

# Intuitionistic Fuzzy $\pi g\gamma^*$ Contra Continuous Mappings

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**Abstract:** This paper is devoted to the study of intuitionistic fuzzy topological spaces. In this paper, we have introduced intuitionistic fuzzy  $\pi g\gamma^*$  contra continuous mappings. Also, we have studied some of their properties of intuitionistic fuzzy  $\pi g\gamma^*$  contra continuous mappings in intuitionistic fuzzy topological spaces. Identified few examples for intuitionistic fuzzy  $\pi g\gamma^*$  contra continuous mappings. Studied the cases of IF contra continuous mapping, IFS contra continuous mapping, IFP contra continuous mapping, IF $\alpha$  contra continuous mapping, IF $\gamma$  contra continuous mapping, IFG contra continuous mapping and IF $\gamma^*$ G contra continuous mapping. Derived some properties of  $cT_{1/2}$  space. Also studied the case in  $gT_{1/2}$  space. Have done some analysis on the composition of two IF $\pi g\gamma^*$  contra continuous mappings.

**Keywords:** Intuitionistic fuzzy topology, intuitionistic fuzzy  $\pi g\gamma^*$  closed set, intuitionistic fuzzy  $\pi g\gamma^*$  continuous mapping and intuitionistic fuzzy  $\pi g\gamma^*$  contra continuous mappings.

## I. INTRODUCTION

The concept of intuitionistic fuzzy sets was introduced by Atanassov[1] using the notion of fuzzy sets. On the other hand Coker[2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy  $\pi g\gamma^*$  contracontinuous mappings and studied some of their basic properties. We provide some relations of intuitionistic fuzzy  $\pi g\gamma^*$  contracontinuous mappings between existing intuitionistic fuzzy continuous and irresolute mappings.

## II. PRELIMINARIES

**Definition 2.1:** [1] An intuitionistic fuzzy set (IFS in short)  $G$  in  $X$  is an object having the form

$$G = \{ \langle x, \mu_G(x), \nu_G(x) \rangle / x \in X \}$$

where the functions  $\mu_G(x): X \rightarrow [0, 1]$  and  $\nu_G(x): X \rightarrow [0, 1]$  denote the degree of membership (namely  $\mu_G(x)$ ) and the degree of non-membership (namely  $\nu_G(x)$ ) of each element  $x \in X$  to the set  $G$ , respectively, and  $0 \leq \mu_G(x) + \nu_G(x) \leq 1$  for each  $x \in X$ . Denote by  $IFS(X)$ , the set of all intuitionistic fuzzy sets in  $X$ .

**Definition 2.2:** [1] Suppose  $G_1$  and  $G_2$  are IFSs of the form

$$G_1 = \{ \langle x, \mu_{G_1}(x), \nu_{G_1}(x) \rangle / x \in X \} \text{ and } G_2 = \{ \langle x, \mu_{G_2}(x), \nu_{G_2}(x) \rangle / x \in X \}.$$

Then

- $G_1 \subseteq G_2$  if and only if  $\mu_{G_1}(x) \leq \mu_{G_2}(x)$  and  $\nu_{G_1}(x) \geq \nu_{G_2}(x)$  for all  $x \in X$
- $G_1 = G_2$  if and only if  $G_1 \subseteq G_2$  and  $G_2 \subseteq G_1$
- $G_1^c = \{ \langle x, \nu_{G_1}(x), \mu_{G_1}(x) \rangle / x \in X \}$
- $G_1 \cap G_2 = \{ \langle x, \mu_{G_1}(x) \wedge \mu_{G_2}(x), \nu_{G_1}(x) \vee \nu_{G_2}(x) \rangle / x \in X \}$
- $G_1 \cup G_2 = \{ \langle x, \mu_{G_1}(x) \vee \mu_{G_2}(x), \nu_{G_1}(x) \wedge \nu_{G_2}(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation  $G_1 = \langle x, \mu_{G_1}, \nu_{G_1} \rangle$  instead of  $G_1 = \{ \langle x, \mu_{G_1}(x), \nu_{G_1}(x) \rangle / x \in X \}$ . Also for the sake of simplicity, we shall use the notation  $G_1 = \{ \langle x, (\mu_{G_1}, \mu_{G_2}), (\nu_{G_1}, \nu_{G_2}) \rangle \}$  instead of  $G_1 = \langle x, (G_1/\mu_{G_1}, B/\mu_{G_2}), (G_1/\nu_{G_1}, B/\nu_{G_2}) \rangle$ .

