



# A New Approach to Formulate $AX=XB$ Hand Eye Calibration Problem using Combination of Data Positions Strategy

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**Abstract:** The ever-changing demands of the production industry can never be fully accomplished using any of the state of the art technologies. Robotics has been doing a wonderful job in every manufacturing aspect from the starting stage to the end in every manufacturing unit. Robots with vision for applications like inspection, welding, component assembly etc. can not only bring a value addition to the industry but also can reduce the production time to a great extent due to their flexible, precise and accurate operations. Robot and vision sensor are always two different components. Efficient working of a Robot with vision is only possible through proper Hand-Eye calibration, which should be completed with utmost care. Calibration is time consuming and is a complex process. A fully automatic calibration process which is accurate can bring down this problem. A strategic way to frame hand eye calibration equations of the form  $AX=XB$  using data collected is proposed in this work. Different methods for solving hand eye calibration problem are also studied using experimental data from a 6 DOF Collaborative Robot (COBOT) in a laboratory environment.

**Keywords:** Hand Eye calibration, Robot vision, Camera calibration,  $AX=XB$ .

## I. INTRODUCTION

The ability to analyse, think and execute a specific task within the stipulated time either with the help of human force or by any other means is a remarkable characteristic and that makes humans different from other animals. There is always some limitation to the human beings both physically and mentally even if they perform any task. An alternative force that can outperform humans has always been as important as developing any new ideas or machines. The evolution of a robot is nothing but the solution to the human alternative problem. The discovery of a vision sensor paved a way to the idea of building a robot with vision [1]. Integrating a vision sensor to any system has always had many benefits and was a stepping stone to develop a robot that can see, analyse, think and perform according to any change in the system environment. This is nothing but an intelligent robot. Flexibility has always been a major advantage of these robots. The ever changing demand of the production industry can be met with these robots [2]. Robot Operating System (ROS) is an accepted open source framework, which can solve almost all the problems come across while developing any robotic systems. It can also be used to ease the effort in integrating a vision sensor to a Robot. Calibration process can be eased with the help of ROS and can cut down setup time of the system. Accurate hand-eye calibration is possible with accurate estimation of two homogenous transformations [3]. That is the homogenous transformation between the camera and the object detected by the camera and the homogenous transformation between the end effector or wrist frame on to which the camera is attached to and the base frame of the Robot. Two different forms of calibration problems can be formulated with these two known homogenous transformation matrices.  $AX=XB$  type of problem [4] and  $AX=YB$  [5] type of hand eye calibration problem.  $AX=YB$  type of problem is usually known as the Robot World Hand Eye calibration problem. In both the problems X is to be found out. It is nothing but the end effector to camera transformation matrix. Matrix A and B varies according to the problem. In  $AX=XB$  problem, A is the transformation between two different robot hand positions and B is the transformation between the camera positions corresponding to the robot hand positions considered. In the case of  $AX=YB$  problem, matrix A is the transformation between the base frame and the robot end effector frame. B is the transformation between the object and the camera. Here there is another matrix Y which is solved along with X. Y is the transformation between the robot base frame and object frame.

To solve the hand eye calibration problems of the forms mentioned above, a number of researches have been conducted. From time to time the problem is well addressed to solve for an accurate answer. The rotations in all the homogenous transformation in the equations are converted into quaternion form and are then solved in all most all the problem solving approaches so far. The solutions can be called as separable solutions, simultaneous solution and iterative solutions. All these solutions by different authors are compared with simulated and experimental data sets by Ihtisham Ali et al [6]. In this study a 6 DOF Articulated Robot, Aubo-i5 is connected with a depth camera, Intel Realsense D435 and the hand eye calibration test has been conducted. The objects selected as calibration objects are Black-white chessboard and ArUco marker. Detailed experiment procedure is explained in the section below.



## II. EXPERIMENT SETUP

A Robot vision is set up in the laboratory environment using a Robot and a camera along with software support. The robot manipulator is AUBO-i5 collaborative robot from AUBO (Beijing) Robotics Technology Co., Ltd [13]. This manipulator is connected to a computer by means of a robotic controller. The communication between robot controller and the computer is established by using an Ethernet cable. Intel Realsense D435 camera is connected to the robot manipulator by means of a threaded rod attached near the end effector without disturbing the motions of the end effector. Camera is connected to the computer via USB cable. The experiment setup is shown in Fig.1.



