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Application Of Soham Transform For Solving Volterra Integral Equations Of First Kind

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Abstract: Volterra Integral equations of First kind arises in many problems of Engineering, Physics, Chemistry, Biology. In this paper we use Soham transform to obtain the solution of linear Volterra integral equation of first kind. Convolution theorem is proved. Some problems are solved using Soham transform, which shows effectiveness of Soham transform for solving linear Volterra integral equation of first kind.

Keywords : Integral transform, Soham transform, Convolution theorem, Volterra Integral equations of first kind.

I. INTRODUCTION

Linear Volterra integral equation of first kind is of the form [1]

f(x)= $\int_0^x K(x,t)u(t)dt$ ....(1)

Here, u is unknown function, K(x,t) is kernel and f(x) is real valued function.
A linear Volterra integral equation is a convolution equation if it is of the form

f(x) $=\int_0^t K(t-s)u(s)ds$ ....(2)

These equations were introduced by Volterra. Further these equations were studied by Traian Lalescu, in his 1908 thesis. In 1911, Lalescu wrote [2] the first book on integral equations.volterra integral equations has many applications in wide range of fields. Some of them are demography, study of viscoelastic material, acturial science through the renewal equations. Soham transform is introduced by D. P. Patil and S. S.Khalcale [3] in October2021. It is defined by the integral equation.

S[f(t)]=P(v)=$\frac{1}{v}\int_0^\infty e^{-v^\alpha t} f(t)dt$, where α is non zero real number.

Now a days many researchers are introducing various types of integral transformers. Receltly in September 2021 Kushare and Patil [4] introduces Kushare transform, for facilitating the process of solving ordinary and partial differential equations inthe time domain. Researchers are alos using the transform for solving various problems. Sudhanshu Agrawal etal [5] used Kamal transform for solving linear volterra integral eqautions of first kind. Recently in January2022 R. S. Sanap and D. P. Patil [6] applied Kusahre transform for Newton's law of cooling. In October2021, Sawi transform is used in Bessel functions by Patil [7]. Further Sawi transform of error function is used to evaluate improper integral by D. P. Patil [8]. Patil [9] used Laplace and Shehu transform in chemical science. Sawi transform and concolution therorm is used for solving wave equation by Patil[10]. D. P. Patil[11] used Magoub transform for obtaining the solution of parabolic boundryvalue problems. Patil [12] also used double Laplace and double Sumudu transform for getting solution of wave equation.

Dualities between various double integral transforms are obtained by D. P Patil[13]. Laplace , Elzaki and Mohgoub transforms are used for solving system of first order and firstdegree differential equations by Kushare and Patil [14]. Dr. Patil [15] also used Aboodh and Mohgoub transform in boundry value problems of system of ordinary differential equations. Recently in April2022, Nikam, Shirshath, Ahre and Patil [16] used Kusahre transform to solve the problems on growth and decay. Patil [17] compared Laplace Sumudu, Aboodh, Elzaki and Mohgoub transform for obtaining solution of boundry value problems. Further Dr.Patil [18] used double Mohgoub transform to solve parabolic boundry value problems. Dr.Patil [19] also used German Astronomar Friedrich Wilhelm Bessel derived Bessel function in 1817 this function are also called as cylinder function.Bessel function is solution of the differential equation. $x^2\frac{d^2y}{dx}+x\frac{dy}{dx}+(x^2-n^2)y=0$ .it is called as Bessel equation.

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In this paper we use Soham transform to solve the linear volterra integral equationsof first kind. Paper is organized as follows. Section 2 is devoted for preliminary.Convolution theorem for Soham tarnsform is used in section 3. Section 4 is used for obtaining Soham transform of Bessel function. Some application in volterra integral equationss of first kind are in section sector 5 is reserved for conclusion.

