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Evaluation of Relation in Discrete Mathematics

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Abstract: This aims to provide the ease of understanding in concept of relation and its types. Relation is one of the important topics of set theory. Relations set and functions are three interlinked concepts. Sets denote the collection of elements whereas functions and relations define the operations performed on sets. The relations define the connection between the two given sets. Also, there are different types of relations defining the connections between the sets.

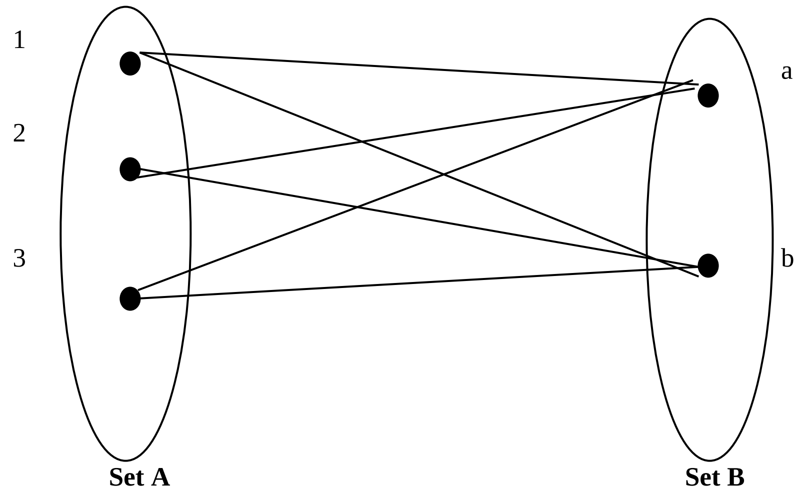
Keywords: Relation theory, relation in discrete mathematics, relation, Basic of discrete mathematics.

I. INTRODUCTION

A relation between two sets is a collection of ordered pairs containing one object from each set. A relation is just a relationship between sets of information provided to us. When a and b values are linked in an equation or inequality, they are related hence, they represent a relation. It is possible to test a graph to see if it represents a function by using the vertical line test. Relations and Functions are the most important topics in algebra. The relation shows the relationship between INPUT and OUTPUT. Whereas, a function is a relation which derives one OUTPUT for each given INPUT. All functions are relations, but vice-versa is not true.

II. RELATIONS

A relation defines the relationship between two different types of information. Relations are the set of ordered pair. It is not communicative in nature. Any subset which is created from a Cartesian product is called a relation. Cartesian product is a relation which is largest or having maximum elements.



Ordered pair means (1,a), (2,a), (2,b), (3,b)

Here Set A (1,2,3) are termed as domain

Set B (a,b) are termed as range

Above shown figure is a Cartesian product because every element of one set is related to every elements of other set.

Thus Cartesian product is denoted by X and it has (1,a), (1,b), (2,a), (2,b), (3,a), (3,b) ordered pair.

Any subset from Cartesian product is also a relation means,

Example 1: (1,a), (2,a), (2,b), (3,b) is a relation
Example 2: (1,b), (2,b), (3,a), (3,b) is a relation
Example 1: (2,a), (3,b), (2,b), (1,a) is a relation

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III.COMPLEMENT OF RELATION

Complement of relation implies all those elements which is not exist in particular relation for which we are finding complement. Complement of a relations is denoted by R'.