The intuitionistic fuzzy sets  $0_- = \{ \langle x, 0, 1 \rangle / x \in X \}$  and  $1_- = \{ \langle x, 1, 0 \rangle / x \in X \}$  are respectively the empty set and the whole set of  $X$ .

**Definition 2.3:** [4] An intuitionistic fuzzy topology (IFT in short) on  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms.

- (i)  $0, 1 \in \tau$
- (ii)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$
- (iii)  $\cup G_i \in \tau$  for any family  $\{ G_i / i \in J \} \subseteq \tau$ .

In this case the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS in short) in  $X$ .

The complement  $G^c$  of an IFOS  $G$  in IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS in short) in  $X$ .

**Definition 2.4:**[4] Suppose  $(X, \tau)$  is an IFTS and  $G = \langle x, \mu_G, \nu_G \rangle$  is an IFS in  $X$ . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int}(G) = \cup \{ G_1 / G_1 \text{ is an IFOS in } X \text{ and } G_1 \subseteq G \},$$

$$\text{cl}(G) = \cap \{ G_1 / G_1 \text{ is an IFCS in } X \text{ and } G \subseteq G_1 \}.$$

Note that for any IFS  $G$  in  $(X, \tau)$ , we have  $\text{cl}(G^c) = [\text{int}(G)]^c$  and  $\text{int}(G^c) = [\text{cl}(G)]^c$ .

**Definition 2.5:**[6] An IFS  $G = \langle x, \mu_G, \nu_G \rangle$  in an IFTS  $(X, \tau)$  is called an

- (i) intuitionistic fuzzy semi open set (IFSOS in short) if  $G \subseteq \text{cl}(\text{int}(G))$ ,
- (ii) intuitionistic fuzzy  $\alpha$ -open set (IF $\alpha$ OS in short) if  $G \subseteq \text{int}(\text{cl}(\text{int}(G)))$ ,
- (iii) intuitionistic fuzzy regular open set (IFROS in short) if  $G = \text{int}(\text{cl}(G))$ ,

**Definition 2.6:** [7] The union of IFROSs is called intuitionistic fuzzy  $\pi$ -open set (IF $\pi$ OS in short) of an IFTS  $(X, \tau)$ . The complement of IF $\pi$ OS is called intuitionistic fuzzy  $\pi$ -closed set (IF $\pi$ CS in short).

**Definition 2.7:**[6] An IFS  $G = \langle x, \mu_G, \nu_G \rangle$  in an IFTS  $(X, \tau)$  is called an

- (i) intuitionistic fuzzy semi closed set (IFSCS in short) if  $\text{int}(\text{cl}(G)) \subseteq G$ ,
- (ii) intuitionistic fuzzy  $\alpha$ -closed set (IF $\alpha$ CS in short) if  $\text{cl}(\text{int}(\text{cl}(G))) \subseteq G$ ,
- (iii) intuitionistic fuzzy regular closed set (IFRCS in short) if  $G = \text{cl}(\text{int}(G))$ .

**Definition 2.8:**[5] An IFS  $G$  of an IFTS  $(X, \tau)$  is an

- (i) intuitionistic fuzzy  $\gamma$ -open set (IF $\gamma$ OS in short) if  $G \subseteq \text{int}(\text{cl}(G)) \cup \text{cl}(\text{int}(G))$ ,
- (ii) intuitionistic fuzzy  $\gamma$ -closed set (IF $\gamma$ CS in short) if  $\text{cl}(\text{int}(G)) \cap \text{int}(\text{cl}(G)) \subseteq G$ .

**Definition 2.9:**[11] Suppose  $G$  is an IFS in an IFTS  $(X, \tau)$ . Then

$$\text{sint}(G) = \cup \{ G_1 / G_1 \text{ is an IFSOS in } X \text{ and } G_1 \subseteq G \},$$

$$\text{scl}(G) = \cap \{ G_1 / G_1 \text{ is an IFSCS in } X \text{ and } G \subseteq G_1 \}.$$

Note that for any IFS  $G$  in  $(X, \tau)$ , we have  $\text{scl}(G^c) = (\text{sint}(G))^c$  and  $\text{sint}(G^c) = (\text{scl}(G))^c$ .

