



LEVEL CONTROL OF TWO INTERACTING CONICAL TANKS USING DE-CENTRALIZED PID CONTROLLER

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Abstract: This paper shows the controller design of a non-linear system. The Non-linear system we took up for the analysing is the Two Tank Interacting Conical System. The controller design is obtained in two parts: first a decoupling matrix is designed in order to minimize the interaction effects. Later, the controller design is obtained for both process and Decoupler block. Our aim is to meet the design specifications for each loop independently. So, we are using De-Centralized PID Controller. Simulation is made to demonstrate the effectiveness of the proposed method using MATLAB software

Keywords—Multi-loop PID Controller, Relative Gain Array, Decentralized controller, Two Tank Conical Interacting System, Decoupling.

1. INTRODUCTION

Most of industrial processes are basically multi-input/multi-output(MIMO) systems. For such MIMO systems, loop interactions can arise and cause difficulties in the controller design. To solve this problem there exist several alternatives that can be used to centralize multivariable control (LQR, LQG and LQ, Robust control, Predictive multivariable control, fuzzy control...) or decentralized control. The last is the most broadly used in the industrial environment. Despite the development of such advanced multivariable controllers, the de-centralized multi-loop PID control using multiple single-input/single-output (SISO) PID controllers remains the standard for controlling MIMO systems with modest interaction because of its simple and failure tolerant structure that are easy to implement and maintain by plant personnel and adequate performance. In most of the industrial chemical process present many challenging problems due to their nonlinear dynamic behavior's. One such non-linear process taken up for study is Interacting Conical systems. Conical tanks provide complete drainage and easy discharge of liquid.

Conical tanks are best suited for food process industries, hydrometallurgical industries, concrete mixing industries and waste water treatment industries. To design Multi-loop controller, control engineer must have clear idea of which manipulated variable is paired with which controlled variable. Relative Array Gain (RGA) is used to find influence between manipulated variable and controlled output at steady state. To control the liquid level and the flow in process tanks are still challenging problem in process industries. With decentralized techniques, a multivariable system with n inputs and n output variables is treated as n mono-variable systems. However, due to process and loop interactions, the design and tuning of multi-loop controllers is much more difficult compared with that of single-loop controllers. Since the controllers interact with each other, the tuning of one loop cannot be done independently. The sacrifice that supposes the invariable performance deterioration of a decentralized control structure when it is compared with a full multivariable control strategy is compensated with certain advantages such as design, easiness of use and hardware simplicity.

Controller consists of decomposing the design problem into two parts: first part is to decoupling the system in order to minimize interaction or to make the system diagonal dominant, then in second part designing the controllers using some decentralized method. The final control system will be the product of the decoupling and the controller matrices. The decouplers, together with single-loop controllers, constitute the multivariable controller. The essence of decoupling is the imposition of a calculation net that cancels the existent process interaction, allowing the independent control of the loops.

2. LITERATURE SURVEY

Some of decentralized methods got to know by literature Survey could be classified under the following groups, showing the interest that it has been risen in the last few years.

First group, that is based on design of some SISO method. These formulas indicate the direction in which the PID parameters have to be de-tuned to compensate the interaction effects when all loops are closed. Method of Shinskey and BLT are one of most cited in literature, this are included in this group. However, with these methods it is difficult to establish a proper design specification in all loops and they can be rather considered of trial and error.

second group includes the works that looks for critical gains of the system, in order to tune the PID controllers. This gain can be obtained by means of proportional controller or by means of relay methods. Not all these methods require a complete model of the systems.

Last method included in the third group uses the whole transfer function matrix considering into account, the interaction effects and the controllers are obtained by means of analytic, numeric or graphics methods. Some methods use optimization algorithms to obtain controllers and others uses pole placement techniques, and some provides on-line tuning formulas. Method showed in this paper comes under this group.

3. PROCESS DESCRIPTION

The two tank conical interacting system consists of two identical conical tanks, i.e. Tank 1 and Tank 2, two identical pumps are used to deliver the liquid flows F_{in1} and F_{in2} to Tank 1 and Tank 2 through the two control valves CV1 and CV2 respectively. These two tanks are interconnected at the bottom through a manually controlled valve MV12 with a valve coefficient β_{12} . F_{out1} and F_{out2} are the two output flows from Tank 1 and Tank 2 through manual control values MV1 and MV2 with valve coefficients β_1 and β_2 respectively.