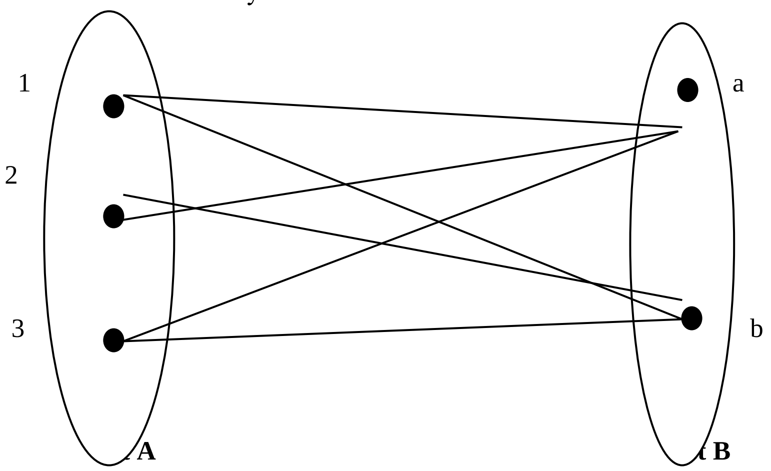

Figure 1: Showing the complement of a relation

Let A X B = (1,a), (1,b), (2,a), (2,b), (3,a), (3,b)
Let R = { (2,b), (1,b), (2,a) } is a relation
R' = { (1,a), (3,a), (3,b) } it is a complement of R
So Complement of relations is [(A X B) – R]

IV. INVERSE OF RELATION

Inverse of relation defines when the ordered pair of any relation has been interchanged. Inverse of a relation is denoted by R-1.

In a above shown figure 1,
Let A X B = (1,a), (1,b), (2,a), (2,b), (3,a), (3,b)
Let R = { (2,b), (1,b), (2,a) } is a relation
R-1 = { (b,2), (b,1), (a,2) }

V. TYPES OF RELATIONS

Various types of relation exist in discrete mathematics. Below each type or relations is explained with set of examples.

Reflexive Relation

A relation R on set A is said to be reflexive if ∀ a E A, (a,a) E R . This implies in reflexive relation all the elements are mapped to itself. To become reflexive pair, one should have diagonal pair in set. No matter if it contains some other pair or not.

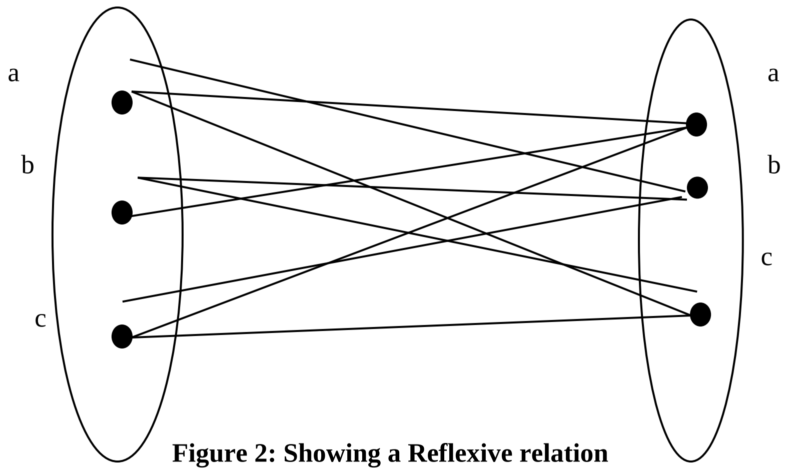

Figure 2: Showing a Reflexive relation

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aa	ab	ac
ba	bb	bc
ca	cb	cc

Table 1: Showing a pair of reflexive relation

The diagonal pair will make reflexive relation.

Example 1	{ (a,a), (b,b), (c,c) }	Reflexive relation (Smallest)
Example 2	{ (a,a), (a,b), (b,b), (c,c) }	Reflexive relation
Example 3	{ (a,a), (b,a), (c,c) }	Not Reflexive relation
Example 4	∅	Not Reflexive relation

Irreflexive Relation

A relation R is said to be irreflexive if a E A, (a,a) ∄ R. Irreflexive relation it should not contain any of the elements which is mapped to itself. It doesn't contain any of diagonal elements to become irreflexive relation.
In above figure 2 and table 1

Example 1	{ (a,a), (b,b), (c,c) }	Not Irreflexive relation
Example 2	{ (a,a), (a,b), (b,b), (c,c) }	Not Irreflexive relation
Example 3	{ (a,a), (b,a), (c,c) }	Not Irreflexive relation
Example 4	∅	Irreflexive relation (Smallest)
Example 5	{ (a,b), (b,a), (c,a) }	Irreflexive relation