**Definition 2.10:**[10] An IFS  $A$  in an IFTS  $(X, \tau)$  is an

- (i) intuitionistic fuzzy generalized closed set (IFGCS in short) if  $\text{cl}(G) \subseteq U$  whenever  $G \subseteq U$  and  $U$  is an IFOS in  $X$ .
- (ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if  $\text{cl}(G) \subseteq U$  whenever  $G \subseteq U$  and  $U$  is an IFROS in  $X$ .

**Definition 2.11:**[11] An IFS  $G$  in an IFTS  $(X, \tau)$  is called an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if  $\text{scl}(G) \subseteq U$  whenever  $G \subseteq U$  and  $U$  is an IFOS in  $(X, \tau)$ .

**Definition 2.12:**[10] An IFS  $G$  in  $(X, \tau)$  is called an intuitionistic fuzzy  $\pi\gamma^*$  closed set (IF $\pi\gamma^*$ CS in short) if  $\text{cl}(\text{int}(G)) \cap \text{int}(\text{cl}(G)) \subseteq U$  whenever  $G \subseteq U$  and  $U$  is an IF $\pi$ OS in  $(X, \tau)$ . The family of all IF $\pi\gamma^*$ CSs of an IFTS  $(X, \tau)$  is denoted by IF $\pi\gamma^*$ C(X).

**Result 2.13:** [10] Every IFCS, IFGCS, IFRCS, IF $\alpha$ CS is an IF $\pi\gamma^*$ CS but the converses may not be true in general.

**Definition 2.14:**[10] An IFS  $G$  is called an intuitionistic fuzzy  $\pi\gamma^*$  open set (IF $\pi\gamma^*$ OS in short) in  $(X, \tau)$  if the complement  $G^c$  is an IF $\pi\gamma^*$ CS in  $X$ .

The family of all IF $\pi\gamma^*$ OSs of an IFTS  $(X, \tau)$  is denoted by IF $\pi\gamma^*$ O(X).

**Definition 2.15:**[5] Suppose  $f$  is a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is called intuitionistic fuzzy contra continuous (IF contra continuous in short) if  $f^1(G) \in \text{IFCS}(X)$  for every  $G \in \sigma$ .

**Definition 2.16:**[6] Suppose  $f$  is a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is called

- (i) intuitionistic fuzzy semi contra continuous (IFS contra continuous in short) if  $f^{-1}(G) \in \text{IFSCS}(X)$  for every  $G \in \sigma$ .
- (ii) intuitionistic fuzzy  $\alpha$  contra continuous (IF $\alpha$ contra continuous in short) if  $f^{-1}(G) \in \text{IF}\alpha\text{CS}(X)$  for every  $G \in \sigma$ .
- (iii) intuitionistic fuzzy pre contra continuous (IFP contra continuous in short) if  $f^{-1}(G) \in \text{IFPCS}(X)$  for every  $G \in \sigma$ .

**Definition 2.17:**[5] A mapping  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is called an *intuitionistic fuzzy  $\gamma$  contra continuous* (IF $\gamma$ contra continuous in short) if  $f^{-1}(G)$  is an IF $\gamma$ CS in  $(X_1, \tau)$  for every  $G \in \sigma$ .

**Definition 2.18:**[9] Suppose  $f$  is a mapping from an IFTS  $(X_1, \tau)$  into an IFTS  $(X_2, \sigma)$ . Then  $f$  is called an intuitionistic fuzzy generalized contra continuous (IFG contra continuous in short) if  $f^{-1}(G) \in \text{IFGCS}(X)$  for every IFOS  $G$  in  $X_2$ .

**Definition 2.19:**[5] An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $\pi\gamma^*cT_{1/2}$  (in short IF $\pi\gamma^*cT_{1/2}$ ) space if every IF $\pi\gamma^*CS$  in  $X$  is an IFCS in  $X$ .