II. PRELIMINARY

Soham transform for the function f(t) of exponential order in the setB defined by:
B={f(t):∃M,k1.k2>0.|f(t)|<Me$^{|t|kj}$, if t ϵ$(-1)^j$ ×{0,∞}
For a given function in the set B,the constant M must be finiteNumber, ,k_1,k_2 may be finite or infinite.
Soham transform denoted by the operation δ(.)defined by the integral equations

δ[f(t)] = P(v)=$\frac{1}{v}\int_0^\infty f(t)\, e^{-v^{\propto}t}dt$, α is non zero real number t ≥0,k_1≤vk_2(2)

- **Inverse soham transform:** Inverse soham transform of f(t) is P(v) theninverse soham transform is **S^{-1}[P(v)]=F(t)**
- **Soham transformation of the elementary function:**

S.N	F(t)	S{F(t)} = P(v)
1)	1	$\frac{1}{v^{\alpha+1}}$
2)	t	$\frac{1}{v^{2\alpha+1}}$
3)	t^2	$\frac{2!}{v^{2\alpha+\alpha+1}}$
4)	t^n, n∈N	$\frac{\Gamma(n+1)}{v^{\alpha n+\alpha+1}}$
5)	e^{at}	$\frac{1}{v(v^\alpha-a)}$
6)	e^{-at}	$\frac{1}{v(v^\alpha+a)}$
7)	sinat	$\frac{a}{v(v^{2\alpha}+a^2)}$
8)	cosat	$\frac{v^\alpha}{v(v^{2\alpha}+a^2)}$

III. CONVOLUTION THEOREM:

The convolution of two function f(t)&g(t) is.

(f*g)(t)=$\int_0^t (t-\tau)g(\tau)d\tau$

Applying Soham transform.

S(f*g)(t) = S[$\int_0^t$ f(t − τ)g(τ)dτ]

$= \frac{1}{v}\int_0^\infty e^{-v^\alpha t}\left(\int_0^t f(t-\tau)g(\tau)d\tau\right)dt$

………(Change order of integration)

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$= \frac{1}{v}\int_0^\infty \int_0^t e^{-v^\alpha t} f(\tau)g(t-\tau)d\tau dt$

$= \frac{1}{v}\int_0^\infty \int_0^t e^{-v^\alpha t} f(\tau)g(t-\tau)dt d\tau$

Put t – τ = b

S(f*g)(t) = $\frac{1}{v}\int_0^\infty f(\tau)\int_0^\infty e^{-v^\alpha(\tau-b)}g(b)dbd\tau$

= v$\{\frac{1}{v}\int_0^\infty f(\tau)e^{-v^\alpha \tau}\,d\tau \frac{1}{v}\int_0^\infty e^{-v^\alpha b}\,g(b)db\}$

S(f*g)(t) = v{F(v)G(v)}

The convolution theorem for Soham transformations:

S(f*g)(t) = v{F(v)G(v)}

IV. Soham Transform Of Bessel's Function:

The Bessel's function of order n is defined as $J_n(t)$:

$$\text{Jn(t)} = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\sqrt{n+r+1}}(\frac{t}{2})^{n+2r}$$

Soham transform of Bessel's function of zero order $J_0(t)$:

For n=0 we have,

$J_0(t) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\sqrt{r+1}}(\frac{t}{2})^{2r} = \sum_{r=0}^{\infty} \frac{(-1)^r}{(r!)^2}(\frac{t}{2})^{2r}$

$=1-\frac{t^2}{2^2}+\frac{t^4}{2^2\times 4^2}-\frac{t^6}{2^2\times 4^2\times 6^2}+\cdots$

Thus,

$L[J_0(t);s] = L[1,s] - \frac{1}{2^2}L[t^2;s] + \frac{1}{2^2\times 4^2}L[t^4;s]$ ...