Fig.1 Experiment setup in a laboratory environment

Among different Robot camera configurations, the selected configuration for the set of experiments is the Eye in Hand configuration. In this configuration the camera is mounted on the robot arm and the target object is viewed directly. This configuration is having advantages like occlusion avoidance, flexibility, and precise sight of scene.



Fig.2 Eye in Hand Configuration using Aubo-i5 Robot and Intel Realsense D435 camera

For a hand eye calibration, camera calibration must be done at first and is called the intrinsic calibration. The intrinsic parameters include the lens focal length, lens distortion and pixel properties such as skewness and resolution. Camera intrinsic calibration is only possible by using a calibration pattern. In this experiment the calibration pattern used is a Black-White chessboard [7]. Whichever pattern is used, the aim is to provide a set of feature points with known 3D positions. By obtaining the correspondences between the feature points in the 2D image plane and their 3D real world from multiple images, the camera parameters can be determined. The pinhole camera model [8] is used in order to find the camera intrinsic matrix. The relationship between cameras natural units into real world measurements is established using intrinsic calibration. That is in simple words pixels of camera and object measurement in millimeters are bounded with some relationship in calibration. OpenCV[12] function find Chessboard Corners is used to detect the chessboard pattern in each image. Once the patterns are detected in each image, calibrate Camera function is used to calculate the calibration matrices. The outputs of the calibrate Camera are the camera matrix and the distortion matrix. After successful calibration these matrices are stored in to the camera configuration file. The next step is the extrinsic calibration procedure. The extrinsic calibration procedure is the hand eye calibration procedure. The object used to detect is the ArUco Marker. It is placed as the fixed object in a specific place on the table. This position needs to be undisturbed throughout the completion of the calibration process. The robot is jogged to different positions and made sure that in these positions the object is completely detected without any disturbance. The position of the Robot is recorded at each position and the co-ordinate of the marker with respected to camera origin is calculated. The camera-marker transformation and robot base-end effector transformation is calculated [9] at each position and given as input to the calibration problem. The transformation between robot base and end effector is calculated using tf package in ROS [11] and aruco\_detect package is used to find out the camera marker transformation.



III. PROBLEM GEOMETRY

The aim of the extrinsic calibration is to formulate a relationship between the camera frame and the end effector frame of the robot. The relationship is a homogenous transformation matrix of the order 4 by 4. Two different formulations are  $AX=XB$  and  $AX=YB$  [10] [11].

A.  $AX=YB$  Problem

A is usually the transformation between Base of the Robot and the end effector or flange of the robot. B is usually the transformation between the tool or end effector and the object frame. X is the transformation between camera and robot end effector and Y is the transformation between the base of the robot and the object frame. The object frame is interpreted as the world frame.  $AX=YB$  Problem geometry is shown in Fig.3.

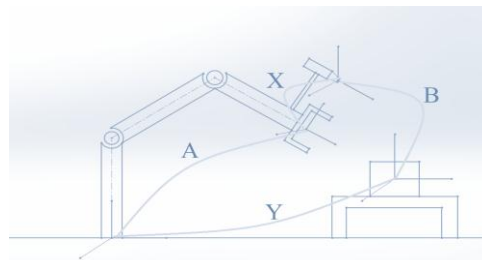


Fig.3  $AX=YB$  Problem Geometry

For a specific position we can directly take the inputs using established robot kinematic model and object detection feature. The inputs are A and B. A is the transformation matrix from base to effector and B is the transformation from object to camera. For different positions a number of  $AX=YB$  problems are formulated and is solved for X and Y. The transformation we are interested in is X, the robot hand to eye transformation.