**Definition 2.20:**[6] An IFTS  $(X, \tau)$  is an intuitionistic fuzzy  $\pi\gamma^*gT_{1/2}$  (IF $\pi\gamma^*gT_{1/2}$ ) space if every IF $\pi\gamma^*CS$  is an IFGCS in  $X$ .

**Definition 2.21:**[7] An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $\pi\gamma^*T_{1/2}$  (in short IF $\pi\gamma^*T_{1/2}$ ) space if every IF $\pi\gamma^*CS$  in  $X$  is an IF $\gamma$ CS in  $X$ .

### III. INTUITIONISTIC FUZZY $\pi\gamma^*$ CONTRA CONTINUOUS MAPPINGS

In this section we introduce intuitionistic fuzzy  $\pi\gamma^*$  contra continuous mappings and studied some of its properties.

**Definition 3.1:** A mapping  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is called an *intuitionistic fuzzy  $\pi\gamma^*$  contra continuous* (IF $\pi\gamma^*$ contra continuous in short) mapping if  $f^{-1}(G)$  is an IF $\pi\gamma^*CS$  in  $(X, \tau)$  for every IFOS  $G$  of  $(X_2, \sigma)$ .

**Example 3.2:** Suppose  $X_1 = \{a, b\}$ ,  $X_2 = \{u, v\}$  and  $T_1 = \langle x_1, (0.3, 0.3), (0.6, 0.6) \rangle$ ,  $T_2 = \langle x_2, (0.3, 0.2), (0.6, 0.7) \rangle$ . Then  $\tau = \{0_-, T_1, 1_-\}$  and  $\sigma = \{0_-, T_2, 1_-\}$  are IFTs on  $X_1$  and  $X_2$  respectively. Define a mapping  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an IF $\pi\gamma^*$ contra continuous mapping.

**Theorem 3.3:** Every IF contra continuous mapping is an IF $\pi\gamma^*$ contra continuous mapping but not conversely.

**Proof:** Suppose  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is an IF contra continuous mapping. Let  $G$  be an IFOS in  $X_2$ . As  $f$  is IF contra continuous mapping,  $f^{-1}(G)$  is an IFCS in  $X_1$ . As every IFCS is an IF $\pi\gamma^*CS$ ,  $f^{-1}(G)$  is IF $\pi\gamma^*CS$  in  $X_1$ . Hence  $f$  is an IF $\pi\gamma^*$ contra continuous mapping.

**Example 3.4:** Suppose  $X_1 = \{a, b\}$ ,  $X_2 = \{u, v\}$  and  $T_1 = \langle x_1, (0.2, 0.2), (0.5, 0.7) \rangle$ ,  $T_2 = \langle x_2, (0.4, 0.2), (0.5, 0.5) \rangle$ . Then  $\tau = \{0_-, T_1, 1_-\}$  and  $\sigma = \{0_-, T_2, 1_-\}$  are IFTs on  $X_1$  and  $X_2$  respectively. Define a mapping  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . The IFS  $B = \langle x_2, (0.4, 0.2), (0.5, 0.5) \rangle$  is IFOS in  $X_2$ . Then  $f^{-1}(B) = \langle x_1, (0.4, 0.2), (0.5, 0.5) \rangle$  is IF $\pi\gamma^*CS$  in  $X_1$  but not IFCS in  $X_1$ . Thus  $f$  is an IF $\pi\gamma^*$  contra continuous mapping but not an IF contra continuous mapping.

**Theorem 3.5:** Every IFS contra continuous mapping is an IF $\pi\gamma^*$  contra continuous mapping but not conversely.

**Proof:** Suppose  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is an IFS contra continuous mapping. Suppose  $B$  be an IFOS in  $X_2$ . Then by hypothesis  $f^{-1}(B)$  is an IFSCS in  $X_1$ . As every IFSCS is an IF $\pi\gamma^*CS$ ,  $f^{-1}(B)$  is an IF $\pi\gamma^*CS$  in  $X_1$ . Hence  $f$  is an IF $\pi\gamma^*$  contra continuous mapping.

