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Binding Energies of Nuclei with and without shell corrections

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Abstract: The liquid drop model describes the bulk properties of nuclei. However there are significant deviations from this near closed shells underestimates the binding energies. It is therefore necessary to include a correction that takes into account of this phenomenon in order to accurately calculate ,for instance ground state masses. In the present work we have calculated the shell corrections for all possible nuclei. Seeger introduced a semi empirical mass formula. We have taken shell corrections term from this formula. Inclusion of this term brings the binding energies very close to experimental values.

INTRODUCTION

The liquid drop model proposed by Bohr and Wheeler[1] in nuclear physicstreats the nucleus as a drop of incompressible nuclear fluid. It was first proposed by George Gamow[2] and then developed by Niels Bohr[3]. The fluid is made of nucleons(protons and neutrons) which are held together by the strong nuclear force. This is a crude model that does not explain all the properties of the nucleus, but does explain the spherical shape of most nuclei. It also helps to predict the binding energy of the nucleus.

Mathematical analysis of the theory delivers an equation which attempts to predict the binding energy of a nucleus in terms of the numbers of protons and neutrons it contains. This equation has five terms on its right hand side. These correspond to the cohesive binding of all the nucleons by the strong nuclear force, the electrostatic mutual repulsion of the protons, a surface energy term, an asymmetry term (derivable from the protons and neutrons occupying independent quantum momentum states) and a pairing term (partly derivable from the protons and neutrons occupying independent quantum spin states).

Shortcomings:It fails to explain extra stability of magic nuclei.It fails to explain magnetic moment and spin of many nuclei.It is not successful in explaining excited states in most of nuclei.The agreement of semi-empirical mass formula with experimentally observed masses and binding energy is poor for lighter nuclei compared to heavy ones.

SHELL MODEL

The shell model[4] is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. When adding nucleons (protons or neutrons) to a nucleus, there are certain points where the binding energy of the next nucleon is significantly less than the last one. This observation, that there are certain magic numbers of nucleons: 2, 8, 20, 28, 50, 82, 126 which are more tightly bound than the next higher number, is the origin of the shell model.

Note that the shells exist for both protons and neutrons individually, so that we can speak of "magic nuclei" where one nucleon type is at a magic number, and "doubly magic nuclei", where both are. Due to some variations in orbital filling, the upper magic numbers are 126 and, speculatively, 184 for neutrons but only 114 for protons. There have been found some semimagic numbers, notably Z=40 giving nuclear shell filling for the various elements; 16 may also be a magic number.

LIQUID DROP MODEL AND SHELL CORRECTION

Semi empirical mass formula given by Von Weizsacker [5] can be used to predict accurately the masses of nuclei which ranges from light nuclei to heavy nuclei .In reality this situation is complicated. The inability of the liquid drop model proposed by Bohr and Wheeler [1] to account for the observed asymmetry in the mass yield curve of binary fission was demonstrated by Cohen and Swiatecki [6]. It does not explain the peaks in Binding energy curve at certain key values of N and Z.

There might be local variation of masses due to effects known as shell effects. Introduction of shell correction explains magicity in the binding energy curve. A.E.L.Deperink[7] has shown that if in addition to an improved version of liquid drop mass formula with modified symmetry and coulomb terms, shell effects are modeled.

METHODOLOGY

The Binding energies have been calculated by using semi empirical formula.

$$E_B = a_v A - a_s A^{\frac{2}{3}} - \frac{a_c Z(Z-1)}{A^{\frac{1}{3}}} - \frac{a_A (A-2Z)^2}{A} + \delta(A,Z)$$

The values of these coefficients are are calculated by “Wapstra”[8] as

$a_v = 14.1 MeV a_s = 13 MeV a_c = .595 MeV a_A = 19 MeV$

Seeger gave a formula for calculating binding energies

$$\Delta M_0(Z,A) = 7.2887\mathrm{Z} + 8.0713(\mathrm{A}-\mathrm{Z}) - \alpha\mathrm{A} + 0.8076\mathrm{Z}^2\mathrm{A}^{-1/3}\left(1 - 0.7636\mathrm{Z}^{-2/3} - 2.29\mathrm{A}^{-2/3}\right) + \gamma A^{2/3} + \left(\beta - \eta A^{-1/3}\right)A^{-1}[(A-2Z)^2 + 2|A-2Z|] - S_{jk}(N'Z')$$