Symmetric Relation

A relation R is said to be symmetric if ∀ a,b E A, (a,b) E R then (b,a) E R. Symmetric relation means all those elements which it contain in an relation, inverse of all those elements should be posses in that same relation to become an symmetric relation except the diagonal element.
In above figure 2 and table 1

Example 1	{ (a,a), (b,b), (c,c) }	Symmetric relation
Example 2	{ (a,a), (a,b), (b,b), (c,c) }	Not Symmetric relation
Example 3	{ (a,a), (b,a), (c,c) }	Not Symmetric relation
Example 4	∅	Symmetric relation (Smallest)
Example 5	{ (a,b), (b,a), (c,a), (a,c), (a,a) }	Symmetric relation

Anti-Symmetric Relation

A relation R is said to be anti-symmetric if ∀ a,b E A, (a,b) E R then (b,a) ∄ R. Anti-symmetric relation means all those elements which it contain in an relation, inverse of all those elements should not posses in that same relation to become an symmetric relation except the diagonal element.

In above figure 2 and table 1

Example 1	{ (a,a), (b,b), (c,c) }	Anti-Symmetric relation
Example 2	{ (a,a), (a,b), (b,b), (c,c) }	Anti-Symmetric relation
Example 3	{ (a,a), (b,a), (c,c) }	Anti-Symmetric relation
Example 4	∅	Anti-Symmetric relation (Smallest)
Example 5	{ (a,b), (b,a), (c,a), (a,c), (a,a) }	Not Anti-Symmetric relation

Asymmetric Relation

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A relation R is said to be asymmetric if ∀ a,b E A, (a, b) E R then (b, a) ∄ R. Anti-symmetric relation means all those elements which it contain in an relation, inverse of all those elements should not posses in that same relation to become an symmetric relation including the diagonal element.

In above figure 2 and table 1

Example 1	{ (a,a), (b,b), (c,c) }	Not Asymmetric relation
Example 2	{ (a,a), (a,b), (b,b), (c,c) }	Not Asymmetric relation
Example 3	{ (a,a), (b,a), (c,c) }	Not Asymmetric relation
Example 4	∅	Asymmetric relation (Smallest)
Example 5	{ (a,b), (b,a), (c,a), (a,c), (a,a) }	Not Asymmetric relation
Example 6	{ (a,b), (b,c), (c,a)	Asymmetric relation

Transitive Relation

A relation R is said to be transitive if ∀ a, b E A, (a, b) E R and (b, c) E R then (a, c) E R. Transitive relation means all elements should posses the transitive property such that if (a, b) exists and (b, c) exists then (a, c) should also exists in an relation to become transitive relation.

In above figure 2 and table 1

Example 1	{ (a,a), (b,a), (a,b), (b,b) }	Transitive relation
Example 2	{ (a,a), (b,b), (c,c) }	Transitive relation
Example 3	{ (a,c), (a,b) }	Transitive relation
Example 4	{ (a,b) }	Transitive relation
Example 5	{ (b,c), (c,a)	Not Transitive relation

To identify whether a relation is transitive relation or not Warshall Algorithm can be used

Warshall Algorithm

Example 1 = { (1,1), (1,3), (2,2), (3,1), (3,2)}

(i) Before applying Warshall algorithm, first find out transitive closure and for this first convert the given set into matrix format by writing 1 if pair exist between them and 0 if no pair exist.

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 \end{array}$$

(ii) Now, find the closure (I, II, III) three columns are created because of 3X 3 matrix. C implies Column and R implies Row. Now for filling (C,I) cell consider the step one matrix and find out that in column 1 where are the no. 1 is written and then writes its relevant row no in cell (C,I) and repeat the same process for (C,II) and (C, III).