**Example 3.6:** Suppose  $X_1 = \{a, b\}$ ,  $X_2 = \{u, v\}$  and  $T_1 = \langle x_1, (0.4, 0.3), (0.5, 0.6) \rangle$  and  $T_2 = \langle x_2, (0.2, 0.2), (0.7, 0.7) \rangle$ . Then  $\tau = \{0_-, T_1, 1_-\}$  and  $\sigma = \{0_-, T_2, 1_-\}$  are IFTs on  $X_1$  and  $X_2$  respectively. Define a mapping  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . The IFS  $B = \langle x_2, (0.2, 0.2), (0.7, 0.7) \rangle$  is IFOS in  $X_2$ . Then  $f^{-1}(B)$  is IF $\pi\gamma^*CS$  in  $X_1$  but not IFSCS in  $X_1$ . Then  $f$  is IF $\pi\gamma^*$ contra continuous mapping but not an IFS contra continuous mapping.

**Theorem 3.7:** Every IFP contra continuous mapping is an IF $\pi\gamma^*$  contra continuous mapping but not conversely.

**Proof:** Suppose  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is an IFP contra continuous mapping. Suppose  $B$  is an IFOS in  $X_2$ . Then by hypothesis  $f^{-1}(B)$  is an IFPCS in  $X_1$ . As every IFPCS is an IF $\pi\gamma^*CS$ ,  $f^{-1}(B)$  is IF $\pi\gamma^*CS$  in  $X_1$ . Hence  $f$  is an IF $\pi\gamma^*$  contra continuous mapping.

**Example 3.8:** Suppose  $X_1 = \{ a, b \}$ ,  $X_2 = \{ u, v \}$  and  $T_1 = \langle x_1, (0.2, 0.3), (0.5, 0.6) \rangle$  and  $T_2 = \langle x_2, (0.3, 0.4), (0.6, 0.6) \rangle$ . Then  $\tau = \{0_-, T_1, 1_-\}$  and  $\sigma = \{0_-, T_2, 1_-\}$  are IFTs on  $X_1$  and  $X_2$  respectively. Define a mapping  $f: (X_1, \tau) \rightarrow (X_2, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . The IFS  $B = \langle x_2, (0.3, 0.4), (0.6, 0.6) \rangle$  is IFOS in  $X_2$ . Then  $f^{-1}(B)$  is  $IF\pi\gamma\gamma^*$ CS in  $X_1$  but not IFPCS in  $X_1$ . Then  $f$  is  $IF\pi\gamma\gamma^*$  contra continuous mapping but not an IFP contra continuous mapping.

**Theorem 3.9:** Every  $IF\alpha$ contra continuous mapping is an  $IF\pi\gamma\gamma^*$ contra continuous mapping but not conversely.

**Proof:** Suppose  $f: (X_1, \tau) \rightarrow (X_2, \sigma)$  is an  $IF\alpha$ contra continuous mapping. Suppose  $B$  is an IFOS in  $X_2$ . Then by hypothesis  $f^{-1}(B)$  is an  $IF\alpha$ CS in  $X_1$ . As every  $IF\alpha$ CS is an  $IF\pi\gamma\gamma^*$ CS,  $f^{-1}(B)$  is an  $IF\pi\gamma\gamma^*$ CS in  $X_1$ . Hence  $f$  is an  $IF\pi\gamma\gamma^*$ contra continuous mapping.

**Example 3.10:** Suppose  $X_1 = \{ a, b \}$ ,  $X_2 = \{ u, v \}$  and  $T_1 = \langle x_1, (0.4, 0.2), (0.6, 0.7) \rangle$ ,  $T_2 = \langle x_1, (0.7, 0.8), (0.2, 0.2) \rangle$  and  $T_3 = \langle x_2, (0.5, 0.6), (0.4, 0.3) \rangle$ . Then  $\tau = \{0_-, T_1, T_2, 1_-\}$  and  $\sigma = \{0_-, T_3, 1_-\}$  are IFTs on  $X_1$  and  $X_2$  respectively. Define a mapping  $f: (X_1, \tau) \rightarrow (X_2, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . The IFS  $B = \langle x_2, (0.5, 0.6), (0.4, 0.3) \rangle$  is IFOS in  $X_2$ . Then  $f^{-1}(B)$  is  $IF\pi\gamma\gamma^*$ CS in  $X_1$  but not  $IF\alpha$ CS in  $X_1$ . Then  $f$  is  $IF\alpha$ G contra continuous mapping but not an  $IF\alpha$ contra continuous mapping.

