IARJSET

ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Impact Factor 7.105 ⁜ Vol. 9, Issue 4, April 2022

DOI: 10.17148/IARJSET.2022.9480

Concepts of Algebra using Maxima A FOSS

Rosemary Varghese[1], Indrani YRL[2]

[1]Assistant Professor, Department of Mathematics, Nrupathunga University, Bengaluru, Karnataka

[2] Assistant Professor, Department of Mathematics, Nrupathunga University, Bengaluru, Karnataka

Abstract: Maxima is one of the free and open source software which is regarded as a computer algebra system. In this paper, we have discussed some of the concepts of algebra and solved some problems using Maxima. It is one of the simple programming which will certainly have its application in other concepts of algebra and is a stepping stone for researchers in algebra. Cumbersome calculations that are very time consuming and iterative in order to find the results can otherwise be obtained in seconds with maxima.

Keywords: Maxima, Free and open source software, Rank, Consistency

1. INTRODUCTION

1.1 Consistency

A system of linear equations, written in the matrix form as AX = B, is consistent if and only if the rank of the coefficient matrix is equal to the rank of the augmented matrix; that is, ρ (A) = ρ ([A : B]).
Algebra is almost confused as being a fancy arithmetic that deals with numbers and variables However, algebra refers to manipulations of more abstract entities. Linear algebra refers to algebraic manipulation of straight lines, vectors, scalars, system of linear equations, and matrices (Basics).
In this section we have discussed the solutions of Homogeneous and Non-Homogeneous system of linear equations that involve various cases namely the trivial and non-trivial solutions with solutions that are unique and infinite.

1.2 Definition : Homogeneous and Non Homogeneous System of Linear Equations

A system of equations in which all the unknown quantities occur in the first degree alone is called a system of linear equations

The system of equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots\dots\dots\dots\dots\dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots\dots\dots\dots\dots\dots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots\dots\dots\dots\dots\dots + a_{3n}x_n &= b_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots\dots\dots\dots\dots\dots + a_{mn}x_n &= b_m \end{aligned} \qquad \dots\dots(1)$$

Where $a_{11}, a_{12}, a_{13} \dots\dots\dots\dots a_{mn}$ and $b_1, b_2, b_3 \dots\dots\dots\dots\dots b_n$ are constants, is called a system of m linear equations in n unknowns.

If $b_1 = b_2 = b_3 \dots\dots\dots = b_n = 0$ in (1) , then the system (1) is said to be homogenuous
If not all $b_1, b_2 \dots\dots\dots, b_m$ are zero then the system(1) is called non homogeneous .
A solution of system of linear equations is the set of values of
$x_1\ x_2, \dots\dots\dots\dots x_n$ which satisfy all the m equations simultaneously

A system of linear equations is said to be consistent if it possesses a solution, otherwise it is said to be inconsistent

The system of equations (1) can be represented in the matrix form as
AX=B

IARJSET ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Impact Factor 7.105 ⋇ Vol. 9, Issue 4, April 2022

DOI: 10.17148/IARJSET.2022.9480

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots\cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} \cdots\cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} \cdots\cdots & a_{mn} \end{pmatrix}_{mXn}$$

$$X = \begin{pmatrix} \mathbf{x_1} \\ \mathbf{x_2} \\ \\ \mathbf{x_n} \end{pmatrix}_{nX1} \qquad B = \begin{pmatrix} \mathbf{b_1} \\ \mathbf{b_2} \\ \\ \mathbf{b_n} \end{pmatrix}_{nX1}$$

The matrix A is called the coefficient matrix of the given system, the matrix X is called the matrix of unknowns and the matrix B is called the matrix of independent terms.

One can clearly see that the homogeneous system has always one set of solutions
$x_1=0$, $x_2=0$, ... $x_n=0$.
That is the homogeneous system is always consistent. This solution is called the trivial solution of the homogeneous system. Any other solution of the homogeneous system is called the non-trivial solution where at least one of the variables is non zero.

1.3 Maxima

Maxima is a symbolic based mathematical software, catering to the needs of the user requiring to perform mathematical calculations including calculus, numerical analysis, systems of linear equations, polynomials, Fourier series and transforms, and various such mathematical models. Maxima can plot functions in two and three dimensions.

The below examples show the working rule of finding the solution of the homogeneous and non-homogeneous system and their corresponding program using maxima.

