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A DOUBLE GENERAL INTRGRAL TRANSFORM FOR THE SOLUTION OF PARABOLIC BOUNDARY VALUE PROBLEMS

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Abstract: Boundary value problems arise in several branches of physics, Engineering, Biological sciences, Health sciences and Environmental sciences. The boundary value problems are of three type's elliptics, parabolic and hyperbolic. These problems can be solved by the variety of methods. To solve these boundary value problems recently many researchers are engaged in applying various types of integral transforms. In this paper, we use recently developed integral transform called as double general integral transform to solve the parabolic boundary value problems.

Keywords: Parabolic boundary value problems, One dimensional heat equation, Double general integral transform.

INTRODUCTION:-

The parabolic boundary value problems, that describes the distribution of heat in a given region over certain time interval is heat equation. The energy transferred from one point to another point is heat. Heat flows from a point of higher temperature to the point of lower temperature. For example the boundary value problems which governs the heat flow in a rod. The heat conduction equation may have numerous solutions unless a set of initial and boundary condition. The boundary conditions are mainly of three types 1. Dirichlet condition 2. Neumann condition 3. Mix boundary condition (Robin condition)

In Neumann conditions$\frac{dt}{dn} = 0$ is homogeneous neumann boundary conditions .It is also called as insulated boundary conditions which staes that heat flow is zero. Robin boundary conditions are of the type $k\left(\frac{dt}{dn}\right) + h\,t = g\,(r,t)$ where k and h are constants, implies that the boundary surface dissipates the heat by convection. If g(r.t)= 0 then it is homogeneous Robins boundary conditions which means that the heat converted by dissipation from the boundary surface into a surrounding maintained at zero temperature.

The other boundary conditions such as heat transfer due to radiation obeying the fourth power temperature law and those associated with change of phase, like melting ablation etc. gives rise to nonlinear boundary conditions. Now a day's many researchers are interested to introduced different types of integral transforms like Laplace, Kamal, sadik, Aboodh, tarig , mohand , mahgoub , rishi etc. Recently in September 2021 S. R. Kushare and D.P Patil and Takate [1] introduced Kushare transform . Further in October 2021 S..S. khakale and D. P. Patil [2] developed new integral transform called as Soham transform . D. G. Kakilj [3] introduced double general integral transform in March 2022. Some researchers are engaged in introducing new integral transforms at the same time some are interested in using those in various fields and various problems. In January 2022 D. P. Patil and R.S. Sanap [4] used kushare transform in Newton's law of cooling in April 2022 D. P. Patil [5] used kushare transform for solving the problems on population growth and decay .In oct. 2021 D. P. Patil [6] used sawi transform in Bessel functions. D. P. Patil [7] used sawi transform of error functions for evaluating improper integrals. Further Laplace transform and shehu transform are used in chemical science by D. P. Patil [8]. Sawi transforms and its convolution theorem is used for solving wave equation by D. P. Patil [9]. D. P. Patil [10] also used double laplace and double sum due transform for obtaining the solution of wave equation. Dualities between double integral transforms are obtained by D. P. Patil [11]. Laplace ,Eizaki and mahgoub transforms are used for solving system of first order and first degree differential equations by Kushare, Takate and Patil [12]. D. P. Patil[13] also solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Furthermore, D. P. Patil [14] study laplace ,Sumudh,

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Elzaki and mahgoub transform comparatively and used them to obtain the solution of boundary value problems. Parabolic boundary value problems are solved by D. P. Patil [15] by using double mahgoub transform. D. P. Patil [16] also obtains the solution of parabolic boundary value problems by using mahgoub transform. By using MAPLE, D. P. Patil [17] classified second order partial differential equations as elliptic parabolic, hyperbolic and ultra hyperbolic. D. P. Patil et al [18] used Anuj transform to solve Volterra integral equations of first kind. Soham transform is used to solve same equations by D. P. Patil et al [19] . Rathi sisters and D. P. Patil used Soham transform for system of differential equations [20]. Recently Zankar , Kandekar and D. P. Patil used general integral transform of error function for evaluating improper integrals[21].

2. Preliminaries

2.1. A double general integral transform for the solution of parabolic boundary value problems

Definition Of double general integral transform :

Let (t) be an integrable function defined for $t \geq 0$, $(s) \neq 0$ and (s) are positive real valued functions then the general integrable transform (s) of $f(t)$ is defined by

$\{f(t)\} = p\,()\int_0^\infty f(t)\, e^{-q(s)t}$dt provided that the integral exists.

Formulae for New general integral transform

Function $f(t)$	New integral transform $T(f(t)) = \mathcal{T}(s)$
1	$\frac{p(s)}{q(s)}$
T	$\frac{p(s)}{(q(s))^2}$
t^α	$\frac{\Gamma(\alpha+1)\,p(s)}{q(s)^{\alpha+1}}$, $\alpha > 0$
Sin t	$\frac{p(s)}{(q(s))^2+\alpha^2}$
sinαt	$\frac{\alpha p(s)}{(q(s))^2+\alpha^2}$
cost	$\frac{p(s)q(s)}{(q(s))^2+1}$
e^t	$\frac{p(s)}{q(s)-1}$,$q(s) > 1$
f'(t)	$q(s)\mathcal{T}(s) - p(s)f(0)$

2.2. Double general integral transform

Definition of the Double general integral transform:

Let $f(x, y)$ be an integrable function defined for the variables x and y in the first quadrant $p_1(s) \neq 0$, $p_2(s) \neq 0$ and$q_1(s)$, $q_2(s)$ are positive real functions; we define the Double general integral transform $T_2\{f(x, y)\}$ by the formula