**Theorem 3.11:** Every  $IF\gamma$ contra continuous mapping is an  $IF\pi\gamma\gamma^*$ contra continuous mapping but not conversely.

**Proof:** Suppose  $f: (X_1, \tau) \rightarrow (X_2, \sigma)$  is an  $IF\gamma$ contra continuous mapping. Suppose  $B$  is an IFOS in  $X_2$ . Then by hypothesis  $f^{-1}(B)$  is an  $IF\gamma$ CS in  $X_1$ . As every  $IF\gamma$ CS is an  $IF\pi\gamma\gamma^*$ CS,  $f^{-1}(B)$  is an  $IF\pi\gamma\gamma^*$ CS in  $X_1$ . Hence  $f$  is an  $IF\pi\gamma\gamma^*$ contra continuous mapping.

**Example 3.12:** Suppose  $X_1 = \{ a, b \}$ ,  $X_2 = \{ u, v \}$  and  $T_1 = \langle x_1, (0.6, 0.6), (0.5, 0.4) \rangle$ ,  $T_2 = \langle x_2, (0.2, 0.3), (0.8, 0.7) \rangle$  and  $T_3 = \langle x_3, (0.6, 0.4), (0.4, 0.6) \rangle$ . Then  $\tau = \{0_-, T_1, T_2, 1_-\}$  and  $\sigma = \{0_-, T_3, 1_-\}$  are IFTs on  $X_1$  and  $X_2$  respectively. Define a mapping  $f: (X_1, \tau) \rightarrow (X_2, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . The IFS  $B = \langle x_2, (0.6, 0.4), (0.4, 0.6) \rangle$  is IFOS in  $X_2$ . Then  $f^{-1}(B)$  is  $IF\pi\gamma\gamma^*$ CS in  $X_1$  but not  $IF\gamma$ CS in  $X_1$ . Then  $f$  is  $IF\pi\gamma\gamma^*$ contra continuous mapping but not an  $IF\gamma$ contra continuous mapping.

**Theorem 3.13:** Every  $IFG$  contra continuous mapping is an  $IF\pi\gamma\gamma^*$  contra continuous mapping but not conversely.

**Proof:** Suppose  $f: (X_1, \tau) \rightarrow (X_2, \sigma)$  is an  $IFG$  contra continuous mapping. Suppose  $B$  is an IFOS in  $X_2$ . Then by hypothesis  $f^{-1}(B)$  is an  $IFG$ CS in  $X_1$ . As every  $IFG$ CS is an  $IF\pi\gamma\gamma^*$ CS,  $f^{-1}(B)$  is an  $IF\pi\gamma\gamma^*$ CS in  $X_1$ . Hence  $f$  is an  $IF\pi\gamma\gamma^*$  contra continuous mapping.

**Example 3.14:** Suppose  $X_1 = \{ a, b \}$ ,  $X_2 = \{ u, v \}$  and  $T_1 = \langle x_1, (0.2, 0.8), (0.2, 0.0) \rangle$  and  $T_2 = \langle x_2, (0.3, 0.8), (0.2, 0.1) \rangle$ . Then  $\tau = \{0_-, T_1, 1_-\}$  and  $\sigma = \{0_-, T_2, 1_-\}$  are IFTs on  $X_1$  and  $X_2$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . The IFS  $B = \langle x_2, (0.3, 0.8), (0.2, 0.1) \rangle$  is IFOS in  $X_2$ . Then  $f^{-1}(B)$  is  $IF\pi\gamma\gamma^*$ CS in  $X_1$  but not  $IFG$ CS in  $X_1$ . Then  $f$  is  $IF\pi\gamma\gamma^*$  contra continuous mapping but not an  $IFG$  contra continuous mapping.