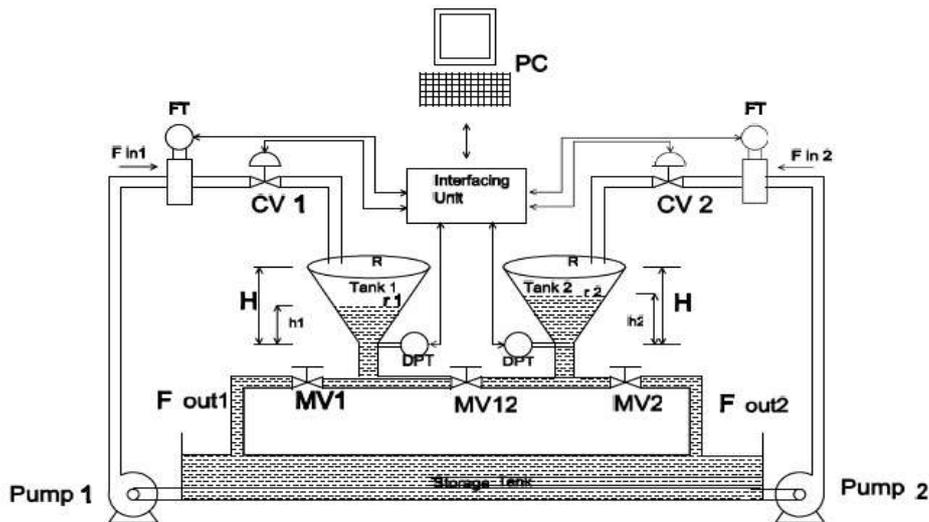


Fig. 1 Schematic diagram of Two Tank Conical Interacting System

In this work, the above process is considered as TITO process in which level h_1 in TANK 1 and level h_2 in TANK 2 are considered as output variables and F_{IN1} and F_{IN2} are considered as respective manipulated Variable. The operating parameters of this process is shown in Table 1

Table. 1 Operating Parameters of Two Tank Conical Interacting System

Parameter	Description	Value
R	Top radius of conical tank	19.25cm
H	Maximum height of Tank1&Tank2	73cm
F_{in1} & F_{in2}	Maximum inflow to Tank1&Tank2	252cm ³ /sec
β_1	Valve coefficient of MV ₁	35 cm ² /sec
β_{12}	Valve coefficient of MV ₁₂	78.28 cm ² /sec
β_2	Valve coefficient of MV ₂	19.69 cm ² /secs

The non-linear equations describing the open loop dynamics of the Two Tank Conical Interacting Systems is derived using the Mass balance equation and Energy balance equation principle. The mathematical model of two tank conical interacting system is given by:

$$\frac{dh_1}{dt} = \left[\frac{F_{IN1} - h_1 \frac{dA(h_1)}{dt} - \beta_1 \sqrt{h_1} - \text{sign}(h_1 - h_2) \beta_{12} \sqrt{|h_1 - h_2|}}{\frac{1}{3} \pi R^2 \frac{h_1^2}{H^2}} \right] \quad \text{----- (1)}$$

$$\frac{dh_2}{dt} = \left[\frac{F_{IN2} - h_2 \frac{dA(h_2)}{dt} - \beta_2 \sqrt{h_2} + \text{sign}(h_1 - h_2) \beta_{12} \sqrt{|h_1 - h_2|}}{\frac{1}{3} \pi R^2 \frac{h_2^2}{H^2}} \right] \quad \text{----- (2)}$$

4. CONTROLLER DESIGN

4.1 Relative Gain Array

The main task in any multi-loop control system design is the pairing of manipulated and controlled variables. The most popular and widely used technique for determining the best controller pairing is the relative gain array (RGA) method. RGA analysis has been widely used in process control to Identify Control structures and to characterize the degree of process interactions between controlled and manipulated variables. RGA presents a new dynamic loop pairing criterion for decentralized control of multivariable processes through defining an effective gain matrix.