(iii) For filling R (R,I) cell consider the step one matrix and find out that in Row 1 where are the no. 1 is written and then writes its relevant column no in cell (R,I) and repeat the same process for (R,II) and (R, III).

(iv) Then find the Cartesian product of C X R by writing ordered pair in each cell I, II, III

(v) Then all the pairs of set present in C X R row must exist in resultant relation to make that relation as transitive.

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(vi) So, here to as per above question to find out a transitive relation include all the pair mentioned at C X R that are { (1,1), (1,3), (3,1), (3,3), (2,2), (3,2), (1,2)

	I	II	III
C	(1, 3)	(2, 3)	(1)
R	(1, 3)	(2)	(1,2)
C X R	(1,1), (1,3), (3,1), (3,3)	(2,2) (3,2)	(1,1) (1,2)

Equivalence Relation
A relation R is said to be equivalence if it is

(i) Reflexive
(ii) Symmetric
(iii) Transitive
In above figure 2 and table 1

Example 1	{ (a,a), (b,b), (c,c), (b,a) }	Not Equivalence relation
Example 2	{ (a,a), (b,b), (c,c) }	Equivalence relation
Example 3	{ (a,c), (a,a), (b,a), (c,a) }	Not Equivalence relation
Example 4	{ ∅ }	Not Equivalence relation
Example 5	{ (a,a), (b,b), (c,c), (b,a), (c,a), (a,c), (a,b) }	Equivalence relation

VI. PARTIAL ORDERING RELATION

A relation R is said to be equivalence if it is

(i) Reflexive
(ii) Anti-Symmetric
(iii) Transitive
In above figure 2 and table 1

Example 1	{ (a,a), (b,b), (c,c), (b,a), (a,b) }	Not Partial Order relation
Example 2	{ (a,a), (b,b), (c,c) }	Partial Order relation
Example 3	{ (a,c), (a,a), (b,b), (b,c), (c,c) }	Partial Order relation
Example 4	{ ∅ }	Not Partial Order relation
Example 5	{ (a,a), (b,b), (c,c), (b,c), (a,c) }	Partial Order relation

VII. HASSE DIAGRAM/POSET DIAGRAM

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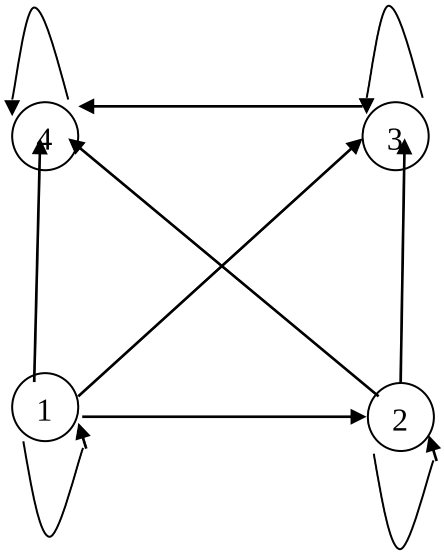
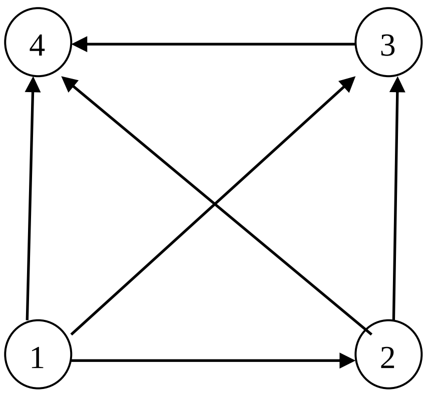
A Hasse diagram is used to find out whether a relation is a partial order relation or not. It is also known as poset diagram. For this we have follow a step wise procedure which is as follows and assume that A with relation R as given below

A = { 1,2,3,4}
R = { (11,), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4) }