$J_0(t) = \frac{1}{v^{\alpha+1}} - \frac{1}{2^2}[\frac{2!}{v^{2\alpha+\alpha+1}}] - \frac{1}{2^2\times 4^2}\left[\frac{4!}{v^{4\alpha+\alpha+1}}\right] - \frac{1}{2^2\times 4^2\times 6^2}\left[\frac{6!}{v^{6\alpha+\alpha+1}}\right]+\ldots$

$= \frac{1}{v^{\alpha+1}}[1 - \frac{1}{2}(\frac{1}{v^{2\alpha}}) - \frac{1}{2^2\times 4^2}(\frac{4\times 3\times 2\times 1}{v^{4\alpha+\alpha+1}}) - \frac{1}{2^2\times 4^2\times 6^2}(\frac{6\times 5\times 4\times 3\times 2}{v^{6\alpha+2+1}})$

$=\frac{1}{v^{\alpha+1}}[1 - \frac{1}{2}(\frac{1}{v^{2\alpha}}) - \frac{1\times 3}{2\times 4}(\frac{1}{v^{2\alpha}})^2 - \frac{1\times 3\times 5}{2\times 4\times 6}(\frac{1}{v^{2\alpha}})^3 + \cdots$

$=\frac{1}{v^{\alpha+1}}\left(1+\frac{1}{v^{2\alpha}}\right)^{-\frac{1}{2}}$

$=\frac{1}{v^{\alpha+1}\times(\sqrt{1+\frac{1}{v^{2\alpha}}})}$

$=\frac{1}{v^{\alpha+1}\times(\sqrt{\left(\frac{v^{2\alpha}+1}{v^{2\alpha}}\right)})}$

$=\frac{1}{v^{\alpha+1}\frac{1}{v^\alpha}(\sqrt{v^{2\alpha}+1})}$

$=\frac{1}{v^{\alpha+1}v^{-\alpha}\left(\sqrt{v^{2\alpha}+1}\right)}$

$=\frac{1}{v\sqrt{v^{2\alpha}+1}}$

$J_0(t) =\frac{1}{v\sqrt{v^{2\alpha}+1}}$

V. APPLICATIONS:

a) Application

Consider Linear volterra Integral Equation of first kind whose Kernel containing exponential function.

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x=$\int_0^x e^{(x-t)}$ u(t)dt ....(1)

Applying the Soham transformation on both side of (1)we have,
S{x} = S{$\int_0^x e^{(x-t)}$ u(t)dt} ....(2)

Using convolution theorem of Soham transform on(2)We have,

$$S\{x\} = vS(e^{\tau})S\{u(x)\}$$
$$\frac{1}{v^{2\alpha+1}} = v\frac{1}{v(v^{\alpha}-1)}S\{u(x)\}$$
$$S\{u(x)\} = \frac{v^{\alpha}-1}{v^{2\alpha+1}}$$
$$= \frac{v^{\alpha}}{v^{2\alpha+1}} - \frac{1}{v^{2\alpha+1}}$$
$$S\{u(x)\} = \frac{1}{v^{\alpha+1}} - \frac{1}{v^{2\alpha+1}} \qquad \text{..................(3)}$$

Operating inverse Soham transform on both sides of (3),

$$u(x) = S^{-1}\left\{\frac{1}{v^{\alpha+1}}\right\} - S^{-1}\left\{\frac{1}{v^{2\alpha+1}}\right\}$$
$$u(x) = 1-x \qquad \text{..............................(4)}$$

which is required exact solution of (1).

b) Application

Consider linear volterra integral equation of first kind whose kernelcontains exponential function.
sinx = $\int_0^x e^{(x-t)} u(t)dt$...........(1)

Applying the Soham transform to both sides of (1),
We have,

S{sinx} = S{$\int_0^x e^{(x-t)} u(t)dt$}...............(2)

Using convolution theorem of Soham transform on (2)
We have,

$$S\{sinx\} = v\, S(e^{\tau})S\{u(x)\}$$
$$\frac{1}{v(v^{2\alpha}+1)} = v\frac{1}{v(v^{\alpha}-1)}S\{u(x)\}$$
$$S\{u(x)\} = \frac{v^{\alpha}-1}{v(v^{2\alpha}+1)}$$
$$S\{u(x)\} = \frac{v^{\alpha}}{v(v^{2\alpha}+1)} - \frac{1}{v(v^{2\alpha}+1)} \text{..............(3)}$$

