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Mathematics in Scientific Environs: A case study of different Applications

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Abstract: Mathematics, will play a very important role in different fields of Science and Engineering Matrices are very important in cryptography, robotics, computer graphics, optics, Economics, Chemistry, Geology and Signal processing etc.

Keywords: Cryptography, Computer Graphics, Optics, Encryption, Decryption, Eigen Values and Eigen Vectors

INTRODUCTION

The growth Mathematics parallel to development of human civilization, Numbers were created for counting and measurements in equalities to compare sizes, functions to express the dependence of one physical or geometrical quantities on the other Mathematics was derived from two Greek words:

Manthanein : to learn and
Techne : an art or Technique

Use of matrices in cryptography

- Cryptography is concerned with keeping communications private.
- Cryptography mainly consists of Encryption and Decryption.
- Encryption is the transformation of data into some unreadable form
- Its purpose is to ensure privacy by keeping the information hidden from anyone for whom it is not intended, even those who can see the encrypted data.
- Decryption is the reverse of encryption it is the transformation of encrypted data back into some intelligible form
- Encryption and decryption require the use of some secret information usually referred to a key.
- Depending on the encryption mechanism used, the same key might be used for both encryption and decryption while for other mechanisms, the keys used for encryption and decryption might be different.

Application of matrix to cryptography

- One type of code, which is extremely difficult to break makes use of a large matrix to encode a message.
- The receiver of the message decodes it using the inverse of the matrix.
- This first matrix, used by the sender is called the encoding matrix and its inverse is called the decoding matrix, which is used by the receiver.

Cryptography

Matrix inverse can provide a simple and effective procedure for encoding and decoding messages. Assign the numbers 1 - 26 to the letters in the alphabet Assign 27 to blank
A B C D E .... X Y Z blank
1 2 3 4 5 24 25 26 27
Any matrix ‘A’ choose elements are positive integers and choose inverse exists can be used as an “encoding matrix”.
A message can be decoded by multiplying with A^{-1}, the ‘decoding matrix’.
Eg: The message secret code corresponds to the sequence
19 5 3 18 5 20 27 3 15 4 5

Encoding matrix $A = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}_{2\times 2}$

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First divided the numbers in the sequence into groups of 2 and use these groups as the columns of matrix B with 2 rows.

$$B = \begin{bmatrix} 19 & 3 & 5 & 27 & 15 & 5 \\ 5 & 18 & 20 & 3 & 4 & 27 \end{bmatrix}_{2\times 6}$$

[Note: We added an extra blank at the end of the message to make the columns come out even]

C = AB

$$C = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 19 & 3 & 5 & 27 & 15 & 5 \\ 5 & 18 & 20 & 3 & 4 & 27 \end{bmatrix}$$

$$C = \begin{bmatrix} 91 & 66 & 80 & 117 & 72 & 101 \\ 24 & 21 & 25 & 30 & 19 & 32 \end{bmatrix}$$

The coded message is

91 24 66 21 80 25 1 1 7 30 72 19 10 132

$$A^{-1} = \frac{1}{4-3} \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$$

Original message is = A^{-1} C

$$= \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 91 & 66 & 80 & 117 & 72 & 101 \\ 24 & 21 & 25 & 30 & 19 & 32 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 3 & 5 & 27 & 15 & 5 \\ 5 & 18 & 20 & 3 & 4 & 27 \end{bmatrix}$$

= B

$\Rightarrow$ 19 5 3 18 5 20 27 3 15 4 5 27

S E C R E T C O D E

Eg: The message 46 84 85 55 101 31 59 64 57 102 99 29 57 38 65 111 122 was encoded with matrix A decode

this message. Here $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

Solution: Since A is 3x3, enter the coded message in the columns of a matrix C with three rows.

$$C = \begin{bmatrix} 46 & 55 & 31 & 57 & 29 & 65 \\ 84 & 101 & 59 & 102 & 57 & 111 \\ 85 & 100 & 64 & 99 & 38 & 122 \end{bmatrix}$$

If B is the matrix containg the uncoded (original) message, then B and C are related by C = AB, To recover B,

multiply by A^{-1}, thus B = A^{-1}C

$$= \begin{bmatrix} 5 & 2 & 1 \\ 2 & -1 & 0 \\ 4 & 1 & -1 \end{bmatrix} \begin{bmatrix} 46 & 55 & 31 & 57 & 29 & 65 \\ 84 & 101 & 59 & 102 & 57 & 111 \\ 85 & 100 & 64 & 99 & 38 & 122 \end{bmatrix}$$

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$$=\begin{bmatrix} 23 & 27 & 27 & 18 & 7 & 19 \\ 8 & 9 & 3 & 12 & 1 & 19 \\ 15 & 19 & 1 & 27 & 21 & 27 \end{bmatrix}$$