B.  $AX=XB$  Problem

A is the transformation between two selected camera positions and B is the transformation between corresponding end effector position. X is the transformation between the camera and the end effector. Fig.3 shows  $AX=XB$  problem geometry. To formulate an  $AX=XB$  problem, two positions of the robot and camera is needed. The input matrices will be effector to base transformations and object to camera transformations. EB1 and EB2 are the Effector to base transformations for the first position and second positions respectively. OC1 and OC2 are the object to camera transformations for the first and Second positions respectively. These four matrices are the input matrices. A and B are calculated using the equation

$$A = EB1 (EB2)^{-1}$$

$$B = (OC1)^{-1} OC2$$

Then  $AX=XB$  is formulated and is solved for X. This is continued for different sets of Positions.

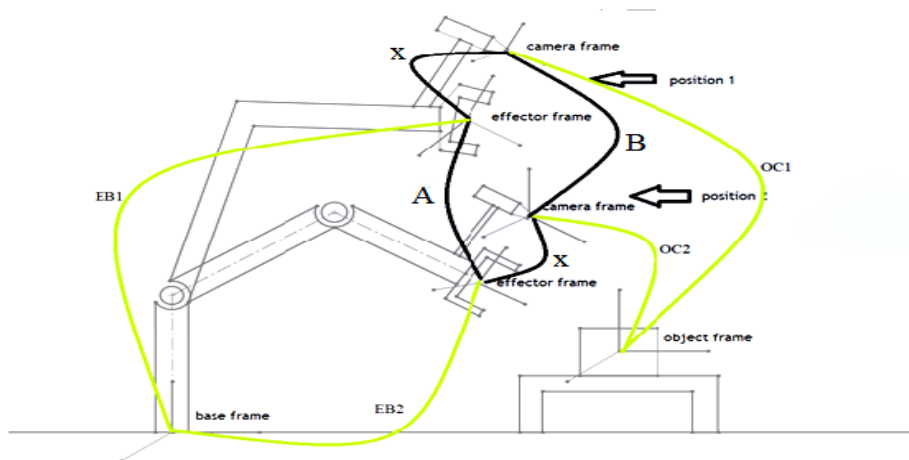


Fig.4  $AX=XB$  Problem Geometry using position 1 and position 2

C. Error estimation

There will be error associated with while solving for the robot effector to camera transformation matrix since we are solving a number of equations. In both the formulations the output at the end is a homogenous transformation matrix.

This matrix has both rotation and translation part. A transformation matrix X can be written as,

$$X = \begin{bmatrix} R_x & T_x \\ 0 & 1 \end{bmatrix}$$



$R_x$  Is a 3 x 3 matrix and  $T_x$  is a 3 x 1 matrix.

There will be rotation error and translation error associated with  $X$  while solving  $AX=XB$  and  $AX=YB$  formulations. The equations will be different for these formulations are they are as follows,

For  $AX=YB$  Problem, Mean rotational error is given by  $\frac{1}{n} \sum_0^{n-1} \|R_A R_X - R_Y R_B\|^2$

Mean translational error is given by  $\frac{1}{n} \sum_0^{n-1} \|R_A t_X + t_A - R_Y t_B - t_Y\|^2$

For  $AX=XB$  Problem, Mean rotational error is given by  $\frac{1}{n} \sum_0^{n-1} \|R_A R_X - R_X R_B\|^2$

Mean translational error is given by  $\frac{1}{n} \sum_0^{n-1} \|R_A t_X + t_A - R_X t_B - t_X\|^2$

#### IV. RESULTS AND DISCUSSIONS

In order to obtain the results of hand eye calibration of a robot, several other results needs to be found out and these results will act as inputs to the following steps. The first calibration is the intrinsic calibration which has been performed in order to find out the camera matrix. This resultant matrix is saved and used as the input for object detection using the same camera. The object detection package will give the transformation matrix between the camera and the object. The transformation between the robot end effector and robot base is also noted making use of the tf package of the ROS. Detailed results are discussed below.