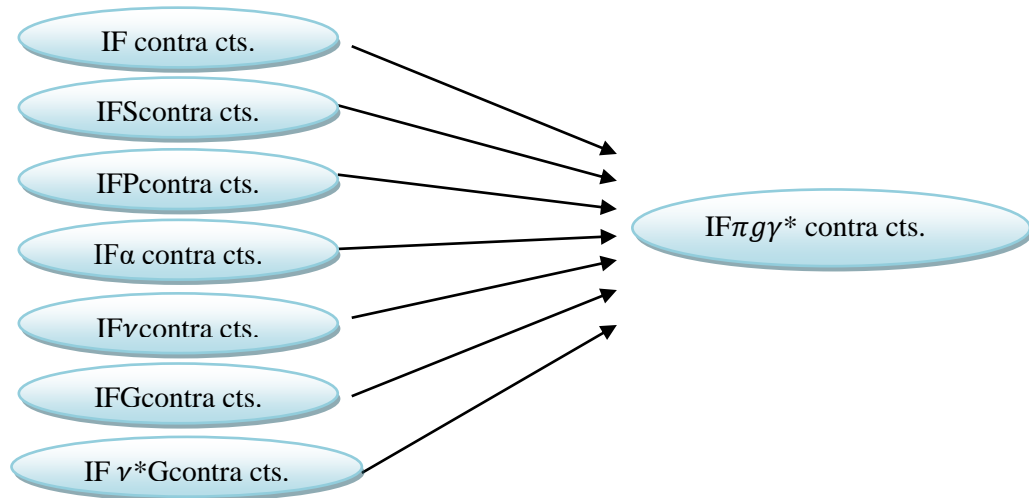
**Theorem 3.15:** Every  $IF\gamma^*G$  contra continuous mapping is an  $IF\pi\gamma\gamma^*$  contra continuous mapping.

**Proof:** Suppose  $f: (X_1, \tau) \rightarrow (X_2, \sigma)$  is an  $IF\gamma^*G$ contra continuous mapping. Suppose  $B$  is an IFOS in  $X_2$ . Then by hypothesis  $f^{-1}(B)$  is an  $IF\gamma^*G$ CS in  $X_1$ . As every  $IF\gamma^*G$ CS is an  $IF\pi\gamma\gamma^*$ CS,  $f^{-1}(B)$  is an  $IF\pi\gamma\gamma^*$ CS in  $X_1$ . Hence  $f$  is an  $IF\pi\gamma\gamma^*$ contra continuous mapping.

**Theorem 3.16:** A mapping  $f: (X_1, \tau) \rightarrow (X_2, \sigma)$  is  $IF\pi\gamma\gamma^*$  contra continuous if and only if the inverse image of each IFCS in  $X_2$  is an  $IF\pi\gamma\gamma^*$ OS in  $X_1$ .

**Proof:** Suppose  $A$  is an IFCS in  $X_2$ . This implies  $A^c$  is IFOS in  $X_2$ . As  $f$  is  $IF\pi\gamma\gamma^*$  contra continuous,  $f^{-1}(A^c)$  is  $IF\pi\gamma\gamma^*$ CS in  $X_1$ . As  $f^{-1}(A^c) = (f^{-1}(A))^c$ ,  $f^{-1}(A)$  is an  $IF\pi\gamma\gamma^*$  OS in  $X_1$ .

The relations between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram ‘cts.’ means continuous.



The reverse implications are not true in general.

**Theorem 3.17:** Suppose  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is an  $IF\pi\gamma^*$  contra continuous mapping, then  $f$  is an  $IF$  contra continuous mapping if  $X_1$  is an  $IF\pi\gamma^*cT_{1/2}$  space.

**Proof:** Suppose  $B$  is an IFOS in  $X_2$ . Then  $f^{-1}(B)$  is an  $IF\pi\gamma^*$ CS in  $X_1$ , by hypothesis. As  $X_1$  is an  $IF\pi\gamma^*cT_{1/2}$  space,  $f^{-1}(B)$  is an IFCS in  $X_1$ . Hence  $f$  is an  $IF$  contra continuous mapping.

**Theorem 3.18:** Suppose  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is an  $IF\pi\gamma^*$  contra continuous mapping then  $f$  is an  $IFG$  contra continuous mapping if  $X_1$  is an  $IF\gamma^*gT_{1/2}$  space.