2. PROBLEMS

Solve the system of non-homogeneous linear equations

2.1 Solve
x+2y+3z=1
9x+18y+30z=3
12x+48y+60z=5
The augmented matrix is

$$(A:B) = \begin{pmatrix} 1 & 2 & 3:1 \\ 9 & 18 & 30:3 \\ 12 & 48 & 60:5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 3:1 \\ 0 & 0 & 3:-6 \\ 0 & 24 & 24:-7 \end{pmatrix} \quad R_2 \to R_2\text{-}9R_1,\ R_3 \to R_3\text{-}12R_3$$

$$\sim \begin{pmatrix} 1 & 2 & 3:1 \\ 0 & 1 & 1:\frac{-7}{24} \\ 0 & 0 & 1:-2 \end{pmatrix} \quad \text{Interchanging } R_2 \text{ and } R_3,\ R_2 \to {}^{R_2}/_{24},\ R_3 \to {}^{R_3}/_{3}$$

Impact Factor 7.105 ⋇ Vol. 9, Issue 4, April 2022

DOI: 10.17148/IARJSET.2022.9480

The rank of the coefficient matrix = the rank of the augmented matrix =number of variables
Unique solution exist
the equivalent system is as follows

x+2y+3z=1
y+z = $\frac{-7}{24}$
z = -2

On back substitution we get x = 43/12 y = 41/24 z = -2

Maxima program :

```
(%i11)  A: matrix(
         [1,2,3],
         [9,18,30],
         [12,48,60]
        );
        b:[1,3,5];
        matsolve(A,b);
(%o9)   matrix(
               [1,    2,    3],
               [9,    18,   30],
               [12,   48,   60]
        )
(%o10)  [1,3,5]
(%o11)  matrix(
               [43/12],
               [41/24],
               [-2]
```

2.2 Solve
x+2y +3z=0
y+5z=0
3x+2y+z=0
x+3z=0

output:

(%o9) [x=0,y=0,z=0]

"The given system has trivial solution "

2.3 Solve
y+x=0
-z-y+x=0
-z+y+3x=0

output:

(%o8) [x=%r7/2,y=-%r7/2,z=%r7]

"The given system has non-trivial that is infinite solution "

2.4 Solve
x+2y-z=3;
3x-y+2z=1;
x-2y+3z=3;
x-y+z=-1;

DOI: 10.17148/IARJSET.2022.9480

output:

(%o11) [x=-1,y=4,z=4]

"The given system has unique solution "

2.5 Solve
-2z+y+x=5
z-2y+x=-2
z+y-2x=4

output:

"The system has no solution"

2.6 Solve
-2z+2y+2x=1
-z+4y+4x=2
2z+6y+6x=3

(%o10) [x=-(2*%r3-1)/2,y=%r3,z=0]

"The given system has infinite solution "

CONCLUSION

Solving for unknowns is one of the basic building blocks and requirement in Mathematics that we often encounter in life. Throughout our discussion we have solved six types of equations and the solutions we obtain depend upon the rank of the coefficient matrix, rank of the augmented matrix and the number of variables.
Applications of Homogeneous and non-homogeneous system of linear equations are practically embedded into our day to day life, be it age related problems, people and work etc.

These simultaneous equations have various approaches to solve for the variables involved.. Here we have solved three equations in three unknowns using the concept of rank of a matrix and its relationship with the number of variables involved.
The same can be extended to n cross n matrix that can be very cumbersome to solve manually and Maxima comes handy in getting us the quick answers irrespective of the number of equations.

REFERENCES:

1. Don Coppersmith Journal: Math. Comp. 62 (1994), 333-350 MSC: Primary 11Y16; Secondary 11-04, 15A06, 15A33
2. MathSciNet review: 1192970
3. Artin, Michael, “Algebra,” Prentice Hall, ISBN 978-0-89871-510-1, 1991.
4. Anton, Howard, “Elementary Linear Algebra,” 5th ed., New York: Wiley, ISBN 0-471-84819-0, 1985
5. Bau III, David, Trefethen, Lloyd N., “Numerical linear algebra, Philadelphia, PA: Society for Industrial and Applied Mathematics,” ISBN 978-0-89871-361-9 , 1995
6. Beauregard, Raymond A., Fraleigh, John B., “A First Course In Linear Algebra: with Optional Introduction to Groups, Rings, and Fields,” Boston: Houghton Mifflin Co., ISBN 0-395-14017-X , 1973.
7. Study on the Linear Algebra and Matrix in Mathematics Snehal Nandkumar Patait, Research Student Prof. Dr. P. G. Sasane, Dept. of Mathematics, P. V. P. College, Patoda H V Geetha, T G Sudha and Harshini Srinivas. “Solution of wave equation by method of separation of variables using Foss Tools Maxima.” In International Journal of Pure and Applied Mathematics, 117, pp.167 – 174. ISSN: 1311-8080, 2017

.