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The last term in the above equation is a shell correction term, which is the function of parameter N and Z defined as

$$N' = \frac{N - N_j}{N_{j+1} - N_j}$$
$$Z' = \frac{Z - Z_k}{Z_{k+1} - Z_k}$$

Here N_j and Z_k are magic numbers
N_j, Z_k =8, 20, 50, 82, 126, 184 and $N_j \leq N < N_{j+1} Z_k \leq Z < Z_{k+1}$
Thus the function S_{jk} is different for different intervals between magic numbers. The formula for S_{jk} is

$$S_{jk}(N', Z') = \xi_j sinN'\pi + \xi_k sinZ'\pi + \nu_j sin2N'\pi + \nu_k sin2Z'\pi + (\phi_{j+}\phi_k)(sinN'\pi)(sinZ'\pi) + \chi$$

The adjustable constants have been determined by method of least squares. The constants are the same for the full range of masses listed from A =19 to A =260.
α= 17.06 MeV β = 33.61 MeV γ= 25.00 MeV η =59.54 MeV
In our calculation we use Von Weisacker's formula [5] to calculate the binding energy as

$$\Delta M(Z, A) = a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{\frac{1}{3}}} - \frac{a_A (A - 2Z)^2}{A} - S_{jk}(N', Z')$$

We have calculated seeger correction terms for isotopes of nuclei with Z =1 to 100. Graphs Z versus $S_{jk}(N', Z')$, N versus $S_{jk}(N', Z')$, A versus $S_{jk}(N', Z')$ are plotted.

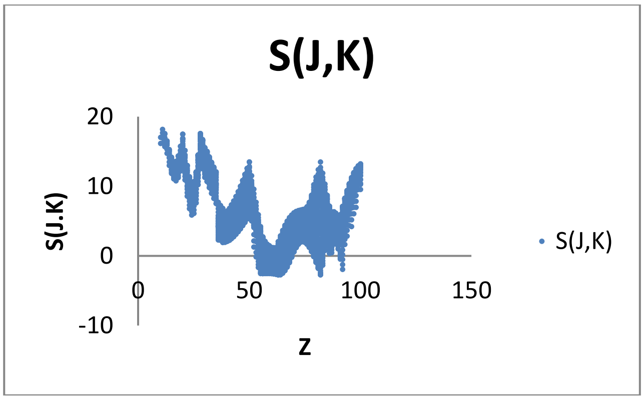

Fig. 1. No of protons (Z) vs. seeger correction S(j,k)
In this figure,we find maximum shell correction at the shell closures i.e. forZ=20,28,50,82. By extrapolating this graph the next magic nucleus can be predicted.

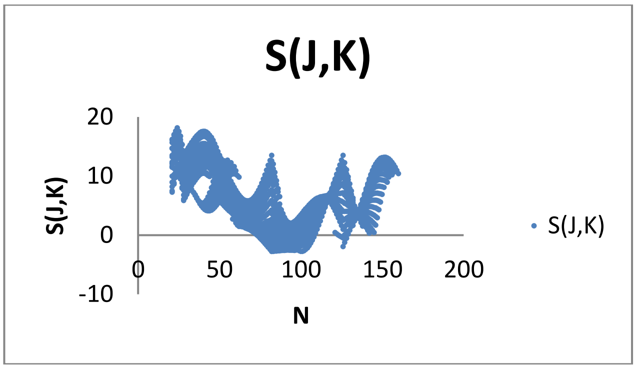

Fig.2. No of neutrons (N) vs. seeger correction S(j,k)
In this graph we find, the increasing trend at N=50, 82,126.We see this trend at N=152 also. This may lead to the next magic and after 126