##### A. Intrinsic Calibration Results

The images of the calibration pattern are captured at a distance of 400-500 mm. The chessboard is placed in different position and orientation. It's always made sure that the object is fully visible from the camera. The corners of the chessboard pattern have been detected using the program. The result is shown below in Fig.5. Along with the real world coordinates these results are taken as the input for the calibratecamera package of the openCV. We obtain the results after calibration as follows:

$$\text{Camera Matrix } K = \begin{bmatrix} 664.73 & 0 & 224.48 \\ 0 & 664.84 & 228.24 \\ 0 & 0 & 1 \end{bmatrix}$$

Distortion Matrix = [0.0355 0.5108 -0.0080 -0.0618 -0.9162]

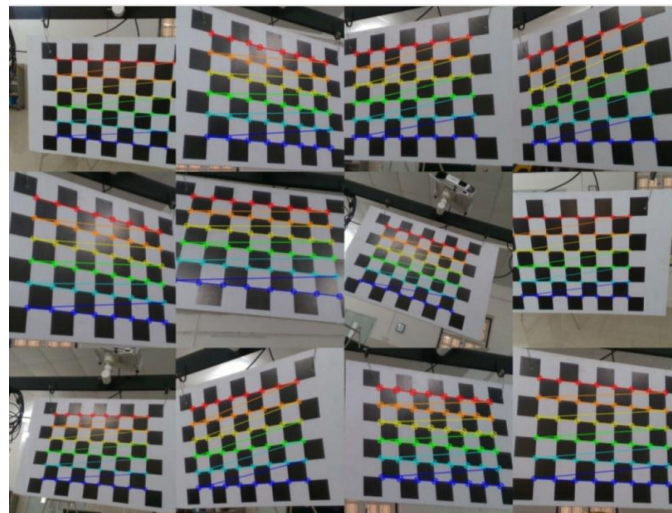


Fig.5 Chessboard Corner Detection

The results are saved on to the camera configuration file in XML format. This is to load the camera matrix into the program during the co-ordinate transformation. The reprojection error is also found out. Re-projection error is the geometric error corresponding to the image distance between a projected point and a measured one. It is used to quantify how closely an estimate of 3D point recreates the point's true projection. The recommended value of re-projection error is less than 0.5. The value of re-projection error is found to be 0.09895 which is within the acceptable limits.

##### B. ArUco Marker Detection Results

ArUco marker has been used as the object to be detected. The marker is placed fixed on the table. The camera is moved at different positions and orientations in order to detect the object. This is by using the teach pendant of the robot. The position of the robot is recorded at each position and the co-ordinate of the marker with respected to camera origin is calculated. The position of the robot and the position of the object is the input for Hand Eye calibration Problem. We use aruco\_detect package for finding



the object position with respect to the camera. The transformation between robot base and end effector is calculated using tf package in ROS.

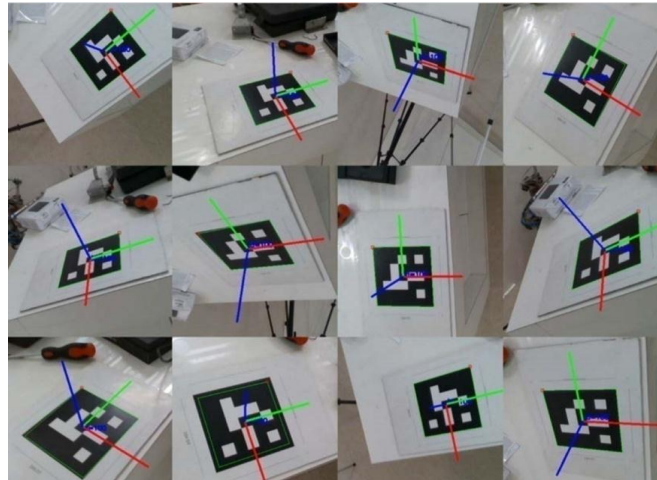


Fig.6 Different results of ArUco detection

The results we obtain for one position is shown below:

Frame\_id; —camera\_color\_optical\_frame

Image\_seq: 14594

Transforms: Fiducial\_id: 100

Transform:

Translation: x: 0.116102897921 y: 0.052133077526 z: 0.541460223379

Rotation: x: -0.744749864533 y: -0.625898732161 z: 0.221027333694 w: 0.06888638557

Robot base to camera attached wrist link Translation: [0.186, -0.523, 0.269]

Rotation: in quaternion [-0.028, 0.990, 0.131, -0.053]

Here rotation is expressed in terms of quaternion this is converted to pure rotation elements

C. Error values of AX=XB and AX=YB problems

The input transformations for a single robot camera position are given below.