Applying operating inverse Soham transform on both side of (3)
We have,

$$S\{u(x)\} = S^{-1}\left\{\frac{v^{\alpha}}{v(v^{2\alpha}+1)}\right\} - S^{-1}\left\{\frac{1}{v(v^{2\alpha}+1)}\right\}$$

u(x) = cosx – sinx ....(4)
Which is the required exact solution of (1)

c) Application

Consider linear volterra integral equation of first kind whose kernel linear in the argument x and t.

$$x^2 = \frac{1}{2}\int_0^x (x-t)u(t)dt \qquad \text{..........................(1)}$$

Applying the soham transform to both sides of (1),We have,

$$S\{x^2\} = \frac{1}{2}S\left\{\int_0^x (x-t)u(t)dt\right\} \text{........................(2)}$$

Using convolution theorem of Soham transform on (2)We have,

$$S\{x^2\} = \frac{1}{2}vS\{\tau\}S\{u(x)\}$$

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$$\frac{2!}{v^{2\alpha+\alpha+1}} = \frac{1}{2} v \frac{1}{v^{2\alpha+1}} S\{u(x)\}$$

$$\dots\dots\dots\dots\dots\dots\dots.\left\{S(x^n) = \frac{\Gamma(n+1)}{v^{2\alpha+\alpha+1}} = \frac{n!}{v^{2\alpha+\alpha+1}}\right\}$$

$$S\{u(x)\} = \frac{4v^{2\alpha+1}}{v^{2\alpha+1}v^{\alpha}v}$$

$$= \frac{4}{v^{\alpha+1}}$$

$S\{u(x)\} = 4\left\{\frac{1}{v^{\alpha+1}}\right\}$ ………………….(3)

Operating inverse Soham transform on both side of (3)We have,

$u(x) = 4\, S^{-1}\left\{\frac{1}{v^{\alpha+1}}\right\}$

$u(x) = 4$ …………………..(4)

Which is the required exact solution of (1)

d) Application

Consider linear volterra integral equation of first kind whose kernelcontaining exponential function.

$x = \int_0^x e^{-(x-t)} u(t)\,dt$ ………………(1)

Applying the Soham transform to both sides of (1),We have,

$S\{x\} = S\left\{\int_0^x e^{-(x-t)} u(t)\,dt\right\}$ …………….(2)

Using convolution theorem of Soham transform on (2),

We have,

$$S\{x\} = v\, S\{e^{-\tau}\} S\{u(x)\}$$

$$\frac{1}{v^{2\alpha+1}} = v \frac{1}{v(v^{\alpha}+1)} S\{u(x)\}$$

$$= \frac{1}{(v^{\alpha}+1)} S\{u(x)\}$$

$$S\{u(x)\} = \frac{v^{\alpha}+1}{v^{2\alpha+1}}$$

$$= \frac{v^{\alpha}}{v^{2\alpha+1}} + \frac{1}{v^{2\alpha+1}}$$

$S\{u(x)\} = \frac{1}{v^{\alpha+1}} + \frac{1}{v^{2\alpha+1}}$ ………………(3)

Operating inverse Soham transform on both side of (3)We have,

$u(x) = S^{-1}\left\{\frac{1}{v^{\alpha+1}}\right\} + S^{-1}\left\{\frac{1}{v^{2\alpha+1}}\right\}$

$u(x) = 1 + x$ …………………(4)

Which is required exact solution of (1).

e) Application

Consider linear volterra integral equation of first kind with unity as a kernel

$x \left\{= \int_0^x u(t)\,dt\right\}$ …………………….(1)

Applying the Soham transform to both sides of (1),We have,

$S\{x\} = S\left\{\int_0^x u(t)dt\right\}$ …………………….(2)

