Transportation Model in Linear Programming Problem: An Approach Towards Minimization of Time and Cost in Reality

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Abstract: The optimization processes in mathematics, computer science and economics are solving effectively by choosing the best element from set of available alternatives elements. The most important and successful applications in the optimization refers to transportation problem (TP), that is a special class of the linear programming (LP) in the operation research (OR). The main objective of transportation problem solution methods is to minimize the cost or the time of transportation. Most of the currently used methods for solving transportation problems are trying to reach the optimal solution, whereby, most of these methods are considered complex and very expansive in term of the execution time. This paper shows the total transportation time problem regarding the time of the active transportation routes. If the multiple optimal solutions exist, it is possible to include other criteria as second level of criteria and find the corresponding solutions. Furthermore, if there is a multiple solution, again, the third objective can be optimized in lexicographic order. The methods of generation of the optimal solution in selected cases are developed.

Keywords: Operation Research, Transportation Problem, LPP, VAM, Optimal Solution.

I. INTRODUCTION

Transportation problem is concerned with the optimal pattern of the product units' distribution from several original points to several destinations. Suppose there are m points of original $A_1, \ldots, A_i, \ldots, A_m$ and n destinations $B_1, \ldots, B_j, \ldots, B_n$. The point $A_i(i = 1, \ldots, m)$ can supply a_i units, and the destination $B_j(j = 1, \ldots, n)$ requires b_j units (see Equation 1).

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_i$$

Whereby, the cost of shipping a unit from A_i to B_j, is computed as c_{ij}. As well as, the problem in determining the optimal distribution pattern consists the pattern for which shipping costs are at a minimum. Moreover, the requirements of the destinations $B_j, j = 1, \ldots, n$, must be satisfied by the supply of available units at the points of origin $A_j, i = 1, \ldots, m$. Mathematically, the transportation problem can be represented as a linear programming model. Since the objective function in this problem is to minimize the total transportation cost as given by

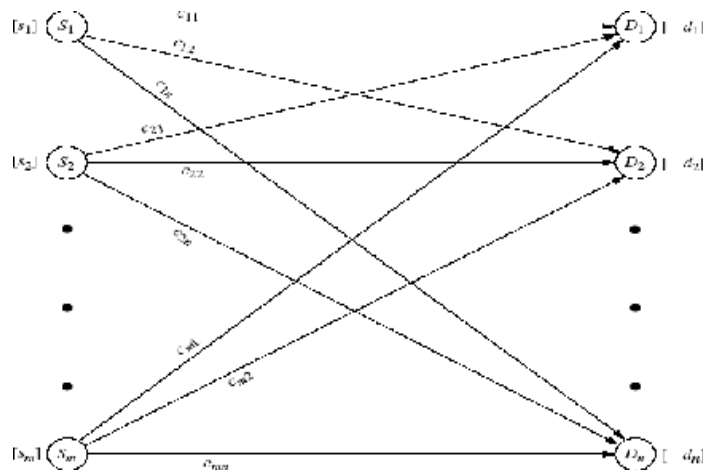$$Z = c_{11} x_{11} \quad c_{12} x_{12} + \cdots + c_{mn} x_{mn}$$

Figure1. Network flow model of the transportation problem.

However, there are several different algorithms to solve transportation problem that represented as LP model. Among these are the known algebraic procedures of the simplex method, which may not be the best method to solve the problem. Therefore, more efficient and simpler procedures have been improved to solve transportation problems. Typically, the standard scenario for solving transportation problems is working by sending units of a product across a network of highways that connect a given set of cities. Each city is considered as a source (S) in that units will be shipped out from, while units are demanded there when the city is considered as a sink (D). In this scenario, each sink has a given demand, the source has a given supply, and the highway that connects source with sink as a pair has a given transportation cost/(shipment unit). Figure 1 visualizes the standard scenario for cities on the highway in the form of a network. As shown by Figure 1, the problem is to determine an optimal transportation scheme that is to minimize the total of the shipments cost between the nodes in the network model, subject to supply and demand constraints. As well as, this structure arises in many applications such as; the sources represent warehouses and the sinks represent retail outlets. Moreover, Ad-hoc networks are designed dynamically by group of mobile devices. In Ad-hoc network, nodes between source and destination act as a routers so that source node can communicate with the destination node

