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# Job Sequencing in Triangular Intuitionistic Fuzzy Environment

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**Abstract:** In this paper sequencing problem is solved with triangular intuitionistic fuzzy numbers. Processing times taken as intuitionistic fuzzy numbers are defuzzified with defined ranking and optimal sequence is obtained. In the last total elapsed time and idle time for each machine is also calculated. Numerical example is given to clear the concept.

**Keywords:** Intuitionistic fuzzy number, Triangular intuitionistic fuzzy number, optimal sequence, ranking of triangular intuitionistic fuzzy number.

## I. INTRODUCTION

In job sequencing problems sequence of jobs is planned to complete the given task in minimum possible time. Sequencing problem consists of processing times of different jobs under different machines. While measuring processing time some errors are committed and to resolve this problem in 1965 Zadeh gave the concept of fuzzy theory. After some time Atanassov [1] introduced intuitionistic fuzzy set which has some advantages over fuzzy sets. By using this concept Li et al.[5] developed value and ambiguity based ranking method for triangular intuitionistic fuzzy numbers and solved multiattribute decision making problems. Mahapatra and Roy [8] gave arithmetic operations for triangular and trapezoidal intuitionistic fuzzy number. Kumar and Kaur [4] proposed new ranking method after studying the limitations of existing method. Shabani and Jamkhaneh[13] worked with generalized intuitionistic fuzzy number and defined their properties. Roseline and Amirtharaj[11] solved assignment problem with the help of trapezoidal intuitionistic fuzzy number by using ranking method based on magnitude of membership and non-membership function. Zhang et al.[15] used normal intuitionistic fuzzy numbers in multi-criteria group decision making. Jia and Zhang[3] solved fuzzy decision making problems with Interval-valued intuitionistic fuzzy numbers. Yogashanthi et al.[14] proposed centroid based ranking for generalized intuitionistic fuzzy numbers and used result for flow shop scheduling problem. After this a comparison was done by Santhi and Selvakumari[12] between interval-valued fuzzy and interval-valued intuitionistic fuzzy by applying them on scheduling problem. Apart from this Das et al.[2] defined robust ranking for intuitionistic trapezoidal fuzzy number and used it in decision making problems. After studying all these papers in this job sequencing problem is solved with trapezoidal intuitionistic fuzzy number.

## II. PRELIMINARIES

**Definition 2.1[1]:**
For universal set $X = \{x_1, x_2, \dots\dots, x_n\}$, an intuitionistic fuzzy set is defined as $A = \{\langle x, \mu_A(x), \theta_A(x)\rangle : x \epsilon X\}$ in which functions $\mu_A(x): X \to [0,1]$ and $\theta_A(x): X \to [0,1]$ called membership function and non-membership function respectively and for every $x \epsilon X$, $0 \le \mu_A(x) + \theta_A(x) \le 1$ always holds.

**Definition 2.2[1]:**
The degree of hesitancy or uncertainty of an element $x$ in $A$ is defined as

$$\pi_A(x) = 1 - \mu_A(x) - \theta_A(x)$$

It is also called intuitionistic fuzzy index. For every , $0 \le \pi_A(x) \le 1$.

**Definition 2.3[1]:**
An intuitionistic fuzzy normal is an intuitionistic fuzzy set in which there exists at least two points $a, b \epsilon X$ which satisfies $\mu_A(a) = 1$ and $\theta_A(b) = 1$.

**Definition 2.4[6]:**
An intuitionistic fuzzy number is an intuitionistic fuzzy set which satisfies following conditions.
i) It is fuzzy normal.
ii) Membership function is convex i.e.

$$\mu_A(\lambda a + (1-\lambda)b) \ge min\big(\mu_A(a), \mu_A(b)\big) \text{ for all } a, b \epsilon R, \lambda \epsilon [0,1]$$

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iii) Non-Membership function is concave i.e.