An important advantage of the RGA method is that it requires minimal process information. i.e. steady-state gains. Another advantage is that the results are independent of both the physical units and are uses the scaling of the process variables. By definition, the relative gain λ_{ij} between i^{th} manipulated variable and the j^{th} controlled variable is defined as:

$$\lambda_{ij} = \frac{\text{open loop gain between } y_i \text{ and } u_j}{\text{closed loop gain between } y_i \text{ and } u_j} \quad \text{----- (3)}$$

$$\lambda_{ij} = \frac{\left(\frac{\partial y}{\partial x}\right)_{u_k}}{\left(\frac{\partial y}{\partial x}\right)_{y_k}} \quad \text{----- (4)}$$

The relative gain between input 1 and output 1 for a TITO system is

$$\lambda_{11} = \frac{\text{gain between } y_i \text{ and } u_j \text{ with } u_2 \text{ constant}}{\text{gain between } y_i \text{ and } u_j \text{ with } y_2 \text{ constant}} \quad \text{----- (5)}$$

The steady state gain between u_1 and y_1 with all other loops open (that is u_2 is constant) is

$$\left(\frac{\partial y_1}{\partial u_1}\right)_{u_2} = G_{11}(0) = k_{11} \quad \text{----- (6)}$$

The steady state relationship between u_1 and y_1 at steady state, with y_2 maintain constant at its set point (using a controller with integral action), is

$$\left(\frac{\partial y_1}{\partial u_1}\right)_{y_1} = G_{11}(0) - \frac{G_{12}(0)G_{21}(0)}{G_{22}(0)} = k_{11} - \frac{K_{12}K_{21}}{k_{22}} \quad \text{----- (7)}$$

The relative gain between u_1 and y_1 is calculated by substituting (5) and (6) in (7) as

$$\lambda_{11} = \frac{k_{11}k_{22}}{k_{11}k_{22} - k_{12}k_{21}} \quad \text{----- (8)}$$

4.2 Decoupling Design

The essence of decoupling is the imposition of a calculation net that cancels the existent process interaction, allowing the independent control of the loops. In decentralized design, the question is not to eliminate interaction, but to take it into account. The objective in decoupling is to compensate for the effect of interactions brought about by cross coupling of the process variables.

In a system with interactions G_1 and G_2 which are not zero, but we can manipulate the controller signal such that the system. appears (mathematically) to be decoupled i.e. the controller output is transformed with a matrix D. which will contain decoupling functions Then the manipulated variables can be written as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} G_{c1} & 0 \\ 0 & G_{c2} \end{bmatrix} \begin{bmatrix} r_1 - y_1 \\ r_2 - y_2 \end{bmatrix} \quad \text{----- (9)}$$

The close loop system equation for basic 2X2 system can be written in matrix form as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = GDG_c \begin{bmatrix} r_1 - y_1 \\ r_2 - y_2 \end{bmatrix} \quad \text{----- (10)}$$

To decouple the system equation $G D G_c$ must be a diagonal matrix. By defining $G_0 = G D G_c$, (10) can be solved for y :

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = [I + G_0]^{-1} G_0 \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \quad \text{----- (11)}$$

Since G , is diagonal, the matrix $[I+G_0]^{-1}G_0$ is also diagonal. Since G_c is already diagonal, $G D$ should also be diagonal:

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \quad \text{-----(12)}$$

From (12), the elements of D can be found as

$$d_{11} = \frac{G_{22}H_1}{G_{11}G_{22}-G_{12}G_{21}} \quad \text{-----(13)}$$

$$d_{22} = \frac{G_{22}H_2}{G_{11}G_{22}-G_{12}G_{21}} \quad \text{-----(14)}$$

$$d_{12} = \frac{-G_{12}}{G_{11}} d_{22} \quad \text{-----(15)}$$

$$d_{21} = \frac{-G_{22}}{G_{22}} d_{11} \quad \text{-----(16)}$$

By assuming

$$H_1 = \frac{G_{11}G_{22}-G_{12}G_{21}}{G_{22}} \quad H_2 = \frac{G_{11}G_{22}-G_{12}G_{21}}{G_{11}} \quad \text{-----(17)}$$

elements of a 2 x 2 system for decoupling matrix try to eliminate interactions from all loops are determined as shown below.