Writing the numbers in the columns of the matrix in sequence and using the correspondence we get 23 8 15

27 9 19 27 3 1 18 12 27 7 1 21 19 19 27

W H O I S C A R L G A U S S

Eg: Encode the message

"THE SUNALSO RISES"

Using $A=\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Solution:

T H E S U N A L S O R I S E S

20 8 5 27 19 2 14 27 1 12 19 15 18 9 19 5 19 27

$$B=\begin{bmatrix} 20 & 5 & 19 & 14 & 1 & 19 & 27 & 9 & 5 & 27 \\ 8 & 27 & 21 & 27 & 12 & 15 & 18 & 19 & 19 & 27 \end{bmatrix}$$

$$AB=\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 20 & 5 & 19 & 14 & 1 & 19 & 27 & 9 & 5 & 27 \\ 8 & 27 & 21 & 27 & 12 & 15 & 18 & 19 & 19 & 27 \end{bmatrix}$$

$$=\begin{bmatrix} 36 & 59 & 61 & 68 & 35 & 49 & 63 & 47 & 43 \\ 44 & 86 & 82 & 95 & 37 & 64 & 81 & 66 & 62 \end{bmatrix}$$

Coded message

36 44 59 86 61 82 68 95 35 37 49 64 63 81 47 66 43 62

Eg: Decode the message

37 52 24 29 73 96 49 69 62 89 36 44 59 86 41 50 22 26 which was encoded with matrix $A=\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Solution : $A^{-1}=\dfrac{1}{3-2}\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}=\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

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$$B = A^{-1}C = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 37 & 24 & 73 & 49 & 62 & 36 & 59 & 41 & 22 \\ 52 & 29 & 96 & 69 & 89 & 44 & 86 & 50 & 26 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 14 & 27 & 9 & 8 & 20 & 5 & 23 & 14 \\ 15 & 5 & 23 & 20 & 27 & 8 & 27 & 9 & 4 \end{bmatrix}$$

7	15	14	5	27	23	9	20	8	27	20	8	5	27	23	9	14	4	27
G	O	N	E		W	I	T	H		T	H	E		W	I	N	D	

Applications of Rank Eigen values and Eigen vectors

- Eigen values and Eigen vectors will play a major role in image enhancements / edge enhancements / digital Filters which kalman filters etc.

Computer graphics:

- To synchronize and enhance both audio as well as video application on far with integrated digital networks.
- Matrices plays a vital role in machine learning with regard to learning strategies.
- Natural Language comprehension regard to lexical / syntactic / referential ambiguity.
- In cinematographic industry for enhancing and reducing a voice depending on theme of the situation we consider a lower order rank for the matrix to be considered to represent voice (Audiometric systems in IOT based sensor environment).
- The Eigen values and Eigen vectors in derandomizing random algorithms such as graph position.
- Special clustering with position determine Eigen values and Eigen vectors of the graph Laplacian matrices which can rap the face.
- Spectrum (collection of Eigen values) of similarity matrix of the data to perform dimensionality reduction.
- Single value decompositions closely related issue of linear Algebra
- The rank of a matrix produces effect on s/n radio (signal by noise ratio)
- Eigen Value problems arising from Markov Process.
- Eigen Value problems arising from Population models. Leslie model.

eg. The Leslie model describes age-specified population growth as follows. Let the oldest age attained by the female in animal population into three age classes of 2 years each. Let the "Leslie Matrix" be

$$L = [L_{jk}] = \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}$$

Where $l1k$ is the average number of daughters born to a single female during the time she in age class k, and $L_{j,j-1}$(j=2,3) is the fraction of female in age class j-1 that will survive and pass in to class j.

What is the number of females in each class after 2,4, 6 years of each class initially consists of 500 females?

Solution: Initially $X_O^T = [500, 500, 500]$

After 2 years

$$X(2) = LX(o) = \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \begin{bmatrix} 500 \\ 500 \\ 500 \end{bmatrix}$$

$$= \begin{bmatrix} 1350 \\ 500 \\ 150 \end{bmatrix}$$

Similarly, after 4 years the number of females in each class is given by

$$X_{(4)}^T = (Lx(2))^T = [750 \ \ 810 \ \ 90]ad$$

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after 6 years we have $X_{(e)}^{T}=(Lx(4))^{T}=[1899\ \ 450\ 243]$

Matrices for financial records:
The matrix method useful in opening and closing balances for any accounting period is very efficient, accurate and less time consuming.

Application of Matrices in optics:
Matrices are used in ray optics during the image formation and YNU ray tracing.
Y - Height of the object.
N - Refractive index.
U - Angle of refraction.

