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Rectangular plate subjected to follower edge load

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Abstract: The vibration and buckling characteristics of rectangular plates subjected to non-conservative follower load are studied using finite element analysis. The first order shear deformation theory is used to model the plate.
In the present work, a numerical has been carried out through finite element method. The problem under investigation has been formulated using energy concepts and Hamilton's principle.
The effects of load bandwidth, boundary condition are considered for the stability behavior of the plate. The results show that the position of intermediate load and load bandwidth has a significant effect on the vibration and buckling of the plate.

Keywords: Rectangular plate, Follower load, Finite element method, Buckling load.

I. Introduction

The follower load is a typical example of non-conservative load. The buckling and vibration of beams, plates subjected to conservative loading has its great importance of the design of aerospace, mechanical and civil engineering structures. When a structure is under follower force whose direction changes according to the deformation of the structure, it may undergo static instability (divergence) or dynamic instability (flutter) depending on system parameters, giving rise to unbounded deformation or growth of vibration without bound. There is a considerable number of papers available on non-conservative instability of beams and columns subjected to follower forces. Though many researchers have given importance to study stability characteristics of plates subjected to uniform edge follower forces, it is worth to mention here that such loads are not very common in practice. Many practical situations demand the behavioral aspects of such structural elements under the action of discontinuous/partial edge follower forces with different non-conservative parameters and acting at any intermediate position.
The origin of follower force can be found in an end rocket thrust applied to flexible missiles and aircraft. The first review of this branch of applied mechanics has been made in book form by Bolotin(1963). A lot of interesting results on the effect of follower forces have been drawn on which the results were brought are greatly and sometimes unduly simplified in Sugiyama, (1976). Bolotin (1963) has extensively studied the non-conservative problems of elastic stability. One of the interesting topics in nonconservative stability problems has been the destabilizing effect of damping (Bolotin, 1963).However, investigations on the stability characteristics of the plates under follower loading are relatively few. Follower force is not only caused by jet or rocket thrust. The onset of brake squeal is completely equivalent to the passage through the stability boundary in Beck's column (Nishiwaki, 1993, Mottershead and Chan, 1996).During manufacture/during the service life of thin structural components, it is almost impossible to avoid the existence of microscopic/macroscopic defects. This eventually affects both the static and dynamic behaviour of such components. In the present work, a numerical has been carried out through finite element route to study the buckling, vibration and instability behaviour of plate subjected to non-conservative loads. The problem under investigation has been formulated using energy concepts and Hamilton's principle. The vibration and stability problems basically consist of a set of homogeneous equations leading to Eigen value problems. The geometric stiffness matrix is highly affected by the presence of flaws and the type of loading and hence affects the flutter instability behaviour. In the present investigation an attempt has been made to study stability characteristics of the plates subjected to uniform intermediate follower edge load having direction control. The effects of intermediate load position and structural damping on instability behaviour have also been considered.

II. Mathematical Formulation

Eight nodded curved isoparametric quadratic element is used to model the plate in the present analysis with five degrees of freedom u, v, w, θ_x and θ_y per node. First order shear deformation theory (FSDT) is used.
If the loading is non-conservative, the loss of stability may not show up by the system going into another equilibrium state but by going into unbounded motion. To encompass this possibility one must consider the dynamic behaviour of the system because stability is essentially a dynamic concept. The instability behaviour can be determined by investigating

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the motion of a system that occurs due to some initial perturbation which turns out the subsequent motion due to the initial perturbation consists of oscillations of increasing amplitude, or is a rapidly increasing departure from the equilibrium state, the equilibrium is unstable; otherwise it is stable.

The strain relation consists of two parts: (1) linear strain terms used in the derivation of elastic stiffness matrix and (ii) non-linear strain for geometrical stiffness matrix.

$$\{\varepsilon\} = \{\varepsilon^{L}\} + \{\varepsilon^{NL}\} \quad (1)$$

Large deflection effects are taken into account by including first order non-inearities in the strain- displacement relations (followings von karman theory) as given below:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \quad (2)$$

$$\varepsilon_y = \frac{\partial u}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \quad (3)$$

$$\gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w \partial w}{\partial x \partial y} \quad (4)$$

Where ε_x, ε_y and ε_{xy} are the strain components in the x-y plane and γ_{xy} is the engineering shear strain. The analogous non- linear effects are including in the beam by using an equation like the first of equations.

The finite element equations are obtained via the principle of virtual work.

Using the standard finite element procedure, the expressions for energies can be written in finite form as:

Strain Energy U_t associated with bending and transverse shear is given by

$$U_1 = \frac{1}{2}\{q\}^T [K]\{q\} \quad (5)$$

$$U_1 = \frac{1}{2}\iiint \left[\{\varepsilon_t\}^T [D]\{\varepsilon_t\} \right] dv \quad (6)$$

And the work done by the initial in-plane stresses and non-linear strain

$$U_2 = \frac{1}{2}\iiint \left[\{\sigma_0\}^T \{\varepsilon_{nl}\} \right] dv \quad (7)$$

$$U_2 = \frac{1}{2}\{q\}^T [K_G]\{q\} \quad (8) \qquad U = U_1 + U_2$$

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The Kinetic energy T of the plate can be expressed as

$$T = \frac{1}{2}\{q\}^T [M]\{\dot{q}\} \quad (10)$$

$$T = \frac{1}{2}\rho \iiint \left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right) dv \quad (11)$$

The extended Hamilton principle has been used to formulate the governing equations, considering the non-conservative (follower) forces. The extended Hamilton principle can be expressed as:

$$\delta \int_{t_1}^{t_2} (T - U)dt + \int_{t_1}^{t_2} \delta W_{nc} dt = 0 \quad (12)$$

δw_{nc} = Variation of the work done by the non-conservative forces, which consists of the two parts: follower forces and damping forces.

$$\delta W_{nc} = \delta W_F + \delta W_D \quad (13)$$

Where W_D and W_F are the work done by the damping force and follower force respectively.

$$\delta W_F = \sum_{n=1}^{N} \delta w_n \left(-P_n \, \theta_{yn} \right) \quad (14)$$

where P_n, w_n and θ_{yn} are the force, deflection and rotation about y axis at the node n, N is the total number of nodes.