$\theta_A(\lambda a + (1-\lambda)b) \le max(\theta_A(a), \theta_A(b))$ for all $a, b \epsilon R, \lambda \epsilon [0,1]$

**Definition 2.5[7]:**

A triangular intuitionistic fuzzy number is an intuitionistic fuzzy number whose membership and non-membership functions are defined as

$$\mu_A(x) = \begin{cases} \frac{x-(a-\alpha)}{\alpha}, x\epsilon[a-\alpha, a] \\ 1, \quad x\epsilon[a,b] \\ \frac{b+\beta-x}{\beta}, \quad x\epsilon[b, b+\beta] \\ 0, \quad otherwise \end{cases}$$

and

$$\theta_A(x) = \begin{cases} \frac{(a-x)}{\alpha'}, x\epsilon[a-\alpha', a] \\ 0, \quad x\epsilon[a,b] \\ \frac{x-b}{\beta'}, \quad x\epsilon[b, b+\beta'] \\ 1, \quad otherwise \end{cases}$$

Where $\alpha, \beta, \alpha', \beta' > 0$.

Triangular Intuitionistic fuzzy number is denoted by $\tilde{A} = [a, b, \alpha; a, b, \alpha']$

## III. ARITHMETIC OPERATIONS ON TRIANGULAR INTUITIONISTIC FUZZY NUMBER [10]

Let $\tilde{A} = [a, b, \alpha; a, b, \alpha']$ and $\tilde{B} = [c, d, \gamma; c, d, \gamma']$ are two triangular intuitionistic fuzzy numbers

i) $\tilde{A} + \tilde{B} = [a+c, b+d, \alpha+\gamma; a+c, b+d, \alpha'+\gamma']$

ii) $\tilde{A} - \tilde{B} = [a-d, b-c, \alpha+\gamma; a-d, b-c, \alpha'+\delta']$

iii) $\lambda\tilde{A} = \begin{cases} ([\lambda a, \lambda b, \lambda\alpha; \lambda a, \lambda b, \lambda\alpha']), \quad if\ \lambda \ge 0 \\ ([\lambda b, \lambda a, -\lambda\beta, -\lambda\alpha; \lambda b, \lambda a, -\lambda\beta', -\lambda\alpha']), \quad if \lambda > 0 \end{cases}$

## IV. COMPARISON OF TWO TRIANGULAR INTUITIONISTIC FUZZY NUMBER [11]

If $\tilde{A} = [a, b, \alpha; a, b, \alpha']$ and $\tilde{B} = [c, d, \gamma; c, d, \gamma']$ are two triangular intuitionistic fuzzy numbers, then

i) $\tilde{A} \ge \tilde{B}$ if and only if $R(\tilde{A}) \ge R(B)$

ii) $\tilde{A} < \tilde{B}$ if and only if $R(\tilde{A}) < R(B)$

iii) $\tilde{A} \approx \tilde{B}$ if and only if $R(\tilde{A}) \approx R(B)$

Where $R(\tilde{A}) = a + b + \frac{1}{4}(\alpha' - \alpha)$ and

$R(\tilde{B}) = c + d + \frac{1}{4}(\gamma' - \gamma)$

## V. ALGORITHM

The sequencing problem is defined as:

| Jobs→ Machines↓ | $J_1$ | $J_2$ | ....... | $J_n$ |
|---|---|---|---|---|
| $M_1$ | $t_{11}$ | $t_{12}$ | ......... | $t_{1n}$ |
| $M_2$ | $t_{21}$ | $t_{22}$ | ....... | $t_{2n}$ |

Here $t_{ij}$ is triangular intuitionistic fuzzy number which denotes time duration taken by $i^{th}$ job on $j^{th}$ machine.

The following algorithm is given to solve the sequencing problem.

Step 1:

By using defined ranking processing times taken as triangular intuitionistic fuzzy number are defuzzified.

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Step 2:
Using Johnson's algorithm Find optimal sequence for given sequencing problem.
Step 3:
Prepare In-Out tables for both machines and calculate total elapsed time.
Step 4:
From above table find idle time of machines.

