bar{I}(q) = \frac{v}{L} \frac{q^3}{(q+u)(q^2 + \frac{R}{L}q + \frac{1}{LC})} \quad \dots (6),$$

where $2\delta = \frac{R}{L}$ and $\omega = \sqrt{\frac{1}{LC}}$

or $\bar{I}(q) = \frac{v}{L} \frac{q^3}{(q+u)(q^2 + 2\delta q + \omega^2)}$

$$\text{or } \bar{I}(q) = \frac{v}{L} \frac{q^3}{(q+u)(q+\beta_1)(q+\beta_2)} \quad \dots (7)$$

where $\omega' = \sqrt{\delta^2 - \omega^2}$, $\delta + \omega' = \beta_1$ and $\delta - \omega' = \beta_2$, $\beta_1 - \beta_2 = 2\omega'$.

$$\text{or } \bar{I}(q) = \frac{v}{L} \left\{ \frac{-uq^2}{(q+u)(-u+\beta_1)(-u+\beta_2)} + \frac{-\beta_1 q^2}{(-\beta_1+u)(q+\beta_1)(-\beta_1+\beta_2)} + \frac{-\beta_2 q^2}{(-\beta_2+u)(-\beta_2+\beta_1)(q+\beta_2)} \right\} \dots (8)$$

Applying inverse Mohand transform, we get

$$I(t) = \frac{v}{L} \left\{ \frac{-ue^{-ut}}{(-u+\beta_1)(-u+\beta_2)} + \frac{-\beta_1 e^{-\beta_1 t}}{(-\beta_1+u)(-\beta_1+\beta_2)} + \frac{-\beta_2 e^{-\beta_2 t}}{(-\beta_2+u)(-\beta_2+\beta_1)} \right\}$$

$$\text{or } I(t) = \frac{v}{L} \left\{ \frac{[\delta - \omega'] e^{-\delta t} e^{\omega' t}}{2\omega' [\delta - \omega' - u]} - \frac{ue^{-ut}}{[\delta + \omega' - u][\delta - \omega' - u]} - \frac{[\delta + \omega'] e^{-\delta t} e^{-\omega' t}}{2\omega' [\delta + \omega' - u]} \right\} \quad \dots (9)$$

This equation (9) is an expression for the current through a series R-L-C network circuit connected to an exponential excitation source at any instant.

When t increases indefinitely, $e^{-\delta t}$ tends to zero, so

$$I(t) = \frac{v}{L} \frac{-ue^{-ut}}{[\delta + \omega' - u][\delta - \omega' - u]}$$

$$\text{or } I(t) = \frac{v}{L} \frac{ue^{-ut}}{[u^2 - \omega^2 - 2\omega']}$$

Analysis of a parallel R-L-C network connected to an exponential excitation current source

The differential equation for a parallel R – L – C network circuit connected to an exponential excitation current source as shown in figure (2) is given by

$$\frac{V(t)}{R} + \frac{1}{L} \int V(t) dt + C \dot{V}(t) = I_0 e^{-ut} \quad \dots (10)$$

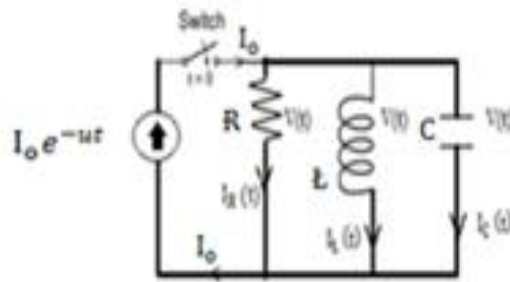


Figure 2: Parallel R-L-C network connected to an exponential excitation current source

Differentiate (10) w.r.t. t and simplifying, we get,

$$\dot{V}(t) + \frac{1}{RC} V(t) + \frac{1}{LC} V(t) = \frac{-I_0 u}{C} e^{-ut} \dots (11)$$

The initial conditions are:

(i) At $t = 0$, $V(0) = 0$.

(ii) Since at $t = 0$, $V(0) = 0$, therefore, (11) gives $\dot{V}(0) = \frac{I_0}{C}$.

Taking Mohand transformation of (11), we get

$$q^2 \bar{V}(q) - q^3 V(0) - q^2 \dot{V}(0) + \frac{1}{RC} \{q \bar{V}(q) - q^2 V(0)\} + \frac{1}{LC} \bar{V}(q) = \frac{-I_0 u q^2}{C(q+u)} \dots (12)$$

Applying conditions: $V(0) = 0$ and $\dot{V}(0) = \frac{I_0}{C}$ and simplifying (12), we get

$$\bar{V}(q) = \frac{I_0}{C} \left[\frac{q}{(q+u)(q^2 + 2 a q + \omega^2)} \right], \text{ where } 2a = \frac{1}{RC} \text{ and } \omega = \sqrt{\frac{1}{LC}}$$

$$\text{or } \bar{V}(q) = \frac{I_0}{C} \left[\frac{q^3}{(q+u)(q^2 + 2 a q + \omega^2)} \right]$$

$$\text{or } \bar{V}(q) = \frac{I_0}{C} \left[\frac{q^3}{(q+u)(q+a_1)(q+a_2)} \right]$$

where $a + \omega' = a_1$ and $a - \omega' = a_2$, $\omega' = \sqrt{a^2 - \omega^2}$, $a_1 - a_2 = 2\omega'$.