The follower force matrix $[K_{NC}]$ in equation (13) is given by $[K_{nc}] = [N]^T [P][N]_d$

Substituting energy expression in the equation (1), the following equilibrium equation for the plate is obtained.

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$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}-P\lfloor[K_G]+\lfloor K_{NC}\rfloor\rfloor\{q\}=0 \quad (15)$$

In equation (15) [M] is mass matrix, [C] is damping matrix, [K] is elastic stiffness matrix, [K_G] is geometric stiffness matrix, [K_{NC}} is the non-conservative loading matrix, {q} is nodal displacement vector and P is the magnitude of the applied load. The matrix [K_G] takes into account all the in-plane forces, including the in-plane (conservative) component of the applied load, while the matrix [K_{NC}] takes into account the non-conservative component of the follower load that is in the direction perpendicular to the unreformed mid plane of the plate. The load width (c) is the physical region over which the load is applied.
The modal transformation is performed by means of the first few normal modes of vibration as follows.

$$\omega^2[M]\{q_0\}+[K]\{q_0\}=0 \quad (16)$$

Where ω is the angular natural frequency of vibration and corresponds to the mode shape of free vibration. Equation (12) is solved for the first few modes of vibration by means of a subspace iteration method. The orders of the finite element matrices are very large and the solution of this equation in its original form may be obtained, particularly for determination of the buckling load subjected to follower load. Hence a modal transformation is applied to equation (16) to reduce its size and to retain only the most dominant modes of vibration. Using the modal transformation technique the numbers of equations have been reduced without significant loss of accuracy.
Equation (10) is an eigenvalue problem with eigen values which are the squares of the natural frequencies of free vibration under follower load P. Equation (16) can be solved by using standard eigenvalue routine for a complex general matrix. The imaginary part of ω corresponds to the exponential increment or decrement of the amplitude of vibration. The system is unstable when any of the value ω of equation (10) has a negative imaginary part. Further, if during the transition from stability to instability of the real part of ω is zero then instability occurs due to divergence. Otherwise instability occurs due to flutter.

III. RESULTS AND DISCUSSIONS

To check the validity of the present model, the follower/flutter loads and follower/flutter frequencies are compared with Adali (1982) and Deolasi (1996) for a plate with C-F-S-S case and subjected to uniformly distributed follower force (c/b=1) at the free edge as shown in table 1. The ratio of breadth to thickness (b/t) is 100, poisson's ratio ν of 0.3 and aspect ratio (a/b) 1 and 0.5 are considered. In the discussion, load width ratio (c/b) and position of load (q/a) are used where γ and λ are non-dimensional load and frequency parameters respectively.

Table 1 Comparison of non-dimensional flutter loads γ_{cr} and non-dimensional flutter frequencies λ_{cr} for an isotropic C-F-S-S plate c/b=1.0 and ν = 0.3

Aspect ratio	Flutter load γ_{cr}			Flutter frequency λ_{cr}		
a/b	Present	Adali (1982)	Deolasi (1996)	Present	Adali (1982)	Deolasi (1996)
1.0	51.23	51.968	52.06	16.92	16.67	16.33
0.5	27.04	27.11	27.20	49.32	49.58	49.30

Tables 2 and 3 show numerical results for C-F-F-F and C-F-S-S square plates having b/t=100, subjected to an intermediate follower edge load with different load positions and load width ratios (c/b) and aspect ratios

Table 2 Non-dimensional critical flutter load for C-F-F-F square plate at different intermediate positions for α =1.0

Load width ratio(c/b)	Non-dimensional critical flutter load (γ_{cr})					
	q/a = 1	q/a=0.8	q/a=0.6	q/a=0.5	q/a=0.4	q/a=0.3
0.2	20.85	20.8572	24.32	32.03	28.56	29.98
0.4	20.75	20.31	24.47	32.89	31.15	32.56
0.6	19.75	19.77	24.64	33.85	35.81	36.82

Table 2 shows the variation of frequency with load for direction control parameter α = 0.8 on rectangular plate having an aspect ratio (a/b) of 2 for different load positions. From the figure it is observed that the minimum flutter load occurs at q/a = 0.6 and it the increases as the load position approaches the fixed edge. It shows that the flutter load significantly changes with the aspect ratio and load direction control parameter. From Table 2 it is observed that the critical load (flutter) gradually decreases with load width ratio up to q/a=0.8 and then increases as the load position approaches the

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fixed edge for C-F-F-F boundary condition. From the table it is observed that the minimum flutter load is observed at q/a = 0.8. From Table it is noticed that the for C-F-S-S boundary condition, the critical load increases with c/b ratio for all the load positions. It can be concluded that the critical flutter load vary significantly with load positions.

IV. CONCLUSION

The follower loading on the plate may lead to flutter type of instability which can be observed due to coalescence of the frequencies of two modes into a complex conjugate pair. Flutter is observed to be more common than divergence under follower loading. The load bandwidth, intermediate load position and load direction control parameter are observed to have significant influence on free vibration, flutter characteristics of the plate having different boundary conditions.

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