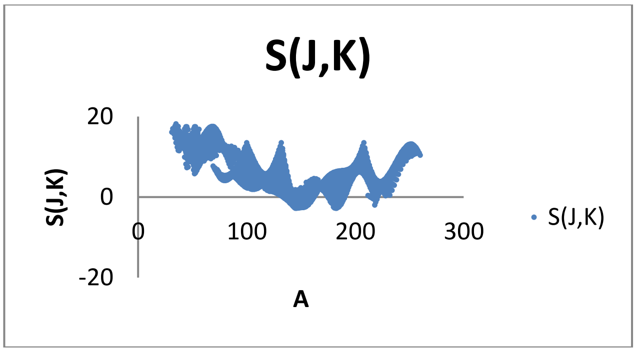

Fig.3. Mass number (A) vs. seeger correction S(j,k)
In this graph, shell correction is more for 68,100,132,208,252 matching with experimental values

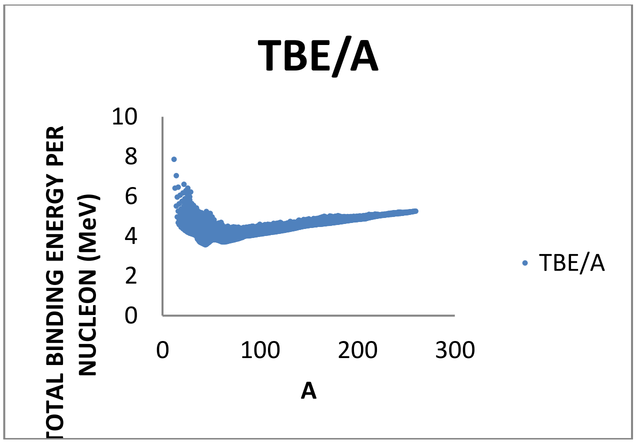

Fig.4. Total binding energy per nucleon (MeV) vs. mass number

We have calculated binding energy from semi-empirical mass formula. We have added seeger shell correction in binding energies. We get total binding energy.
Experimental graphs are shown below

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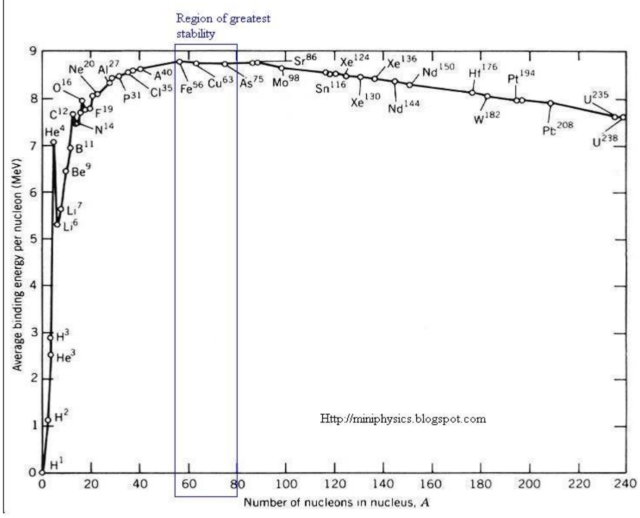

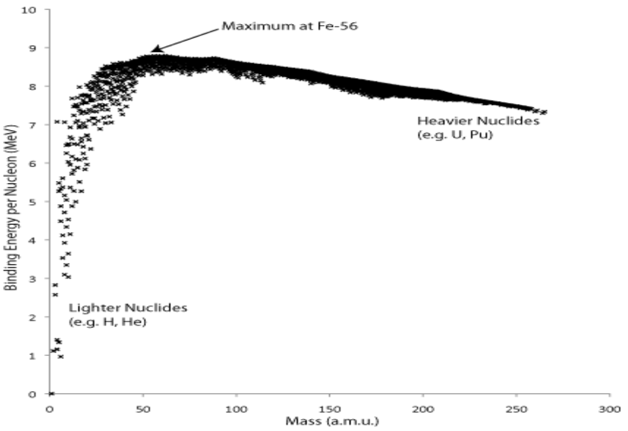

CONCLUSION

Liquid drop model of the nucleus structure explains all the binding energies but cannot explain the existence of magic nuclei. Shell corrections play very important role in nuclear structure. We have calculated total binding energy for 2810 isotopes.The theoretical graph between binding energy per nucleon versus mass number using seeger correction term is in good agreement with experimental graph.

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