IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Vol. 8, Issue 5, May 2021

DOI: 10.17148/IARJSET.2021.8534

Vibration Characteristic of unstiffened and stiffened skew plates under In-plane loadings

A.K.L.Srivastava[1]

Professor, Department of Civil Engineering, N.I.T. Jamshedpur, India[1]

Abstract: The vibration behaviour of unstiffened and stiffened skew plate subjected to uni-axial and bi-axial in-plane loadings is studied. As the applied in-plane loading is not uniform, the resultant plate in-plane stresses are evaluated from the plate membrane analysis. Using these stress distributions, partial differential equations governing the stability of skew plates are derived. The first order shear deformation (FSDT) theory is considered in the present formulation. Effects of skew angles, boundary conditions, number of stiffeners and uniaxial and biaxial types of loadings on the buckled vibration behaviour of the unstiffened and stiffened skew plate are investigated.

Keywords: Vibration, unstiffened and stiffened skew plate, Uniaxial loading, inplane loading

I. INTRODUCTION

Skew plates are important structural element in modern engineering structures such as aircrafts, space vehicles, missiles and many complex structures. Quite often, these structures are subjected to partial edge loading, concentrated loading. To understand the performance of these structural systems, study of vibration characteristics of the structural element in the prebuckling of the skew plates due to inplane uniaxial, biaxial, partial edge loading, concentrated loadings is important. Free vibration behaviour forms a significant part of the analysis of the static and dynamic behaviour of plates. Fundamental frequency of transverse vibration of a rectangular anisotropic plate of a discontinuously varying thickness has been studied by Guitierrez and Laura [1]. Buckling of plate girder webs under partial edge loading was studied by Rockery and Baghichi [2]. There exist 21 possible combinations of simple boundary conditions for rectangular plates as stated by Leissa [3]. Results were found in the literature for all 21 cases for isotropic plates not having in-plane forces. Published results exist for very few cases where in-plane forces are present. This lack of results is even more serious in the case of vibrating plates subjected to in-plane shear loading. Exception to this statement is the excellent papers by Massonet [4] and Dickinson and his co-workers [5-7]. Mssonet considered the case of simply supported rectangular plate subjected to in-plane shear loading, while Dickinson has obtained very accurate frequency equations for different combination of boundary conditions and has even tackled the rather formidable problem of non-uniform in-plane shearing forces. Vibration of rectangular plates subjected to in-plane forces by the finite strip method has been studied by Chan and Foo [8].

A good deal of references is also available in the literature on stability and vibration of stiffened plates. The bending, stability and vibration of stiffened plates are well documented in the book by Troitsky [9]. Aksu [10] has presented a variational principle in conjunction with the finite difference method for the free vibration analysis of uni-directionally and crossed stiffened plates. Shastry and Rao [11] have analyzed the free vibration of stiffened plate. Mukherjee and Mukhopadhayay [12] have investigated the free vibration of eccentrically stiffened plates. Bapu Rao *et al.* [13] have also reported their work on experimental determination of frequencies with real time holographic technique for skew stiffened cantilever plates. The large amplitude free flexural vibration of thin, elastic orthotropic stiffened plate is studied by Pratap and Vardan [14]. The boundary condition considered is either immovable or movable. The solution is obtained on the basis of a single term vibration mode shape by making use of Galerkin method. Mizusawa *et al.* [15, 16] have studied the effect of the arrangement of stiffening beams, skew angles and stiffness parameters on the vibration characteristics of the skew stiffened plates by using the Rayleigh-Ritz method with B-spline functions as the coordinate functions.

II. PROPOSED ANALYSIS

The governing equations for the buckling, vibration of unstiffened and stiffened skew plates subjected to in-plane harmonic edge loading are developed. The presence of non-uniform external in-plane loads, boundary conditions, stiffeners with and without cutouts in the plate induce a non-uniform stress field in the structures. This necessitates the determination of the stress field as a prerequisite to the solution of the problems like vibration, buckling and dynamic stability behaviour of stiffened plates. As the thickness of the structure is relatively smaller, the determination of stress field reduces to the solution of a plane stress problem in the plate skin and stiffeners (where the thickness and breath are small compared to length). The stiffened plates are modeled and the governing equations are solved by finite element method.