$$\text{or } \bar{V}(q) = \frac{I_0}{C} \left\{ \frac{-uq^2}{(q+u)(-u+a_1)(-u+a_2)} + \frac{-a_1 q^2}{(-a_1+u)(q+a_1)(-a_1+a_2)} + \frac{-a_2 q^2}{(-a_2+u)(-a_2+a_1)(q+a_2)} \right\}$$

Applying inverse Mohand transform, we get

$$V(t) = \frac{I_0}{C} \left\{ \frac{-ue^{-ut}}{(-u+\beta_1)(-u+\beta_2)} + \frac{-\beta_1 e^{-\beta_1 t}}{(-\beta_1+u)(-\beta_1+\beta_2)} + \frac{-\beta_2 e^{-\beta_2 t}}{(-\beta_2+u)(-\beta_2+\beta_1)} \right\}$$

$$\text{or } V(t) = \frac{I_0}{C} \left\{ \frac{[\delta - \omega'] e^{-\delta t} e^{\omega' t}}{2\omega'[\delta - \omega' - u]} - \frac{ue^{-ut}}{[\delta + \omega' - u][\delta - \omega' - u]} - \frac{[\delta + \omega'] e^{-\delta t} e^{-\omega' t}}{2\omega'[\delta + \omega' - u]} \right\} \dots (13)$$

This equation (13) is an expression for the potential across a parallel R-L-C network connected to an exponential excitation current source at any instant.

When t increases indefinitely, $e^{-\delta t}$ tends to zero, so

$$V(t) = \frac{I_0}{C} \frac{-ue^{-ut}}{[\delta + \omega' - u][\delta - \omega' - u]}$$

$$\text{or } V(t) = \frac{I_0}{C} \frac{ue^{-ut}}{[u^2 - \omega^2 - 2\omega']}$$

IV. RESULT AND CONCLUSION

In this paper, we have successfully obtained the response of network circuits connected to exponential excitation source. This paper exemplified the application of Mohand Transform for obtaining the response of network circuits connected to exponential excitation source. This paper brought up the Mohand Transform as a simple and effective technique for analyzing the network circuits connected to exponential excitation source.

REFERENCES

- [1]. Network Analysis by G.K. Mithal, 14th edition, 2012.
- [2]. Network Analysis M. E. Van Valkenburg, 3rd Edition, 2014.
- [3]. Theory and Problems of Laplace Transforms by Murray R. Spiegel, McGraw - Hill.

- [4]. Sunil Shrivastava, Introduction of Laplace Transform and Elzaki Transform with Application (Electrical Circuits), International Research Journal of Engineering and Technology (IRJET) ,volume 05, Issue 02 , Feb. 2018.
- [5]. P. Senthil Kumar, S. Vasuki, Applications of Aboodh Transform to Mechanics, Electrical Circuit Problems, International Journal for Research in Engineering Application & Management, Vol-04, Issue-06, Sep 2018.
- [6]. Dinesh Verma, Yuvraj Singh, Rohit Gupta, Response of Electrical Networks with Delta Potential via Mohand Transform, International Research Journal of Innovations Engineering and Technology, volume 2, issue 2, pp. 41-43, February 2020.
- [7]. Rohit Gupta, Loveneesh Talwar, Rahul Gupta, Analysis Of Network Circuit With Steady Voltage Source, And With Steady Current Source Via Convolution Method, International Journal Of Scientific & Technology Research, Volume 8, Issue 11, November 2019.
- [8]. Rohit Gupta, Rahul Gupta, Sonica Rajput, Convolution Method for the Complete Response of a Series L-R Network Connected to an Excitation Source of Sinusoidal Potential, International Journal of Research in Electronics and Computer Engineering, IJRECE Vol. 7, issue 1 (January- March 2019), pp. 658-661.
- [9]. Rohit Gupta, Rahul Gupta, Matrix method for deriving the response of a series L- C- R network connected to an excitation voltage source of constant potential, Pramana Research Journal, Volume 8, Issue 10, 2018, pp. 120-128.
- [10]. Rohit Gupta, Rahul Gupta, Sonica Rajput, Response of a parallel L- C- R network connected to an excitation source providing a constant current by matrix method, International Journal for Research in Engineering Application & Management, Vol-04, Issue-07, Oct 2018.
- [11]. Network Analysis and synthesis by Dr. J. S. Chitode and Dr. R.M. Jalnekar.
- [12]. Mohand M Abdelrahim Mahgoub, The New Integral Transform Mohand Transform, Advances in Theoretical & Applied Mathematics, Vol 12, Number 2 (2017).