II. MATERIALS AND METHODS

Transportation model: In a transportation problem, we are focusing on the original points. These points may represent factories to produce items, and to supply a required quantity of the products to a certain number of destinations. This process must be done successfully in such a way as to maximize the profit or minimize the cost transportation. Therefore, the places of production and supply are collected as the original points and the destinations respectively. Sometimes the original and destinations points are also termed as sources and sinks. However, to illustrate a typical transportation model, suppose that m factories supply certain items to n warehouses. As well as, let factory i ($i = 1, 2, …, m$) produces a_i units, and the warehouse j ($j = 1, 2, …, n$) requires b_j units. Furthermore, suppose the cost of transportation from factory i to warehouse j is c_{ij}. The decision variables x_{ij} is being the transported amount from the factory i to the warehouse j. Typically, our objective is to find the transportation pattern that will minimize the total of the transportation cost (see Table 1).

Table 1: The model of a transportation problem

Origins (Factories)	Destinations (Warehouses)			Available	
	1 2 n				
1	c_{11}	c_{12}	c_{1n}	a_1	
2	c_{21}	c_{22}	c_{2n}	a_2	
…	...	...	...	...	
m	c_{m1}	c_{m2}	c_{mn}	a_m	
Required	b_1	b_2	b_n		

Algorithms for solving: there are several algorithms for solving transportation problems which are based on different of special linear programming methods, among these are:
1. Northwest Corner method
2. Minimum cost method
3. Vogel's Approximation Method

III. SOLUTION METHODS

Basically, these methods are different in term of the quality for the produced basic starting solution and the best starting solution that yields smaller objective value. In this study, we used the Vogel's approximation method, since it generally produces better starting solutions than other solving methods; as well we have used the BCM solution steps

Vogel's Approximation Method (VAM):
Generally, VAM can be summarized by the following three main steps:
i. The result of subtracting the smallest unit cost element in the row/column (cell) from the immediate next smallest unit cost element in the same row/column is determining a penalty measure for the target row/column.
ii. This step includes the following sub-steps:
a. Identify the row or the column that includes the largest penalty.
b. Break ties arbitrarily.
c. As much as possible, the lowest cost row/column (cell) in the row or column should be allocated with the highest difference.

d. Adjust the supply and demand, and then cross out the satisfied row or column.
e. If a row and column are satisfied simultaneously, then only one of them is crossed out, as well the remains rows or columns are assigned to supply as zero (demand).
iii.   Finally, the result should be computed as follows:
a. If a row or a column is assigned as zero supply, or demand remains uncrossed out, then stop the process.
b. If one row/column with positive supply (demand) remains uncrossed out, then determine the basic variables in the row/column by the lowest cost method, and then stop.
c. If all the uncrossed out rows and columns have (remaining) zero supply and demand then determine the zero basic variables by the lowest cost method and stop.
d. Otherwise, go to step (i).

The Best Candidates Method (BCM):
BCM process includes three steps, these steps are shown as follows:
Step1: Prepare the BCM matrix, If the matrix unbalanced, then the matrix will be balanced without using the added row or column candidates in solution procedure.
Step2: Select the best candidates, that is for minimizing problems to the minimum cost, and maximizing profit to the maxumum cost. Therefore, this step can be done by electing the best two candidates in each row. If the candidate repeated more than two times, then the candidate should be elected again. As well as ,the columns must be checked such that if it is not have candidates so that the candidates will be elected for them. However, if the candidate is repeated more than one time, the elect it again.
Step3: Find the combinations by determining one candidate for each row and column, this shoud be done by starting from the row that have the least candidates, and then delete that row and column. If there is situation that have no candidate for some rows or columns, then directly elect the best available candidate. Repeat Step 3 by determining the next candidate in the row that started from. Compute and compare the summation of candidates for each combination. This is to determine the best combination that gives the optimal solution.

An Example
Problem: Finding the optimal cost for the problem of assigning five jobs to five persons.
Solution:
Step1: in this problem, the matrix is unbalanced, where the total of supply is not equal to the total of demand (Table 2). Here we add a dummy row to make supply is equal to the demand, so the transportation costs in this row will be assigned to zero (Table 3).