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Vol. 8, Issue 5, May 2021

DOI: 10.17148/IARJSET.2021.8534

The governing equations for specified problems like vibration, static and dynamic stability are as:.

Vibration with in-plane load:

$$[M]\{\ddot{q}\}+[[K_b]-P[K_G]]\{q\}=\{0\} \quad (1)$$

Or

$$[[K_b]-\alpha P_{cr}[K_G]-\omega^2[M]]\{q\}=0 \quad (2)$$

Finite Element Formulation

In the present analysis, the plate is modelled with nine noded isoparametric quadratic elements where the contributions of bending and membrane actions are taken into account. One of the advantages of the element is that it includes the effect of shear deformation and rotary inertia in its formulation. The formulation has been generalized in such a manner that the stiffener can be placed anywhere within the plate element and need not be situated on the nodal lines as in the approach given in Mukherjee and Mukhopadhyay [12]. This provides considerable flexibility in the mesh generation.

According to the Reissener- Mindlin plate theory, the displacements of the plate can be fully described by the components of the vector

$$U=\{u \quad v \quad w \quad \theta_x \quad \theta_y\}^T \quad (3)$$

where u and v are the displacements in the plane of the plate, w is the out of plane deflection, θ_x and θ_y are the rotations of the normal to the un deformed mid surface in the x-z and y-z plane, respectively.

where the strain in the middle plane of the plate are

$$\varepsilon_o=\begin{Bmatrix}\varepsilon_x^o\\ \varepsilon_y^o\\ \nu_{xy}^o\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u}{\partial x}\\ \dfrac{\partial v}{\partial y}\\ \dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\end{Bmatrix}=\begin{bmatrix}\dfrac{\partial}{\partial x} & 0 & 0 & 0 & 0\\ 0 & \dfrac{\partial}{\partial y} & 0 & 0 & 0\\ \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} & 0 & 0 & 0\end{bmatrix} \quad (4)$$

Numerical methods like finite element method (FEM) are preferred. for problems involving complex in plane loading and boundary conditions as analytical methods are not easily adaptable. The formulation is based on Mindlin's plate theory, which will allow for the incorporation of shear deformation. The plate skin and the stiffeners/composite are modelled as separate elements but the compatibility between them is maintained.

The nine noded isoparametric quadratic elements with five degrees of freedom (u, v, w, θ_x and θ_y) per node have been employed in the present analysis.

The coordinates at a point within the element are approximated in terms of its nodal co-ordinates as follows:

$$x=\sum_{r=1}^{9} N_r x_r \quad \text{and} \quad y=\sum_{r=1}^{9} N_r y_r, \quad (5)$$

$$u=\sum_{r=1}^{9} N_r u_r, \qquad v=\sum_{r=1}^{9} N_r v_r, \qquad w=\sum_{r=1}^{9} N_r w_r$$

$$\theta_x=\sum_{r=1}^{9} N_r \theta_{xr} \quad \text{and} \quad \theta_y=\sum_{r=1}^{9} N_r \theta_{yr} \quad (6)$$

To account for the higher shear deformations, a higher order displacement field is employed. The displacement model is derived out of a power series expansion of the mid surface displacements in the power of the thickness co-ordinate as:

Using the isoparametric coordinates, the element stiffness matrix is expressed as

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Vol. 8, Issue 5, May 2021

DOI: 10.17148/IARJSET.2021.8534

$$[K_b]_p = \int_{-1}^{+1}\int_{-1}^{+1} [B_p]^T [D_p] [B_p] \, |J_p| \, d\xi \, d\eta \tag{7}$$

The element mass matrix can be expressed in iso parametric coordinate as

$$[M_p]_e = \int_{-1}^{1}\int_{-1}^{1} [N]^T [m_p] [N] \, |J_p| \, d\xi \, d\eta \tag{8}$$

The geometric Stiffness matrix can be derived.

$$[K_G]_p = \iiint [B_{GP}]^T [\sigma_p] [B_{GP}] dx\,dy\,dz \tag{9}$$

When expressed in isoparametric coordinates the geometric stiffness matrix can be expressed as:

$$[K_G]_P = \int_{-1}^{+1}\int_{-1}^{+1} [B_{Gp}]^T [\sigma_P] [B_{Gp}] \, |J_p| \, d\xi \, d\eta \tag{10}$$

The geometric stiffness of the stiffener element can be expressed as:

$$[K_G]_S = \int_{-1}^{+1} [B_{GS}]^T [\sigma_S] [B_{GS}] \, |J_S| \, d\xi \tag{11}$$

be similar with appropriate changes for the co-ordinate variables.