## VI. NUMERICAL ILLUSTRATION

**Example 6.1:**
Five jobs are to be processed through two machines and processing time of each job which is taken as triangular intuitionistic fuzzy number is presented in following table. Find optimal sequence of jobs and total elapsed time.

| Jobs→ Machines↓ | A | B | C | D | E |
|---|---|---|---|---|---|
| $M_1$ | $\begin{Bmatrix}(1,2,3);\\(2,3,5)\end{Bmatrix}$ | $\begin{Bmatrix}(0,2,3);\\(0,2,3)\end{Bmatrix}$ | $\begin{Bmatrix}(4,6,8);\\(1,6,6)\end{Bmatrix}$ | $\begin{Bmatrix}(5,6,7);\\(3,6,10)\end{Bmatrix}$ | $\begin{Bmatrix}(3,6,7);\\(1,7,10)\end{Bmatrix}$ |
| $M_2$ | $\begin{Bmatrix}(1,0,2);\\(2,0,2)\end{Bmatrix}$ | $\begin{Bmatrix}(3,4,5);\\(2,3,4)\end{Bmatrix}$ | $\begin{Bmatrix}(4,6,7);\\(3,5,6.5)\end{Bmatrix}$ | $\begin{Bmatrix}(2,2.5,4);\\(2.5,3,6)\end{Bmatrix}$ | $\begin{Bmatrix}(2.4,5,6);\\(3,5,7)\end{Bmatrix}$ |

**Solution:**
Step 1: Using defined ranking defuzzify processing time of each job .

| Jobs→ Machines↓ | A | B | C | D | E |
|---|---|---|---|---|---|
| $M_1$ | 2.5 | 2.625 | 5.75 | 6.5 | 6.125 |
| $M_2$ | 1.75 | 3.75 | 5.3125 | 3 | 4.175 |

Step 2: using Johnson's algorithm optimal sequence is calculated.
Among all processing times minimum time is 1.75 which is corresponding to job first job on second machine, so this job is placed first from last.

| | | | | |
|---|---|---|---|---|
| - | - | - | - | A |

Leaving job A, for remaining jobs the shortest time is 2.625 which is for second job on first machine, so second job will be taken at first position.

| | | | | |
|---|---|---|---|---|
| B | - | - | - | A |

Leaving both jobs A and B, for remaining jobs the shortest time is 3 which is for fourth job on second job, so put that in second last position.

| | | | | |
|---|---|---|---|---|
| B | - | - | D | A |

Leaving these jobs from remaining jobs minimum time is 4.175 which is for fifth job on second machine, so put that job in third position.

| | | | | |
|---|---|---|---|---|
| B | - | E | D | A |

Final sequence is

| | | | | |
|---|---|---|---|---|
| B | C | E | D | A |

Step 3:

| Job Sequence | $M_1$ | | $M_2$ | |
|---|---|---|---|---|
| | Time in | Time out | Time in | Time out |
| B | 0 | $\begin{Bmatrix}(2,3,5);\\(0,2,3)\end{Bmatrix}$ | $\begin{Bmatrix}(0,2,3);\\(2,3,6)\end{Bmatrix}$ | $\begin{Bmatrix}(5,7,10);\\(5,7,12)\end{Bmatrix}$ |
| C | $\begin{Bmatrix}(2,3,5);\\(0,2,3)\end{Bmatrix}$ | $\begin{Bmatrix}(4,8,11);\\(1,8,12)\end{Bmatrix}$ | $\begin{Bmatrix}(8,11,12);\\(1,8,12)\end{Bmatrix}$ | $\begin{Bmatrix}(8,17,19);\\(13,17,18.5)\end{Bmatrix}$ |

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| | | | | |
|---|---|---|---|---|
| E | $\{(8,11,12); (8,11,12)\}$ | $\{(7,18,21); (14,18,22)\}$ | $\{(7,18,21); (2,14,22)\}$ | $\{(17,23,27); (4,23,29)\}$ |
| D | $\{(7,18,21); (14,18,22)\}$ | $\{(20,25,29); (5,20,32)\}$ | $\{(20,25,29); (5,25,32)\}$ | $\{(14,22.5,33); (6,28,38)\}$ |
| A | $\{(12, 25,29); (5,20,,32)\}$ | $\{(13,22,33); (22,28,37)\}$ | $\{(14,28,33); (6,28,38)\}$ | $\{(15,22.5,36); (8,30,42)\}$ |

Step 4:
Total Elapsed time: $\{(15,22.5,30); (22.5,30,42)\} \approx 25.75$ hours
Idle time for machine $M_1$: 2.25 hours
Idle time for machine $M_2$: 7.7625 hours

## VII. CONCLUSION

In this Job sequencing problem is solved with triangular intuitionistic fuzzy numbers by using proposed ranking methods and arithmetic operations of these fuzzy numbers. Total elapsed time is calculated without defuzzify the processing times.

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