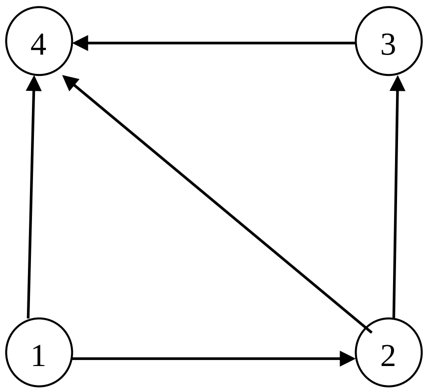
(i) Convert a partial order relation to hasse diagram by first converting the above given relation into graph format



(ii) Now simplify the graph by removing reflexive edges (self loop) and transitive edges because we already know that they exist due to relation is in partial order relation.



(Here, we have removed self loop)



(here, we removed edge (1,3) because (1,2) and (2,3) already exist so we removed transitive edge)

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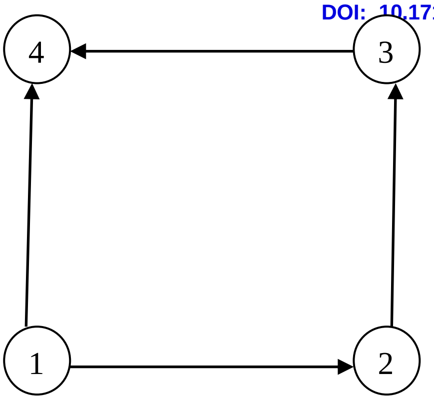
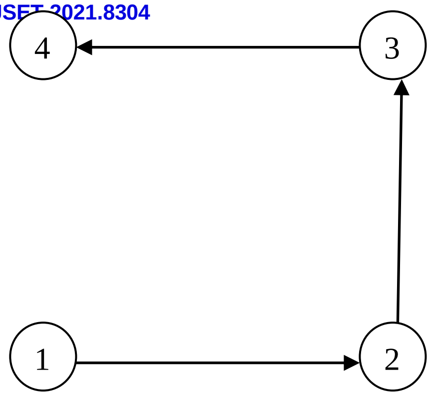
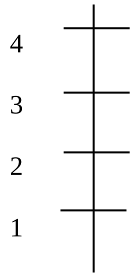


(here, we removed edge (2,4) because (2,3) and (3,4) already exist so we removed transitive edge



(here, we removed edge (1,4) because (1,2), (2,3) and (3,4) already exist so we removed transitive edge)

(iii) Now convert this in linear order as hasse diagram moves from bottom to top and it is non directed graph and its show link by order of nodes.



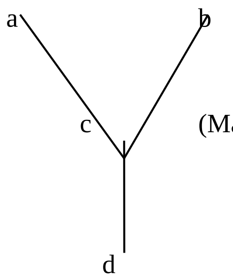
(iv) The above shown diagram is a hasse diagram.

VIII. MAXIMAL ELEMENT

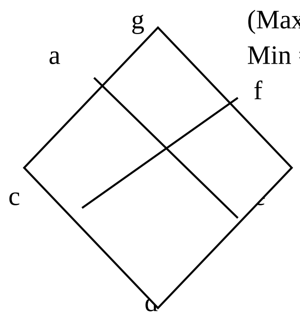
If in a POSET diagram, an element is not related to any other element or an element which is not less than any other element of the POSET. An element over which no path exists which is an dead end.

IX. MINIMAL ELEMENT

If in a POSET diagram, no element is related to any other element or an element which is not greater than any other element of the POSET. An element under which no path exists which is a starting point.



(Maximal = a,c)



(Max= g
Min = d)

There will always one element exists as Maximal and Minimal element. They will never be NULL.

X. MAXIMUM ELEMENT

If in a POSET diagram, an element is maximal element and every element is related to that element in POSET.

XI. MINIMUM ELEMENT

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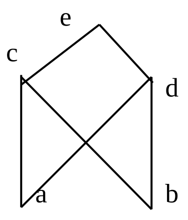
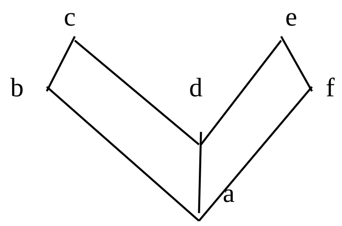
If in a POSET diagram, an element is minimal element and every element is originated from that element or it is related to every element in POSET.