The equivalent nodal forces are given by

$$\{F_e\} = \iint [N]^T P_o(t) \, |J| \, d\xi \, d\eta \tag{12}$$

The intensity of loading within the patch is assumed to be uniform. In such situations it becomes necessary to obtain the equivalent nodal forces when a concentrated load is acting within the element. Again, the equivalent nodal forces are expressed as:

$$P_r = P_o [N]^T |J| \tag{13}$$

The equations are solved using the technique proposed by Corr and Jennings [ ] where the matrices [*K*], [*M*] and [*KG*] are stored in single array according to skyline storage algorithm. In all the cases, the stiffness matrix [*K*] is factorized according to Cholesky's decomposition technique. With this, the solution for displacement is simply obtained by its forward elimination and backward substitution techniques. These displacements components are used to find out the stress field. These stresses are used to calculate the geometric stiffness matrices. The solution of equations go through a number of operations. Moreover it requires a number of iterations to get the solution since these equations come under the category of eigenvalue problem. In such cases, the solution of eigen vector and eigen value is more than one where the different solutions correspond to different modes of vibration or different modes of buckling. The mode which gives lowest value of the eigen value is quite important and it is known as fundamental mode.

III.RESULTS AND DISCUSSIONS

A finite element code has been written in FORTRAN 90 considering possible boundary conditions and loading cases. The code is capable of analysing the following

The stiffened plate structures in practice are seldom subjected to uniform loading at the edges. Cases of practical interest arise when the in-plane stresses are caused by patch, triangular, point or any arbitrary forces acting along the boundaries. The non-uniform loading and boundary conditions cause non-uniform in-plane stress distribution within the structure. Non-uniform stress distribution may also be caused due to material and geometrical discontinuities in the structures.

Majority of the model parameters and results are presented in non-dimensional form to make them independent of the plate size, thickness, material properties, etc for the convenience of the analysis. The non-dimensionalisation of different parameters like vibration, buckling and excitation frequency for dynamic stability analysis is taken as given below:

- Frequencies of vibration (ω) $\varpi b^2 \sqrt{\rho t / D}$

IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
Vol. 8, Issue 5, May 2021
DOI: 10.17148/IARJSET.2021.8534

- Frequencies of excitation (Ω) $\quad \bar{\Omega}\, b^2 \sqrt{\rho\, t/D}$

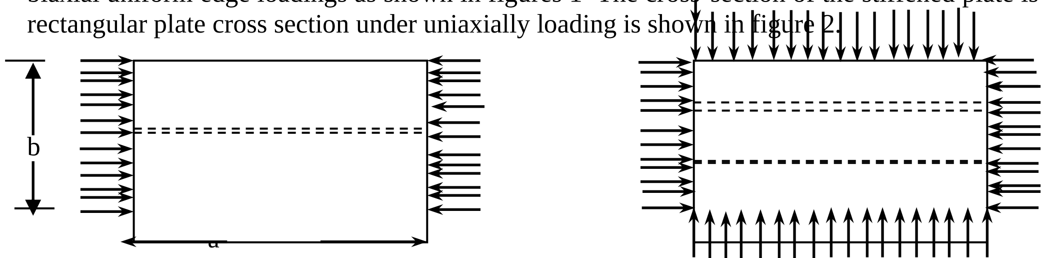
Where D is the plate flexural rigidity, D = $E t^3 / 12(1-\nu^2)$, P is the applied load, P_{cr} is the buckling load, ρ is the density of the plate material and t is the plate thickness. In addition, certain quantities are expressed as the ratio of that quantity to some reference quantity.