Object frame to camera frame transformation matrix

$$\begin{bmatrix} 0.11879539 & 0.901824444 & -0.415451956 & 0.116102898 \\ 0.96272754 & -0.207010886 & -0.174075203 & 0.052133078 \\ -0.242988351 & -0.379287709 & -0.892803167 & 0.541460223 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Effector frame to Base frame transformation matrix

$$\begin{bmatrix} -0.994522 & -0.041554 & -0.112276 & 0.186 \\ -0.06932 & 0.96411 & 0.256412 & -0.523 \\ 0.097604 & 0.262348 & -0.961768 & 0.269 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Different datasets have been used to formulate AX=XB and AX=YB problems. Fig.7 shows different error values for the problems formulated. Mean rotation error for AX=YB is found to be 0.088727 and for AX=XB is found to be 0.056161. Mean translation error for AX=YB is found to be 1.02812 and for AX=XB is found to be 0.014533.

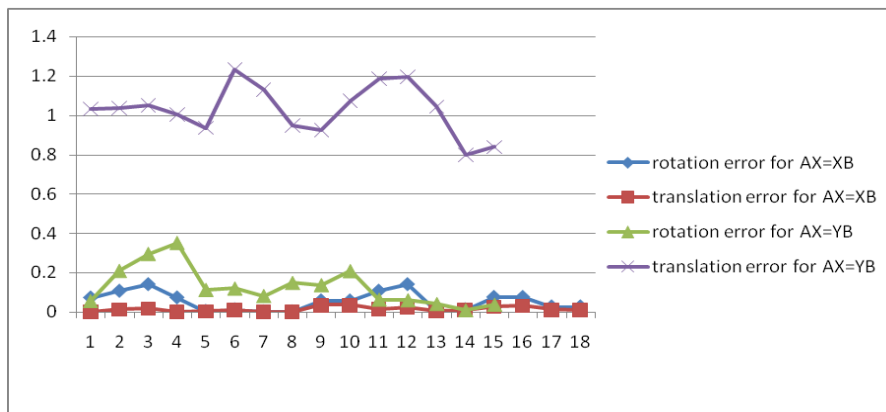


Fig.7 Errors for AX=XB and AX=YB Problems



All the scenarios 2 different position of camera and robot needs to be there so as to formulate a classical  $AX=XB$  hand eye calibration problem. A new strategy is proposed here that can be used so as to generate more number of equations using less camera robot positions. The steps will be simplified & same dataset can be used for formulating  $AX=YB$  problem statement also.

V. STRATEGY FOR  $AX=XB$  PROBLEM FORMULATION

The proposed strategy can be used to automatically generate  $AX=XB$  problem using the available dataset. For a general  $AX=XB$  problem, there is need for 2 different positions of the robot connected with a camera that is 10 positions will generate 5 equations. The foremost step in the strategy is to collect different transformations as the input dataset. These datasets can be used to formulate the problems. At first permutation of 4 data positions has been used to generate  $AX=XB$  equations. That is 4 positions will generate 12 equations when the permutation is used. Among these equations two data points will generate two equations. This can be explained using position1 and position 2. Using permutation there will be an equation formed by selecting position 1 and position 2 in order (1,2) and opposite order (2,1) of the same data position will generate another equation. This paved the way to study the error formed by these equations. If the errors are same, there is no need to formulate two different equations for the same dataset points. That is permutation approach can be neglected and combination approach can be selected.