Maximum = e
Minimum = NULL
Maximal = e
Minimal = a, b

Maximum = NULL
Minimum = a
Maximal = c, e
Minimal = a

XII. UPPER BOUND

Let B be a subset of set A. An element x E A is in upper bound of B if (y,x) E POSET ∀ y E B. Simply Upper bound means a node which is on higher side/upper side from all those given nodes.

XIII. LOWER BOUND

Let B be a subset of set A. An element x E A is in lower bound of B if (x,y) E POSET ∀ y E B. Simply Lower bound means a node which is on lower side/bottom side from all those given nodes.

XIV. LEAST UPPER BOUND

A minimum element in an upper bound is known as least upper bound. This element is also known as supremum or Join. Symbol to denote least upper bound is V.

XV. GREATEST LOWER BOUND

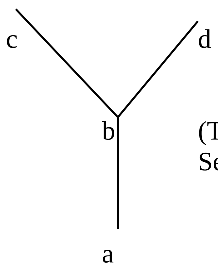
A maximum element in an lower bound is known as greatest lower bound. This element is also known as infimum or meet. Symbol to denote least upper bound is $\wedge$

XVI. MEET SEMI LATTICE

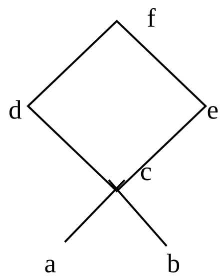
In a POSET if greatest lower bound/MEET/infimum exists for every pair of elements, then that POSET is called as Meet Semi Lattice. It is a binary operation thus it applies between two nodes.

XVII. JOIN SEMI LATTICE

In a POSET if lowest upper bound/JOIN/supremum exists for every pair of an elements, then that POSET is called as Join Semi Lattice. It is a binary operation thus it applies between two nodes.



(True example of Meet Semi Lattice)



(It is not a Meet-Semi Lattice because (a,b) has no lower bound)

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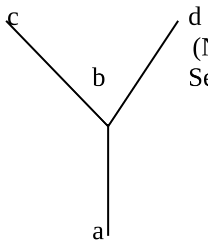
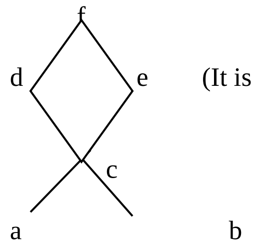


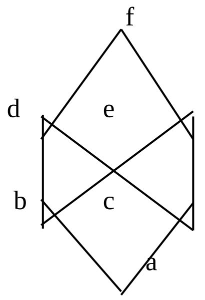
(Not example of Join Semi Lattice)



(It is a Join-Semi Lattice)

XVIII. LATTICE

In a POSET if lowest upper bound/JOIN/supremum and greatest lower bound/MEET/infimum both exists for every pair of an elements, then that POSET is called as Lattice. It is a binary operation thus it applies between two nodes.



Find	For edge (d,e)	For edge (b,c)
Upper Bound	f	e,f,d
Lower Bound	a,b,c	a
Least Upper Bound	f	NULL
Greatest Lower Bound	NULL	a

XIX. CONCLUSION

The Analysis of Relations in Discrete Mathematics is a very wide content that it can be used in various area to solve from very basic to scientific problems. Relations in discrete mathematics are any association or link between elements of one set. Thus relation has a property to map one value to more than one value. In general, a relation is any subset of the Cartesian product of its domain and co-domain. Relations can also be represented using a directed graph. Relations may exist between objects of the same set or between objects of two or more sets. A relation may have possibility to have more than one output for any given input. With this feature it can be applied to day-to life as well.

XX. ACKNOWLEDGEMENTS

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XXI. REFERENCES

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