The dimensionless buckling load coeffeicient $(k = N_{cr} b^2 / \pi^2 D)$ and fundamental frequency $\omega_0 = b^2 / \pi^2 \sqrt{\rho h / D}$) of simply supported isotropic skew plate.

Assuming a general case of several longitudinal ribs and denoting

- EI $_S$ the flexural rigidity of a stiffener at a distance (D_x) from the edge y = 0, the stiffener parameter terms
- δ and γ are defined as: $\delta = A_S / bt$ = Ratio of cross-sectional area of the stiffener to the plate, where A_S is the area of the stiffener.
- $\gamma = E I_S / bD$ = Ratio of bending stiffness rigidity of stiffener to the plate, where I_S is the moment of inertia of the stiffener cross-section about reference axis.

The basic configuration of the problem considered here is a unstiffened and stiffened plate subjected to uniaxial and biaxial uniform edge loadings as shown in figures 1 The cross-section of the stiffened plate is shown in figure 1. Skew rectangular plate cross section under uniaxially loading is shown in figure 2.



Uni-axial loading (b) Bi-axial loading

Figure 1 Plate subjected to inplane uniform edge loading

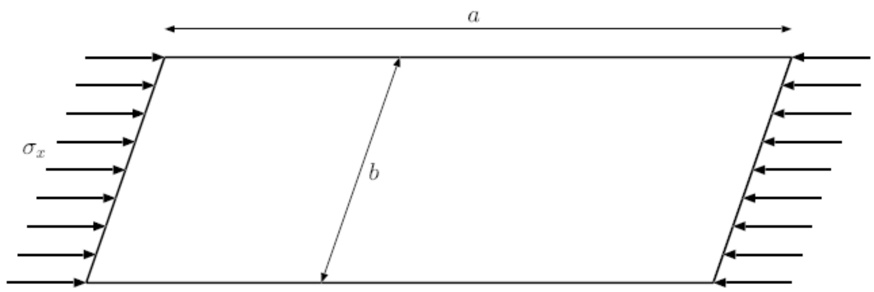


Figure 2: Skew rectangular plate under uniaxially loading

Free Vibration Analysis of stiffened plates

Validation studies of concentrically stiffened clamped square plate

A square plate clamped in all edges having a centrally placed concentric stiffener as presented by Nair & Rao [17] using a package stift1, Mukharjee [12] , Mukhopadhyay [18], and Seikh [19] using FEM , semi analytical method, and spline finite strip method respectively has been analyzed presently in table 1. Seikh [19] has given results neglecting and including mass moment of inertia which has been validated in present results marked as Present (1) for M.I. Neglecting and as mass moment of inertia including. The first six frequencies are compared. The agreement is excellent. In Mukhopadhyay [18] in-plane displacement is not considered in the analysis so results causes slightly varying. Table 1 also present convergence study showing good convergence of results.

Plate size = 600mm x 600 mm $\qquad$ Plate thickness = 1.0 mm,

Poisson's ratio = 0.34 $\qquad$ Mass density = 2.78e-6 Kg / mm^3

E = 6.87 X10^{-7} N / mm^2 , $\qquad$ As = 67.0 mm ^{2} , Is = 2290 mm ^{4}

Js = 22.33 mm ^{4}

Table 1 Frequency in (rad / s) of clamped stiffened plate with a concentric Stiffener

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Vol. 8, Issue 5, May 2021

DOI: 10.17148/IARJSET.2021.8534

Source	Mode No	1	2	3	4	5	6
Present	6 x 6	318.62	404.53	474.30	541.56	727.81	771.47
	8 x 8	317.36	401.66	472.34	538.26	719.23	763.23
	10 x 10	317.00	400.84	471.74	537.32	716.35	759.54
Present	10 x 10	317.00	400.84	471.74	537.32	716.35	759.54
Nair and Rao [17]		317.54	400.12	472.23	537.14	714.14	760.17
Mukharjee [12]		322.34	412.23	506.87	599.34	772.15	860.93
Mukhopadhyay [18]		305.12	382.34	454.76	519.17	696.18	741.15
(1) Seikh [19]		316.85	400.35	471.69	536.95	716.04	759.14
(2) Seikh [19]		316.85	400.35	471.68	536.94	716.02	759.12

The validation studies of buckling Parameter for simply supported and clamped unstiffened skew plates has been presented in table 2 in the form of buckling load parameter. The results compare well with those of Mizusawa *et. al.*[7]. The buckling load based on the proposed approach has, however, indicated higher values with the increase of skew angle, which suggest that the mesh divisions need to be increased with the increase of skew angle for more accurate results

Table 2 Buckling Parameter for simply supported and clamped unstiffened skew plates.

