

# Vibrational Analysis of Plates Attached with Stiffeners

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**Abstract:** The equation of motion yields the characteristic equations for natural frequencies, buckling loads, and their corresponding mode shapes. The effects of stiffener position and number, aspect ratios, boundary conditions, stiffener parameters, and stiffener parameters on the buckling load parameter and fundamental frequency of stiffened plates are studied. The eccentricity of the stiffeners gives rise to axial and bending displacement in the middle plane of the plate.

**Keywords:** Vibration, Buckling, Equispaced stiffener, Stiffened plate.

## I. INTRODUCTION

The equation of motion is used to derive the characteristic equations for natural frequencies, buckling loads, and their corresponding mode shapes. The effects of stiffener position and number, aspect ratios, boundary conditions, stiffener parameters, and stiffeners parameters on the buckling load parameter and fundamental frequency of stiffened plates are investigated. Stiffened structures are widely used in aircraft, ship, bridge, building, and some other engineering activities.

For the analysis of free vibration of uni-directionally and cross-stiffened plates, Aksu [1] used a variational principle in combination with the finite difference method, taking into consideration in-plane inertia and in-plane displacements in both directions.

Shastri and Rao [2] have used the 3 noded conforming element and refined beam-bending element for arbitrary oriented stiffeners.

Olson and Hazell [3] used the finite element method to implement a critical analysis on a clamped integrally stiffened plate. Using the real-time holographic technique, the mode shapes and frequencies were determined experimentally. The effect of rib stiffness changes on different modes has been investigated.

For a few orientations, Sundersan et al. [4] investigated the effect of partial edge compression on the buckling behaviour of angle ply plates. For the analysis of free vibration of uni-directionally and cross-stiffened plates, Aksu [5] used a variational principle in accordance with the finite difference method, taking into account in-plane inertia and in-plane displacements in both directions.

Sheikh and Mukhopadhyay [6] applied the spline finite strip method to the free vibration analysis of stiffened plates of arbitrary shapes.

The problem of vibration and buckling of rectangular stiffened plates subjected to in-plane uniform and non-uniform edge loading is presented in this paper.

The Galerkin method was used by Diez et al. [7] to investigate the effect of combined normal and shear in-plane loads. Singh and Dey [8] used a difference-based variational approach to investigate transverse vibration of rectangular plates subjected to in-plane forces in various combinations. The problem of vibration and buckling of rectangular stiffened plates subjected to in-plane uniform and non-uniform edge loading is presented in this paper. The plate is modeled with a nine noded isoperimetric quadratic element in this study, which takes into account the contributions of bending and membrane actions.

## II. MATHEMATICAL FORMULATION

The governing equations for the buckling and vibration of stiffened plates subjected to in-plane harmonic edge loading are developed. The nine noded isoparametric quadratic elements with five degrees of freedom ( $u$ ,  $v$ ,  $w$ ,  $\theta_x$  and  $\theta_y$ ) per node have been employed in the present analysis. Using the isoparametric coordinates, the element stiffness matrix, mass matrix, geometric matrix are expressed as:

$$[K_b]_p = \int_{-1}^{+1} \int_{-1}^{+1} [B_p]^T [D_p] [B_p] |J_p| d\xi d\eta \quad (1)$$

$$[K_G]_P = \int_{-1}^{+1} \int_{-1}^{+1} [B_{Gp}]^T [\sigma_P] [B_{Gp}] |J_p| d\xi d\eta \quad (2)$$

The governing equations for specified problems like vibration, static and dynamic stability are as: Vibration with in-plane load:

$$[M] \{\ddot{q}\} + [[K_b] - P[K_G]] \{q\} = \{0\} \quad (3)$$

where  $[K_b]$ ,  $[K_G]$ ,  $[M]$  are overall elastic stiffness, geometric stiffness, and mass matrices respectively,  $\{q\}$  is the displacement vector.

As shown in Figure 1, the basic configuration of the problem is an unstiffened and stiffened plate subjected to various uniform and non-uniform edge loadings. The cross-section of the stiffened plate is shown in figure 2 for rectangular cross-section.

### III. RESULT AND DISCUSSION

The problem considered here consists of a rectangular plate (a x b) with stiffener subjected to various types of loading.