Applications of Laplace Transform:

- Easier than solving differential equations.
- Used to describe system behavior
- we assume LTI systems
- Uses S-domain instead of frequency domain
- Circuit analysis
- provides the general solution to any arbitrary wave (note just LRC
- Transient
- Sinusoidal steady state response (Phasors)
- Signal processing
- Communications

Laplace Transform convergence

- The Laplace transform does not converge to a finite value for all signals and all values of S.
- The values of S for which Laplace transform converges is called Region of convergence (ROC)

Always include ROC in your solution!

Example: $f(t)=e^{-at}ut$

$$F(s)=\int_{-\infty}^{\infty} f(t)e^{-st}\,dt$$

$$=\frac{1}{-S-a}e^{(-S-a)t}\Big|_{o+}^{\infty}; \quad \text{note: } S=\delta+ju$$

$$=\frac{-1}{S+a}e^{-(\delta+jw+a)t}\Big|_{o+}^{\infty}$$

$$=\frac{-1}{S+a}e^{-(\delta+a)t}e^{-jwt}\Big|_{o+}^{\infty} \Rightarrow \operatorname{Re}(S+a)>t$$

$$=\frac{1}{S+a}; \operatorname{Re}(s)>-a$$

Shifting and Time differentiation
Shifting in S-domain
Given $X(S)=L\{x(t)\}$ what is the inverse
Laplace transform of X (S + So)?

$$X(S+So)=\int_{o^-}^{\infty} x(t)\,\bar{e}^{(S+So)t}dt$$

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$$= \int_{o^-}^{\infty} \left(x(t)\bar{e}^{Sot}\right)\bar{e}^{St}dt$$

$$= L\left\{\bar{e}^{Sot}x(t)\right\}$$

$$L^{-1}\left\{X(S+So)\right\} = \bar{e}^{Sot}x(t)u(t)$$

$$\bar{e}^{Sot}x(t)u(t) \underset{\leftrightarrow}{L} X(S+So)$$

Differentiation in t

Given $X(S) = L\{x(t)\}$ What is the Laplace transform of x' (t)?

$$L\left\{\frac{dx(t)}{dt}\right\} = \int_{o^-}^{\infty} \frac{dx(t)}{dt}\bar{e}^{st}dt$$

$$u = \bar{e}^{st} \quad du = -S\bar{e}^{st}dt$$

$$dx = \frac{dx(t)}{dt}dt$$

$$L\left\{\frac{dx(t)}{dt}\right\} = \int_{o^-}^{\infty} u\,dv = uv\Big|_{o^-}^{\infty} - \int_{o^-}^{\infty} v\,du$$

$$= \bar{e}^{st}x(t)\Big|_{o^-}^{\infty} - \int_{o^-}^{\infty} x(t)\left(-S\bar{e}^{st}\right)dt$$

$$= o - x\left(e^-\right) + s\int_{o^-}^{\infty} x(t)\bar{e}^{st}dt$$

$$\frac{dx(t)}{dt}u(t) \underset{\leftrightarrow}{L} S \times (S) - x\left(o^-\right)$$

Application of Laplace

Consider an RL circuit with R = 4, $L = \frac{1}{2}$

Find $1(t) if\ v(t) = 12u(t)$

$$L\frac{di(t)}{dt} + Ri(t) = v(t)$$

$$0.5\frac{di(t)}{dt} + 4i(t) = v(t)$$

$$(0.5S + 4) = I(S) = V(S)$$

$$H(S) = I(S)/V(S) = \frac{1}{0.5S + 4} \Rightarrow I(S) = H(S)V(S)$$

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$$V(t)=12u(t)\leftrightarrow V(S)=\frac{12}{S}$$
$$I(S)=H(S)\,V(S)$$
$$=\frac{24}{S(S+8)}=\frac{K_1}{S}+\frac{K_2}{S+8}$$
$$K_1=[(s-p_1)I(s)]\Big|_{p_1=0}=3$$
$$K_2=[s-p_2\,IS]\Big|_{p_2=-8}=3$$
$$\Rightarrow I(s)=\frac{3}{S}+\frac{-3}{S+8}$$
$$\Leftrightarrow i(t)=3u(t)-3\overline{e}^{8t};t>0$$

Partial Traction Expansion
(No repeated Poles / Roots) Example

Using Mat Lab: $\frac{8x^3+3x-21}{x^3-7x-6}$

Mat Lab code:

p = 3.000

r =	3.000
	1.000
	4.000
	-2.000
	-1.000
k =	[ ]

$$b=[8\ 3-21]$$
$$a=[1\ 0-7\ -6]$$
$$[r,p,k]=residue(b,a)$$
$$\frac{b(s)}{a(s)}=\frac{r_1}{s-p_1}+\frac{r_2}{s-p_2}+\therefore+\frac{r_n}{s-p_n}+k(s)$$
$$\frac{8x^2+3x-21}{(x+2)(x-3)(x+1)}=\frac{1}{x+2}+\frac{1}{x-3}+\frac{4}{x+1}$$

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