Using convolution theorem of Soham transform on (2)

We have,

$$S\{x\} = v\, S\{1\} S\{u(x)\}$$

$$\frac{1}{v^{2\alpha+1}} = v \frac{1}{v^{\alpha+1}} S\{u(x)\}$$

$$\frac{v^{\alpha+1}}{v^{2\alpha+1}v} = S\{u(x)\}$$

$$S\{u(x)\} = \frac{v^{\alpha+1}}{v^{\alpha+1}v^{\alpha}v}$$

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$$S\{u(x)\} = \frac{1}{v^{\alpha+1}} \quad \ldots\ldots\ldots\ldots\ldots(3)$$

Operating inverse Soham transform on both side of (3)We have,

$$u(x) = S^{-1}\left\{\frac{1}{v^{\alpha+1}}\right\}$$

$$u(x) = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Which is required exact solution of (1).

f) Application

Consider linear volterra integral equation of first kind whose kernel contain unknown function with linear in the argument x and t

$$\sin x = \int_0^x u(x-t)u(t)dt \quad \ldots\ldots\ldots.(1)$$

Applying Soham transform to both side of (1) we get,

$$S\{\sin x\} = S\{\int_0^x u(x-t)u(t)dt\} \quad \ldots\ldots\ldots\ldots.(2)$$

Using convolution theorem of Soham transform on (2)
we have,

$$\frac{1}{v(v^{2\alpha}+1)} = v.S\{u(x)\}S\{u(x)\}$$

$$[S\{u(x)\}]^2 = \frac{1}{v(v^{2\alpha}+1)}$$

$$S\{u(x)\} = \pm\sqrt{\frac{1}{v^2\,(v^{2\alpha}+1)}}$$

$$= \pm\frac{1}{v\sqrt{1+v^{2\alpha}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Operating inverse Soham transform on both sides of (3)
We have,

$$u(x) = \pm S^{-1}\left\{\frac{1}{v\sqrt{1+v^{2\alpha}}}\right\}$$

$$u(x) = \pm Jo(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Which is the required exact solution of (1).

g) Application

Consider linear volterra integral equation of first kind whose kernel containing Bessel's function of zero order.

$$\sin x = \int_0^x Jo(x-t)u(t)dt \quad \ldots\ldots\ldots\ldots.(1)$$

Applying Soham transform to both side of (1) we get,

$$S\{\sin x\} = S\{\int_0^x Jo(x-t)u(t)dt\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Using convolution theorem of Soham transform on (2) we have,

$$\frac{1}{v(v^{2\alpha}+1)} = v.S\{Jo(x)\}S\{u(x)\}$$

$$\frac{1}{v(v^{2\alpha}+1)} = v\{\frac{1}{v(v^{2\alpha}+1}\}S\{u(x)\}$$

$$\frac{1}{v(v^{2\alpha}+1)} = \{\frac{1}{\sqrt{1+v^{2\alpha}}}\}S[u(x)]$$

$$\frac{\sqrt{1+v^{2\alpha}}}{v(v^{2\alpha}+1)} = S[u(x)]$$

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$S[u(x)] = \frac{\sqrt{1+v^{2\alpha}}}{v[(\sqrt{1+v^{2\alpha}})(\sqrt{1+v^{2\alpha}})}$

$$S[u(x)] = \frac{1}{v\sqrt{1+v^{2\alpha}}} \dots \dots \dots \dots (3)$$

Operating inverse Soham transform on both sides of (3),
We have,

$u(x) = S^{-1}\{\frac{1}{v\sqrt{1+v^{2\alpha}}}\}$

$\{u(x)\} = Jo(x) \dots\dots\dots\dots(4)$

Which is required exact solution of (1).

VI. CONCLUSION

We successfully used **SOHAM TRANSFORM** in Volterra integral equation of first kind. To obtained the solution of linear Volterra integral equation of first kind this shows that such that is useful and effective in solving the problems of linear Volterra integral equation of first kind.