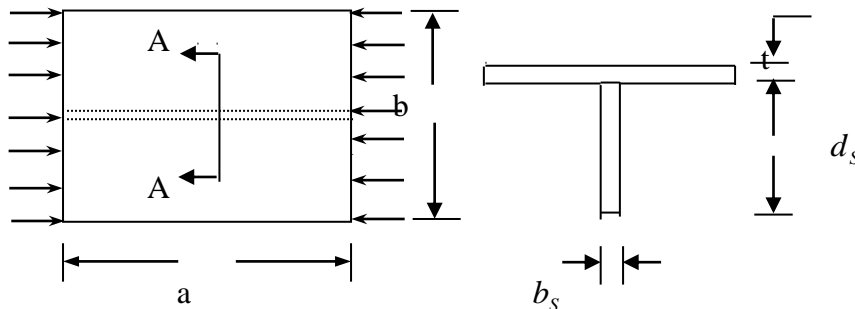


Figure 1 Stiffened plate cross section

#### 3.1 Vibration and buckling studies

The variation of frequency parameter with in-plane load intensity factor of a plate of various aspect ratio and edge condition compressed uniformly on all edges ( $N_x = N_y = N$  &  $N_{xy} = 0$ ) is studied in this section. The fundamental frequency parameter for plate having different boundary conditions (SSSS, CCCC, SCSS) are studied in Table 1. For CCCC plate, there is a steep increase in frequencies with increasing aspect ratios. The natural frequencies are found to increase with decreased magnitude of compressive in-plane forces.

Table 1 Variation of frequency parameter with in-plane load intensity factor of a late of various aspect ratio and edge condition compressed uniformly on all edges ( $N_x = N_y = N$  &  $N_{xy} = 0$ ),  $V = 0.3$ .

a/b	Boundary Condition	Buckling Parameter $\lambda$	Frequency parameter at $P/P_{cr}$				
			0	0.2	0.5	0.8	0.995
1	SSSS	1.99	19.73	17.65	13.95	8.82	0.438
	CCCC	5.30	35.98	32.28	25.65	16.32	0.815
	SCSS	2.66	23.64	21.26	16.76	10.62	0.53
2	SSSS	1.24	12.34	11.03	8.72	5.56	0.270
	CCCC	3.92	24.57	22.06	17.54	11.18	0.558
	SCSS	1.34	12.91	11.55	9.14	5.78	0.286

The variation of buckling and frequency parameters with  $P/P_{cr}$  for rectangular stiffened plate with 1, 2, 3 longitudinal equispaced stiffeners (Stiffener parameters  $\delta = 0.1$  and  $\gamma = 5$ ) subjected to in-plane uniform bi-axial for various aspect ratios ( $a/b = 1, 1.5, 2$ ) and boundary conditions (SSSS, CCCC, SCSS) are studied in detail and shown in table 2 for the sake of getting various interlinking results of number of stiffeners and aspect ratios.

Table 2 Variation of frequency parameter with in-plane load intensity factor of stiffened plate with three equispaced stiffeners having ( $\delta = 0.1$  and  $\gamma = 5$ ) compressed uniformly on all edges.

$$(\sigma_x = \sigma_y = \sigma, \tau_{xy} = 0)$$

a/b	Boundary Condition	$\lambda$	Frequency parameter At Load intensity factor $P/P_{cr}$						
			-2	-1	-0.5	0	0.25	0.5	0.8
1	SSSS	8.27	66.59	52.21	48.54	40.78	36.28	31.13	23.52
	CCCC	18.96	--	---	99.40	88.85	82.80	75.88	54.01
	SCSS	13.24	92.66	78.63	70.51	61.27	56.05	50.27	32.53
2	SSSS	2.08	24.42	19.78	17.13	13.99	12.11	9.89	6.26
	CCCC	7.19	49.84	41.18	35.96	29.67	25.87	21.29	13.69
	SCSS	3.39	31.53	25.77	22.33	18.24	15.80	12.91	8.17

#### IV. CONCLUSION

The stability resistance increases with increase of restraint at the edges for all types of loading, stiffener parameters and plate aspect ratios. The stability resistance increases with increase of number of stiffeners. The variation of buckling load with the position of the concentrated load on the edges is more pronounced for the stiffened plates of the smaller aspect ratios. The buckling load parameter of unstiffened plates simply supported along all the edges increase as the loads are nearer to the support. For plates with small aspect ratios, the boundary condition on the loaded edge has the significant effect on the load required to cause elastic stability.

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