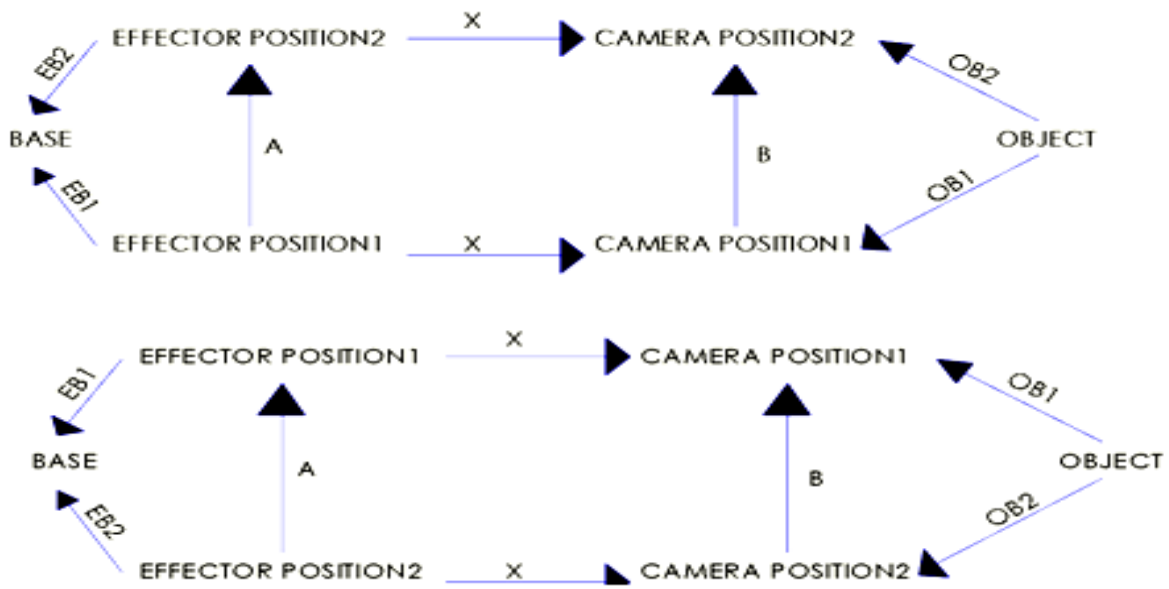


Fig.8  $AX=XB$  Problem formulation using 2 positions in (1,2) and opposite order (2.1)

Fig.8 shows position 1 and position2 formulates two different  $AX=XB$  problem. For the first order that is by taking position 1 at first and position 2 next, we can formulate  $AX=XB$  problem. Then we take position 2 at first and position 1 next, and formulate other  $AX=XB$  problem. These two problems are solved for X and the errors are calculated. In this the X value is already found out using different data positions. That X value is put in to these  $AX=XB$  problems for position 1 and position 2 and errors are calculated. In this study different positions are taken in order and in opposite order and the errors are compared.

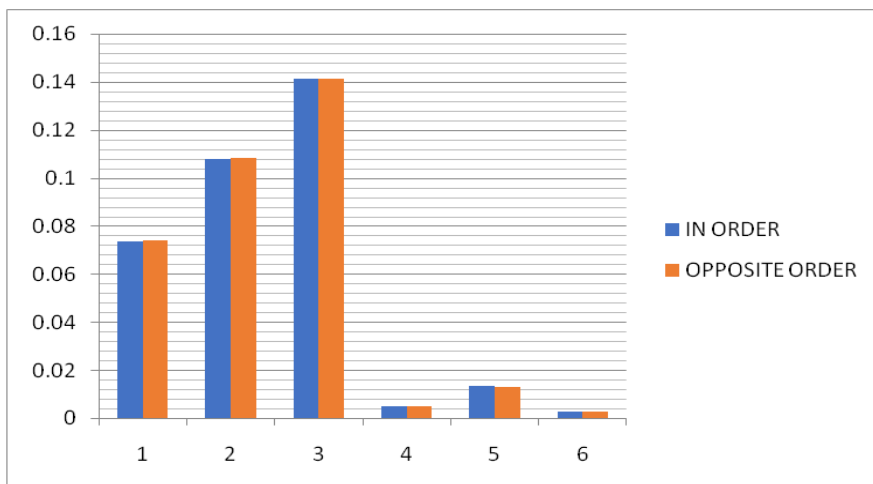


Fig.9 comparison of rotation error for in and opposite order of positions, X axis – Number of position sets, Y-axis- Rotation Error



Fig.9 and Fig.10 shows comparison of rotation error and translation error for the  $AX=XB$  problem generated using two different positions in and opposite order for a total of 6 equations. While comparing the results it has been found out that the rotation error and translation error are same for the same positions even if the order is varied and  $AX=XB$  equation is formulated. This also verifies that the permutation of datasets can be omitted and combination of data sets can be used to generate  $AX=XB$  problems. That is N number of datasets can be used to generate  $NC^2$  number of equations.