$$d_{11} = 1 \quad \text{-----(18)}$$

$$d_{12}(s) = -\frac{g_{12}(s)}{g_{11}(s)} \quad \text{-----(19)}$$

$$d_{21}(s) = -\frac{g_{21}(s)}{g_{22}(s)} \quad \text{-----(20)}$$

$$d_{22}(s) = 1 \quad \text{-----(21)}$$

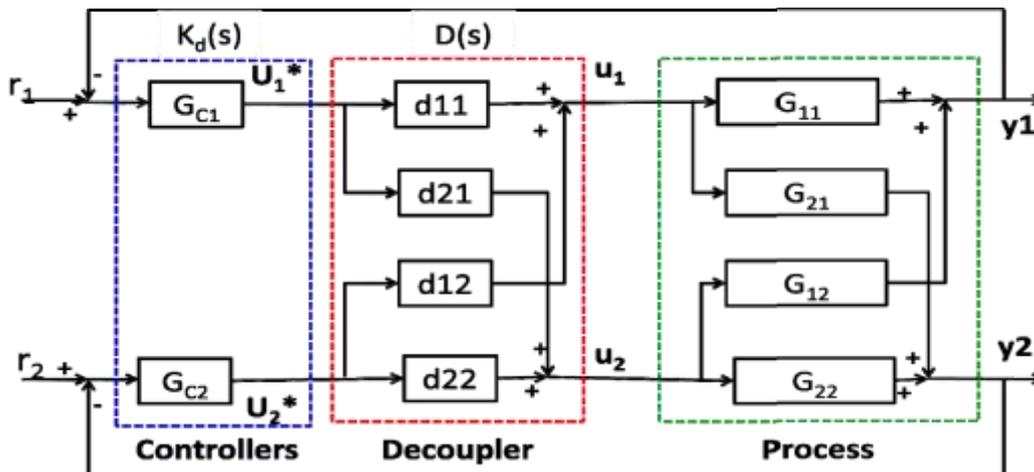


Fig. 2 General 2 x 2 system with decouples and single-loop controllers

The design of a decentralized control system with a decoupling matrix can be done combining a diagonal controller $K_d(s)$ with a block compensator $D(s)$, so that the controller manipulates the variable u_i^* instead of the u_i , as can be appreciated in Fig. 2, for the 2 x 2 case. With this configuration the controller sees the process as a set of n completely independent processes or with the interaction reduced.

5. MODELLING OF SYSTEM

5.1 Model of Two Interacting Conical Tank System

In order to model the TTCIS experimentally, transfer function Matrix Model is utilized. Since the TTCIS is a TITO system, the transfer function matrix is represented as follows:

$$\begin{bmatrix} h_1(s) \\ h_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \quad \text{-----(22)}$$

where $G_{ij}(s)$ are assumed as first order with dead time system as given in (23)

$$G_{ij}(s) = \frac{K_{ij}e^{-T_{dij}s}}{\tau_{ij}s+1} \quad \text{-----(23)}$$

The transfer matrix parameters K_{ij} , T_{dij} , τ_{ij} and are estimated empirically using two-point method. The estimated parameters of the model are summarized in Table II

Table II Estimated Parameters for model

	K	r (Seconds)	Ta(Seconds)
$G_{11}(s)$	0.1365	487.5	117.5
$G_{12}(s)$	0.11075	705	154
$G_{21}(s)$	0.0905	822	92
$G_{22}(s)$	0.129	715.5	158.5

5.2 Decoupler for Two Interacting Conical Tank System

The elements of Decoupler for Two Interacting Conical Tank System are determined from equations (18) to (21) by substituting the parameters as shown in TABLE II.