REFERENCES

[1] Shudhanshu Aggarwal,Nidhi Sharma,Raman Chauhan; Applicationsof Aboodh Transform for solving linear Volterra integral equations offirst kind, International jouranal of Research in Advent technology, Vol.6, No.12 Dec 2018, PP.3745-3753.

[2] Brunner Hermann Volterra Integral equation: An introduction to theory and Applications, Cambridge monographs on Applied and computational mathematics Cambridge UK, Cambridge University press, 2017.

[3] D.P.Patil, S.S.Khakale,The new integral transform, “Sohamtransform”, International Journal of Advances in Engineering andmanagement, Vol.3, issue 10, Oct. 2021, PP 126-132.

[4] S.R.Kushare, D.P. Patil and A.M. Takate, The new integral transform, “Kushare Transform,’’ International Journal of Advances in Engineering and Management Vol.3, Issue.9 Sept 2019 PP.1589- 1592.

[5] Sudhanshu Aggarwal, Nidhi Sharma, Raman Chauhan, Application ofKamal transform for solving Linear Volterra integral equation of first kind, International Journal of Research in Advant Technology, Vol.6, No.8, Aug 2018, PP 2081-2088.

[6] R.S. Sanap and D.P.Patil, Kushare integral transform for Newtons lawof Cooling, International Journal of Advances in Engineering and management Vol.4,Issue 1, January 2022, PP.166-170.

[7] D.P. Patil, Applications of sawi transform in Bessel Functions, AayushiInternational Interdisciplinary Research Journal Special issue No. 86, PP 171-175.

[8] D.P.Patil, Applications of sawi transform of error function for evaluating improper integrals, Vol.11,Issue.20, June(2021), PP 41-45.

[9] D.P.Patil, Applications of integral transform in chemical sciences, Aayushi International Interdisciplinary Research Journal, Special Issue No. 88,PP 437-441.

[10] D.P.Patil, sawi transform and convolution theorem for initial Boundary Value Problems(wave equation), Journal of Research and Development, Vol.11, Issue 14, June 2021, PP 133-136.

[11] D.P.Patil, Application of Mahgoub transform in parabolic boundary value problems, International Journal of Current AdvancedResearch, Vol.9 Issue 4(c), April 2020 PP 21949-21957.

[12] D.P.Patil, Solution of wave equation by double Laplace and double sumudu transform, Vidyabharti International InterdisciplinaryResearch Journal, Special Issue IV CIMs 2021, Aug. 2021, PP.135-138.

[13] D.P.Patil, Dualities between double integral transforms, International Advanced Journal in Science, Engineering and Technology Vol.7, Issue 6, June 2020, PP 74-82.

[14] S.R.Kushare, D.P.Patil,A.M.Thakare, Comparison between Laplace, Elzaki and Mabgoub transform for solving System of First order and First degree differential equations, Vidyabharti International Interdisciplinary Research Journal, Special Issue IVCIMS 2021, Aug.2021, PP 139-144.

[15] D.P.Patil, Aboodh and Mahgoub transform in boundary Valueproblems of System of ordinary differentral equations, Research in science, community and Technology, Vol.6, Issue 1, June 2021, PP 67-75.

[16] D.P.Patil, P.S.Nikam, S.D. Shirsath, A.T. Aher, Kushare transform for Solving the problems on growth and decay, Journal of Engineering Technologies and Innovative Research (JETIR), Vol.9,Issue.4 April 2022, PP h317-h323.

[17] D.P.Patil, Comparative study of Laplace, Sumudu, Aboodh, Elzaki and mahgoub transform and applications in boundary value problems, International Journal of Research and Analytical Reviews, Vol.5,Issue 4,Dec.2018, PP 22-26.

[18] D.P.Patil, Double mahgoub transform for the solution of parabolic boundary value Problems, Journal of mathematics and statistics, Vol.4, Issue 2, PP 28-36.