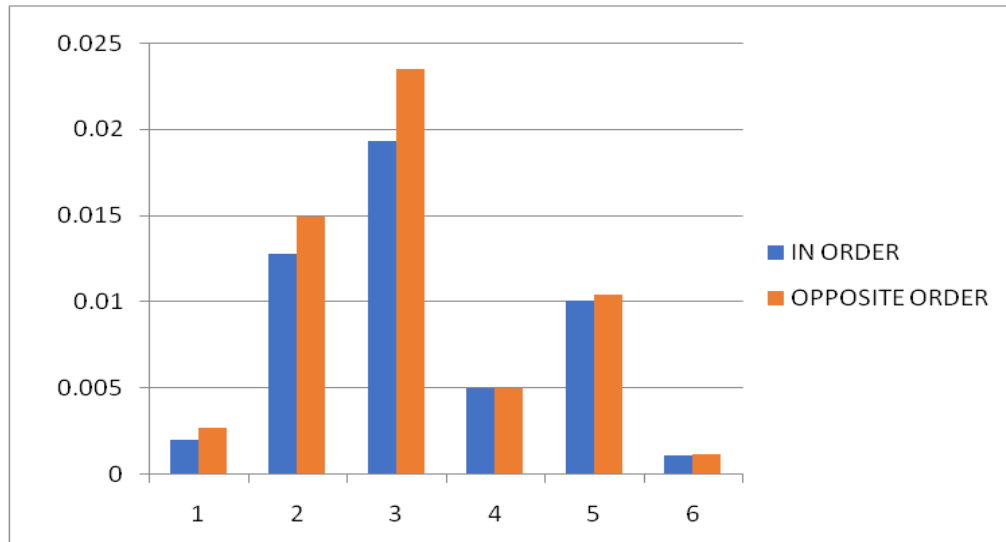


Fig.10 comparison of Translation error for in and opposite order of positions,  
X axis – Number of position sets, Y-axis- Rotation Error

## VI. CONCLUSION

Majority of the industrial robots are working as efficient as human beings. These robots are not flexible compare to the human labours. Teach and playback method is usually employed in these robots at the present scenario. To achieve more flexibility without reducing the efficiency vision integration is a need of the lime light. The vision integration is a complex and time consuming process. This thesis tries to solve this problem by exploring the hand eye calibration of a 6 DOF robot manipulator using real time experiment dataset. The basic step to be fulfilled before setting a vision robot system to any industrial environment is the calibration between two different systems the camera and the robot. Since this thesis works on this problem, it is having relevance in any field of application that uses a vision robotic system.

For almost all the applications the steps are alike. First the object is detected by the camera then camera gives signal to the robot to do specified task. If the system is in the assembly line as a pick and place robot, then the robot will pick the object detected. If it is a welding application, the welding rod is moved through the path specified in the object. For all this applications the object coordinate needs to be converted into robot coordinate system. The object can only be detected by the camera. A relation must be established between the camera and the robot. This is through the robot hand eye calibration which is completed in this work. Intrinsic and extrinsic calibrations are completed and best type of hand eye calibration problem is formulated and solved by framing the equations in the form  $AX=XB$  and  $AX=YB$ . It has been found that  $AX=XB$  problem solution is having less error. Low rotation error and translation error.

A new strategic method is also proposed in this thesis to formulate  $AX=YB$  type calibration problem using the combination of dataset positions. This method is found to be efficient and neglects the need of taking more input data points for calibration. That is there is no need to formulate 2 different equations for the same set of points since they have almost same error in solution. Using the experiment data set for different camera robot positions, the established relation is found to be intact by manually entering a program in which an object is detected by the camera and the robot end effector will move to the position of the object only with the input from the camera.

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## BIOGRAPHIES



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