Boundary Conditions	Skew Angle	Mizusawa *et.al* [7]	Present
Simply Supported	0	4	4
	15	5.90	6.17
	30	10.08	10.20
Clamped	0	10.07	10.08
	15	--	10.94
	30	13.53	13.94
	45	20.05	21.79

Skew stiffened plate

After getting the values for unstiffened skew parallelogrammic plates, the analysis is done for parallelogrammic skew stiffened plates with skew angles from 0^0 to 45^0. The parameter for stiffened plates (δ = 0.1, γ = 10 and GJ_s/Db = 0.0). The all edges simply supported and also clamped plates have been considered and results presented in table 3. The plate is subjected to uniform compression in x direction.

Table 3 Buckling Parameter for SSSS and CCCC skew stiffened plates.

Boundary Conditions	Skew Angle	Present
SSSS	0	16.00
	30	20.90
	45	29.90
CCCC	0	30.8
	30	37.1
	45	56.3

If done with various mesh sizes, it is observed that the results of all edges simply supported plate converged better than those having all edges clamped. This is due to the fact that the rotation along the clamped edges has to be released to obtain consistent results as this would induce the additional complexity of introducing the transformation of displacements along the skew axis.

After this study, the effect of variation of the bending stiffness of the stiffeners is studied for skew stiffened plates. The stiffeners are placed at the centre and properties taken as: (δ = 0.1, and GJ_s/Db = 0.0). The ratio of γ is varied. The aspect ratio of plate is 1. The result is presented in table 5. Now the effect of varying torsional stiffness of the stiffeners on a centrally stiffened skew plate is studied. The stiffener properties are as: (δ = 0.1, γ = 10) and GJ_s/Db varying from 0 to 5.0 and the result is presented in table 4. In the formulation of skew stiffened plates, the mesh division is independent of the location of the stiffeners. Hence for some mesh sizes, the stiffeners followed the element boundary, while in some other dimensions it remains away from the element boundary. Convergence study has also been done. It

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology
Vol. 8, Issue 5, May 2021
DOI: 10.17148/IARJSET.2021.8534

is observed that the values converges to the correct values regardless of the location of the stiffeners within the element. It is also observed that the results of the rectangular plates converge earlier than those of skew plates.

Table 4 Buckling Parameter for SSSS skew stiffened plates with varying γ .

Skew Angle	γ	Present
0	5	10.93
	10	16
45	5	20.60
	10	29.89

Table 5 Buckling Parameter for SSSS skew stiffened plates with varying torsional stiffness of the stiffener. Poisson's ratio = 0.3

Skew Angle	GJ_s/Db	Present
0	0	16.00
	2.5	17.15
	5	17.15
45	0	29.89
	2.5	40.40
	5	45.49

IV. CONCLUSION

An isoparametric quadratic stiffened plate bending element has been introduced. The element has been successfully employed for the solution of vibration and stability problems of concentric and eccentric stiffened plates with/without cutout. The element has performed extremely well in free vibration analysis of concentric as well as eccentric stiffened plates for all shapes-rectangular and skew. The isoparametric Mindlin type element has the desirable effect of shear locking. The present formulation accounts for the eccentricities of the middle plane of the plate element and the centroidal axes of the stiffeners with respect to neutral surfaces. The performance of eccentrically stiffened plate has been found to be excellent. The comparison with the results of previous investigators has revealed a good agreement amongst them.

The stability resistance increases with increase of restraint at the edges for all types of loading, stiffener parameter and plate aspect ratios. The stability resistance increases with increase of number of stiffeners.