$$d_{11}(s) = 1$$

$$d_{12}(s) = \frac{(-395.6819s - 0.81135)e^{-36.5s}}{705s + 1}$$

$$d_{21}(s) = \frac{(-501.9593s - 0.70155)e^{-66.5s}}{855s + 1}$$

$$d_{22}(s) = 1$$

5.3 De-Centralized PID Controller Parameters

The De-Centralized PID controller is Designed as per the block diagram shown in Fig.2 using Simulink Toolbox in MATLAB. The desired values of the controlled variables are given as set point. The outputs of the controllers u_1 and u_2 are manipulated variables which are given to the de-coupler instead of directly to the TTCIS process. The decoupler will eliminate the interaction. This results in formation of two independent single loop PID controllers. Tuning of PID controller 1 in loop1 will not affect h_2 and tuning of PID controller 2 in loop2 will not affect h_1 . Both the controllers are tuned optimally using RGA independently using ISE as a performance metrics.

TABLE III De-Centralized PID controller parameters

Controller	K_p	K_i	K_d
G_{c1}	18.7928	0.0591	85.7283
G_{c2}	25.9071	0.0575	42.7175

6. SIMULATION RESULTS

The response for set point tracking and regulatory problems are shown in the Below Graph. The values of PID controller parameters used to control the process in simulation are shown in Table III.