Table 2: The proposed method –process 1

Plant	D1	D2	D3	D4	Supply
S1	10	30	25	15	14
S2	20	15	20	10	10
S3	10	30	20	20	15
S4	30	40	35	45	12
Demand	10	15	12	15	*52 51*

Table 3: the proposed method –process 2

Plant	D1	D2	D3	D4	Supply
S1	10	30	25	15	14
S2	20	15	20	10	10
S3	10	30	20	20	15
S4	30	40	35	45	12
S5	0	0	0	0	1
Demand	10	15	12	15	*52 52*

Step2: By using BCM, we determine the best combination that will produce the lowest total weight of the costs, where is one candidate for each row and column. The result from applying BCM shown in (Table 4).

Table 4: the proposed method –process 3

Plant	D1	D2	D3	D4	Supply
S1	10	30	25	15	14
S2	20	15	20	10	10
S3	10	30	20	20	15
S4	30	40	35	45	12
S5	0	0	0	0	1
Demand	10	15	12	15	52 52

Step3: We identify the row with the smallest cost candidate from the choosen combination. Then we allocate the supply and the demand to the variable with the least unit cost in the selected row and column. Also, we should adjust the supply and the demand by crossing out the row/column to be then assigned to zero (Table 5). If the column is assigned to zero, then we elect the cost in the cell (d3,s3) because it has a lower cost than the determined cell (d3,s4) from the choosen combination (Table 6) .

Table 5: the proposed method –process 4

Plant	D1	D2	D3	D4	Supply
S1	10	30	25	15	14
S2	20	15	20	10	10
S3	10 **10**	30	20	20	15 5
S4	30	40	35	45	12
S5	0	0	0	0	1
Demand	10	15	12	15	52 52
	0				

Table 6: the proposed method –process 5

Plant	D1	D2	D3	D4	Supply
S1	10	30	25	15	14
S2	20	15	20	10	10
S3	10 **10**	30	20 **5**	20	15 5 0
S4	30	40	35	45	12
S5	0	0	0	0	1
Demand	10	15	12	15	52 52

0   7

Step4: We elect the next least cost from the choosen combination and repeat Step 3 until all columns and rows are exhausted (Table 7).

Table 7: the proposed method –process 6

Plant	D1	D2	D3	D4	Supply
S1	10	30	25	15 **14**	14 0
S2	20	15 **10**	20	10	10 0
S3	10 **10**	30	20 **5**	20	15 5 0
S4	30	40 **5**	35 **7**	45	12 0
S5	0	0	0	0 **1**	1 0
Demand	10	15	12	15	52 52
	0	5	7	1	

We found that the result by using BCM as:
15*14 + 15*10 +10*10 + 20*5 + 40*5 + 35*7 = 1005

V. RESULTS AND DISCUSSION

Actually, using BCM based on the best candidate's election is to minimize the number of combinations solutions. As we can obtain the combinations without any intersect means, such that; one candidate for each row and column. Efficiently, BCM works to obtain the optimal solution or the closest to optimal solution than other available methods. Since it has a better starting solution.

VI. CONCLUSION

The time of transport might be significant factor in several transportation problems. The efficiency of transportation have been introduced as an aggregate of the time and the quantity of goods on active transport operations, longest time on single active transport operation, total time on all active transport operations, total quantity of goods with a longest time of transport etc. The question arises from this whether to perform optimization at the same time for more objectives, with eventual priorities for each, or to treat single objective problems, which are of the great significance for particular problem. In this paper, the single objective problem of total time minimization has been exposed having in mind active transport operations and is defined as Algorithm for its solution. To start from the point of multiple optimal solution for specific total time of transport, it is further suggested the minimization of some other objective while keeping minimal total time of transport. In that way, it is passed to the two objective problems with strong lexicographic order for the determined objectives where the absolute priority is given to the total time of transport. Furthermore, if there is a multiple solution, the third objective can be optimized which bears the next level of significance etc.

Defined process of solving the problem of minimization of total time on active transport operations is necessary even for solving each multi objective transport problem which involves mentioned objective. At the firs step of solving any multi objective problem, it is necessary to determine optimal solutions for each objective separately, that is to treat single objective problems and then to keep searching for Praeto-Optimal solutions for each multi objective problem.

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