The fundamental frequency of the skew plate decreases in the pre-buckled regime and then increases in the post-buckled regime with the increase of in-plane loadings for different skew angles, support conditions, number of stiffeners and load ratios. The fundamental frequency of the skew plate increases with the increase of skew angle. The frequency of the skew plate is same for all the types of loadings when the plate is unloaded. The frequency of skew plate increases with the increase of the edge restraints and it is higher for all the edges clamped and lowers for all the edges simply supported. The frequency of the skew plate is lower for higher load ratio as well as decreases with higher rate than the lower load ratio.

REFERENCES

1. Guitierrez, R.H., Laura, P.A.A. Fundamental frequency of transverse vibration of a rectangular anisotropic plate of discontinuously varying thickness. *Journal of Sound and Vibration.* 248 (3), 573-577, 2001.
2. Rockery, K. C. and Bagchi, D. K., Buckling of plate girder webs under partial edge loading, *International Journal of Mechanical Science*. 12 (1), 61-81, 1970
3. Leissa, A.W. 1969, NASA SP-160. Vibration of plates.
4. Massonet, C, The relation between the normal modes of vibrating plates and stability of elastic systems, *Bulletin Cours laboratories Essais Construction Giene Civile*, 1940
5. Dickinson, S. M. Lateral vibration of rectangular plates subjected to in-plane forces, *Journal of Sound and Vibration* 16, 465-472, 1976.
6. Bassily, S.F and Dickinson, S.M. Buckling and lateral vibration of rectangular plates subjected to in-plane loads- a Ritz approach, *Journal of Sound and Vibration* 24, 219-239, 1972.
7. Bassily, S.F and Dickinson, S.M, Vibration of rectangular plates subjected to arbitrary in-plane loads- a perturbation approach. *Journal of Applied Mechanics* 40, 1023-1028, 1973

IARJSET

ISSN (Online) 2393-8021
ISSN (Print) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Vol. 8, Issue 5, May 2021

DOI: 10.17148/IARJSET.2021.8534

8. Chan H.C and Foo, O. Vibration of rectangular plates subjected to in-plane forces by the finite strip method, *Journal of Sound and Vibration*, 64(4), 583-588, 1979.
9. Troitsky, M.S., *Stiffened Plates: Bending, stability and vibrations*, Elsevier scientific publishing company, Oxford, New York., 1976
10. Aksu, G. , Free vibration of stiffened plates including the effect of in plane inertia, *Journal of Applied Mechanics, Trans of ASME*, 49, 206-212, 1982.
11. Shastry, B.P. and Rao, G.V., Vibration of thin rectangular plates with arbitrary oriented stiffeners, *Computers and Structures*, 7, 627-635, 1977.
12. Mukherjee, A. and Mukhopadhyay, M., Finite element free vibration of eccentrically stiffened plates, *Computers and Structures*, 30 (6), 1303-1317, 1987.
13. Bapu Rao, M. N., Guruswami, P., Venketeswara Rao, M. and Pavitran, S. Studies on vibration of some rib-stiffened cantilever plates. *Journal of Sound and Vibration*, 57 (3), 389-402, 1978.
14. Pratap, G. and Vardan, T.K., Large amplitude flexural vibration of stiffened plates. *Journal of Sound and Vibration*, 57 (4), 583-593, 1978.
15. Mizusawa, T., Kajita, T. and Naruoka, M. Vibration of stiffened skew plates using B-spline functions. *Computer and Structures*, 10, 821-826, 1979.
16. Mizusawa, T. and Kajita, T., Vibration and buckling of skew plates with edges elastically restraints against rotation. *Computer and Structures*, 22, 987-994, 1986.
17. Nair, P. S. and Rao, M. S., On vibration of plates with varying stiffener length. *Journal of Sound and Vibration*, 95(1), 19-29, 1984.
18. Mukherjee, A. and Mukhopadhyay, M., Finite element free flexural vibration analysis of plates having various shapes and varying rigidities, *Computer and Struct*., 23, 807-812, 1986.
19. Sheikh, A.H. and Mukhopadhyay, M., Free vibration analysis of stiffened plates with arbitrary planform the general spline finite strip method, *Journal of Sound and Vibration*, 162 (1), 147-164, 1993.