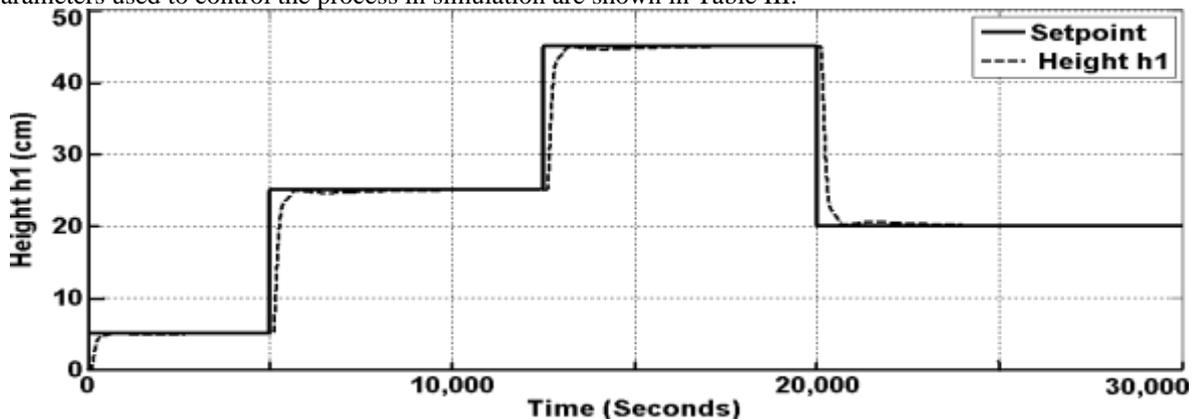
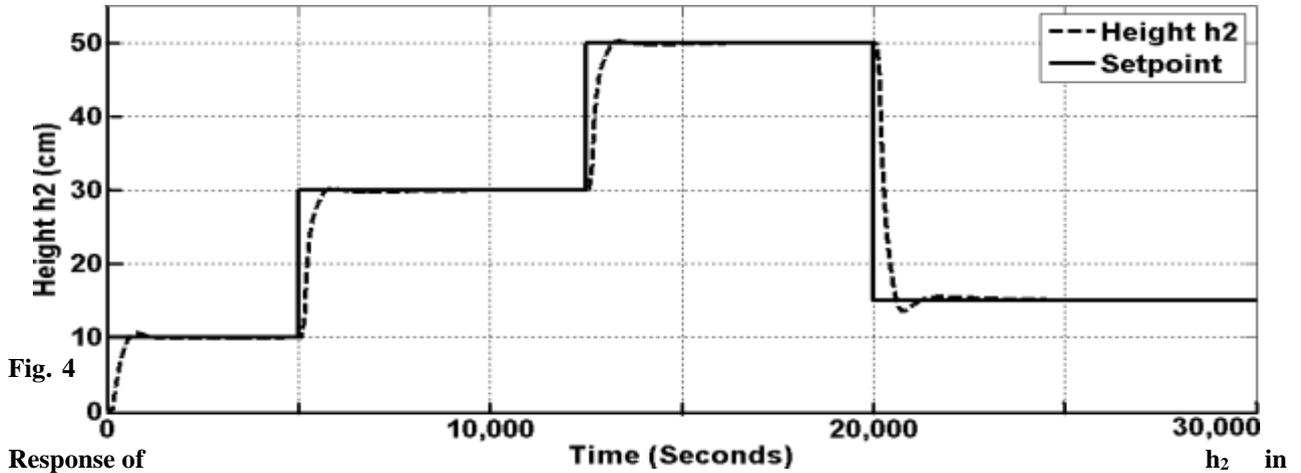
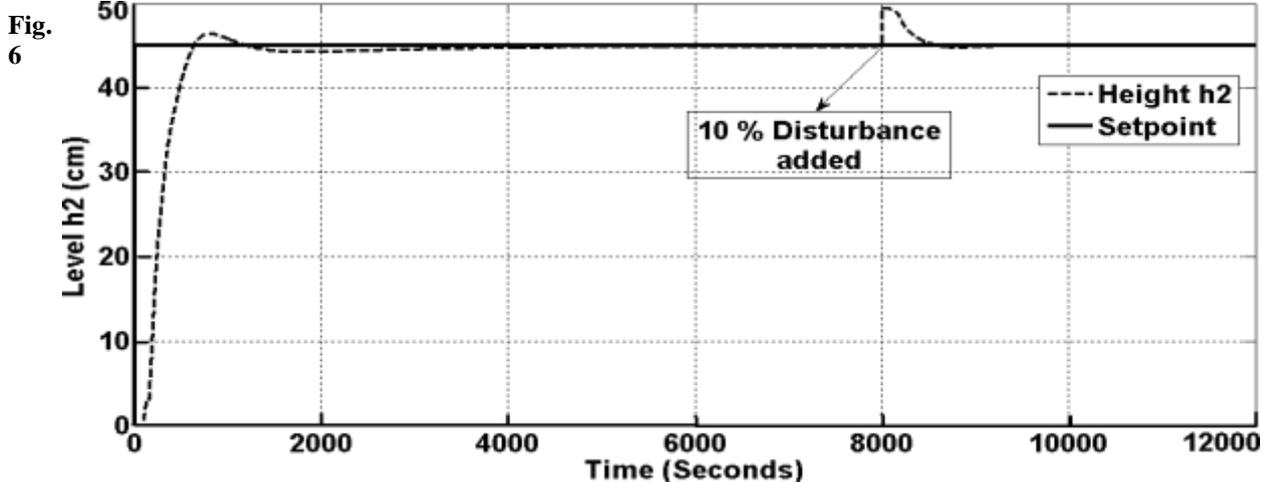
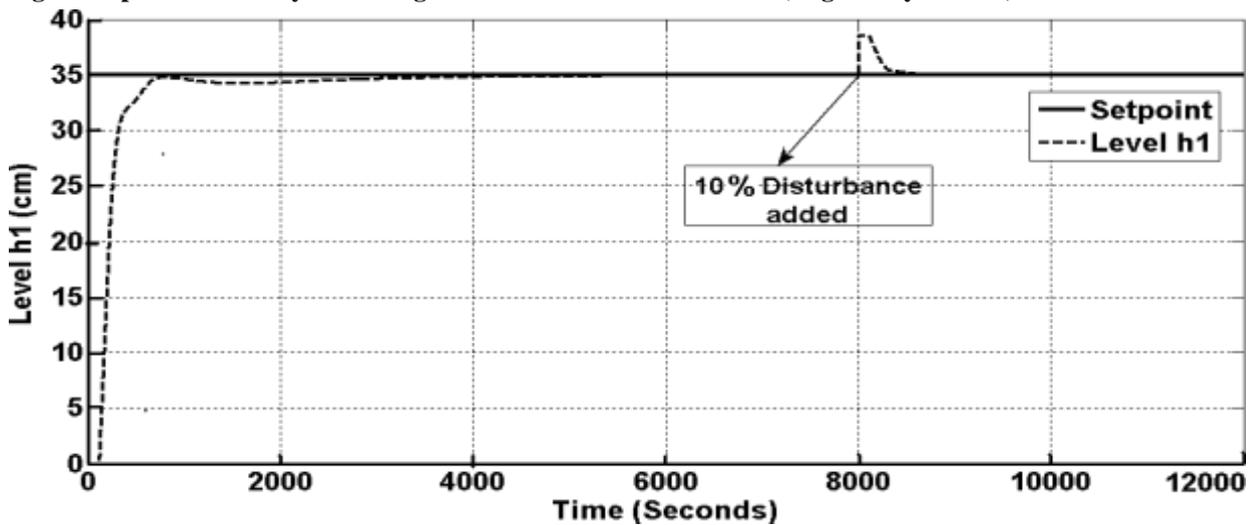