**Proof:** Suppose  $B$  is an IFOS in  $X_2$ . Then  $f^{-1}(B)$  is  $IF\pi\gamma^*$ CS in  $X_1$ , by hypothesis. As  $X_1$  is an  $IF\gamma^*gT_{1/2}$  space,  $f^{-1}(B)$  is an  $IFGCS$  in  $X_1$ . Hence  $f$  is an  $IFG$  contra continuous mapping.

**Theorem 3.19:** Suppose  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is an  $IF\pi\gamma^*$  contra continuous mapping and  $g : (X_2, \sigma) \rightarrow (X_3, \delta)$  is  $IF$  continuous, then  $g \circ f : (X_1, \tau) \rightarrow (X_3, \delta)$  is an  $IF\pi\gamma^*$  contra continuous mapping.

**Proof:** Suppose  $B$  is an IFOS in  $Z$ . Then  $g^{-1}(B)$  is an IFOS in  $X_2$ , by hypothesis. As  $f$  is an  $IF\pi\gamma^*$  contra continuous mapping,  $f^{-1}(g^{-1}(B))$  is an  $IF\pi\gamma^*$ CS in  $X_1$ . Hence  $g \circ f$  is an  $IF\pi\gamma^*$  contra continuous mapping.

**Theorem 3.20:** Suppose  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is an  $IF\pi\gamma^*$  continuous mapping and  $g : (X_2, \sigma) \rightarrow (X_3, \delta)$  is  $IF$  contra continuous, then  $g \circ f : (X_1, \tau) \rightarrow (X_3, \delta)$  is an  $IF\pi\gamma^*$  contra continuous mapping.

**Proof:** Suppose  $B$  is an IFOS in  $X_3$ . Then  $g^{-1}(B)$  is an IFCS in  $X_2$ , by hypothesis. As  $f$  is an  $IF\pi\gamma^*$  continuous mapping,  $f^{-1}(g^{-1}(B))$  is an  $IF\pi\gamma^*$ CS in  $X_1$ . Hence  $g \circ f$  is an  $IF\pi\gamma^*$  contra continuous mapping.

**Theorem 3.21:** Suppose  $f : (X_1, \tau) \rightarrow (X_2, \sigma)$  is a mapping from an  $IFTS X$  into an  $IFTS X_2$ . Then the following conditions are equivalent if  $X$  is an  $IF\pi\gamma^*cT_{1/2}$  space.

- (i)  $f$  is an  $IF\pi\gamma^*$  contra continuous mapping
- (ii) If  $B$  is an IFOS in  $X_2$  then  $f^{-1}(B)$  is an  $IF\pi\gamma^*$ CS in  $X_1$
- (iii)  $f^{-1}(int B) \supseteq cl(int(cl(f^{-1}(B))))$  for every IFS  $B$  in  $X_2$ .

**Proof:** (i)  $\Rightarrow$  (ii): is obviously true.

(ii)  $\Rightarrow$  (iii): Suppose  $B$  is any IFS in  $X_2$ . Then  $int(B)$  is an IFOS in  $X_2$ . Then  $f^{-1}(int(B))$  is an  $IF\pi\gamma^*$ CS in  $X_1$ . As  $X_1$  is an  $IF\pi\gamma^*cT_{1/2}$  space,  $f^{-1}(int(B))$  is an IFCS in  $X_1$ . Thus  $f^{-1}(int(B)) = cl(f^{-1}(int(B))) \supseteq cl(int(cl(f^{-1}(B))))$ .

(iii)  $\Rightarrow$  (i): Suppose  $B$  is an IFCS in  $X_2$ . Then its complement  $B^c$  is an IFOS in  $X_2$ . By hypothesis  $f^{-1}(int(B^c)) \supseteq cl(int(cl(f^{-1}(B^c))))$ . This implies  $f^{-1}(B^c) \supseteq cl(int(cl(f^{-1}(B^c))))$ . Hence  $f^{-1}(B^c)$  is an  $IF\alpha$ CS in  $X_1$ . As every  $IF\alpha$ CS is an  $IF\pi\gamma^*$ CS,  $f^{-1}(B^c)$  is an  $IF\pi\gamma^*$ CS in  $X_1$ . Thus  $f^{-1}(B)$  is an  $IF\pi\gamma^*$ CS in  $X_1$ . Hence  $f$  is an  $IF\pi\gamma^*$  contra continuous mapping.

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