Fig. 3 Response of h_1 in system using De-Centralized PIDcontroller (set point tracking)



Response of system using De-Centralized PIDcontroller (set point tracking)

Fig. 5 Response of h_1 in system using De-Centralized PIDcontroller (Regulatory control)



Response of h_2 in system using De-Centralized PIDcontroller (Regulatory control)

**CONCLUSION**

In this paper, Designing and Implementation of De-Centralized PID controller for a Two Tank Conical Interacting System is done. The Simulation results confirms that the controller designed gives satisfactory response for a given set point. Also the controller effectively rejects the disturbance and brings back the output to the desired set point.

REFERENCES

- 1) P.J. Campo, M. Morari, Achievable closed-loop properties of systems under decentralized control: conditions involving the steady-state gain, *IEEE Trans. Automat. Control* 39 (1994) 932–943.
- 2) J. Lee, W. Cho, T.F. Edgar, Multi-loop PI controller tuning for interacting multivariable processes, *Comput. Chem. Eng.* 22 (1998) 1711–1723.
- 3) Shinskey, *Process Control Systems*, MacGraw-Hill, 1995
- 4) Luyben, W.L., *Practical distillation control*, Editor W.L Luyben, 1992
- 5) Niederlinski, A, A heuristic Approach to the design of linear multivariable interacting control systems". *Automatica*, Vol. 7, 1971, pp.691-701.
- 6) Zhuang, M, Atherton, D, PID controllers design for a TITO system, *IEE Proc. Control Theory Appl.* Vol. 141, 1994, pp. 111-120.
- 7) Halevy, Y, Palmor, Z, Efrati, T, Automatic tuning of decentralized PID controllers for MIMO processes, *J. Proc. Cont.* Vol. 7, 1997, pp 119-128, Elsevier Science, Ltd.
- 8) Toh, K., Devanathan, R, An expert Autotuner for Multiloop SISO Controllers, *Control Eng. Practice*, Vol. 1, 1993, pp 999-1008.
- 9) Shiu, S.J, Hwang, S, Sequential design method for multivariable decoupling and multiloop PID controller, *Ind. Eng. Chem. Res.*, Vol. 37, 1998, pp. 107-119.
- 10) Wang, Q, Lee, T, Fung, H, Independent design of multi-loop controller taking into account multivariable interaction, *Journal of Chemical Engineering of Japan*, Vol. 33, 2000, pp. 427-439.
- 11) Zhang, Y, Wang, Q, Astrom, K., Dominant Pole Placement for Multi- Loop Control Systems, *American Control Conference*, Chicago, USA, June 29, 2000.
- 12) Ho, W.K., T.H. Lee, O.P. Gan, Tuning of multiloop PID controllers based on gain and phase margins specifications. *13th IFAC World Congress*, 1995, pp 211-216.
- 13) Deshpande, B, *Multivariable process Control*, Instrument Society of America, 1989
- 14) B.Wayne Bequette, *Process Control- modeling, Design, and Simulation*,
- 15) Prentice-Hall, inc., USA, 2003 Thomas E. Marlin, *Process Control – Designing Process and Control Systems for Dynamic Performances*, McGraw Hill International Edition, 2000
- 16) Priyanka Singh; Prasad L B. "A Comparative Performance Analysis of PID Control and Sliding Mode Control of Two Link Robot Manipulator". *International Research Journal on Advanced Science Hub*, 2, 6, 2020, 43-54. doi: 10.47392/irjash.2020.35