$T_2\{$ f(x, y)$\}= (s)=p_1(s)p_2(s)\int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y)} f(x, y)$ dx dy

Provided that the integral exists for some $q_1(\;)$, $q_2(s)$

2.3. Properties of Double general integral transform:

a) Linearity property:

$T_2\{$ a f(x, y) +b g(x, y)$\}$=a$T_2\{$ f(x, y)$\}$ + b $T_2\{$ g(x, y)$\}$

b) Shifting property:

IfT_2 $\{(x,\ )\}=\mathcal{T}$ $(s\ )$then

$T_2\{\ e^{-(ax+by)}$f(x, y)$\}=\mathcal{T}$ $(s,\ a,\ b)$=$p_1\ (s)p_2$(s)$\int_0^\infty \int_0^\infty e^{-((q_1(s)+a)x+(q_2(s)+b)y)}\ f(x,y)$ dx dy

c) Change of scale property:

If T_2 $\{(x,\ )\}=\mathcal{T}$ $(s\ )$ then T_2 $\{f(x, y\ )\}$ $=\frac{1}{ab}\mathcal{T}$ $(s,\ a,\ b)$

2.4. Formulae of some elementary functions

In this section we shall derive some formulae for some elementary functions by using double general integral transform.

Function $f(x,y)$	Double general integral transform $T_2\{f(x, y\ )\}$
1	$\frac{p_1\ (s)p_2(s)}{q_1(s)\ q_2(s)}$
$exp(ax+by)$	$\frac{p_1\ (s)p_2(s)}{(q_1(s)-a)\ (q_2(s)-b)}$
$exp(i\ (ax+by))$	$\frac{p_1\ (s)p_2(s)}{(q_1(s)-ia)\ (q_2(s)-ib)}$
$cosh(ax+by)$	$\frac{1}{2}[\frac{p_1(s)p_2(s)}{(q_1(s)-a)\ (q_2(s)-b)}+\frac{p_1(s)p_2(s)}{(q_1(s)+a)\ (q_2(s)+b)}]$
$sinh(ax+by)$	$\frac{1}{2}[\frac{p_1(s)p_2(s)}{(q_1(s)-a)\ (q_2(s)-b)}-\frac{p_1(s)p_2(s)}{(q_1(s)+a)\ (q_2(s)+b)}]$
$cos(ax+by)$	$\frac{1}{2}[\frac{p_1(s)p_2(s)}{(q_1(s)-ia)\ (q_2(s)-ib)}+\frac{p_1(s)p_2(s)}{(q_1(s)+ia)\ (q_2(s)+ib)}]$
$sin(ax+by)$	$\frac{1}{2}[\frac{p_1(s)p_2(s)}{(q_1(s)-ia)\ (q_2(s)-ib)}-\frac{p_1(s)p_2(s)}{(q_1(s)+ia)\ (q_2(s)+ib)}]$
$(xy)^n$, $n>0$	$\frac{(\Gamma(n+1))^2 p_1\ (s)p_2(s)}{(q_1(s)\ q_2(s))^{n+1}}$
$x^m y^n$, $m>0$, $n>0$	$\frac{\Gamma\ (m+\ 1)\Gamma\ (n+\ 1)(p_1\ (s)p_2(s)}{(q_1(s))^{m+1})(q_2(s))^{n+1}}$

2.5. Useful theorems

In this section we state some useful theorems.

Theorem 1: Let $(x,\ y)$ be a function of two variables. If the first ordered partial derivative $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$exists and f(0, y) be given.$p_1(s)$, $p_2(s)$, $q_1(s)andq_2(s)$ are positive real functions then

$T_2\{\frac{\partial f}{\partial x}(x,y)\}=-p_1(s\ )T\{f(0,y)\}+q_1(s)T_2\{f(x,y)\}$

where, $T\{f\ (0,\ y)\}$ is the new general integral transform of the $f(0,\ y)$.

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Theorem 2: Let (x, y) be a function of two variables. If the first ordered partial derivative $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists and f$(x$,0) be given. p_1(s), $p_2(s)$, $q_1(s) and q_2(s)$ are positive real functions then

$T_2\{\frac{\partial f}{\partial y}(x, y)\} = -p_2(s)\, T\{f(x,0)\} + q_2(s) T_2\{f(x, y)\}$

where, $T\{f(x,0)\}$ is the new general integral transform of the $f(x,0)$.

3. Applications

Now we will state and prove following theorems.

Theorem 3: Let (x, y) be a function of two variables. If the first and second ordered partial derivative $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ are exists and f(0,y), f_x(0, y) be given.

p_1(s), $p_2(s)$, $q_1(s) and q_2(s)$ are positive real functions then

$T_2\{\frac{\partial^2}{\partial x^2} f(x, y)\} = -p_1(s)\, [T\{f_x(0, y)\} + q_1(s) T\{f(0, y)\}] + q_1(s)^2 T_2\{f(x, y)\}$

where, $T\{f_x(0, y)\}$, $T\{f(0, y)\}$ is the new general integral transform of the f_x(0, y), f (0, y) respectively

Proof: Applying transform on second order partial derivative with respect to x

$$T_2\{\frac{\partial^2}{\partial x^2} f(x, y)\} = T_2\{\frac{\partial}{\partial x} f_x(x, y)\}$$

$$\therefore\ T_2\{\frac{\partial^2}{\partial x^2} f(x, y)\} = -p_1(s)\, T\{f_x(0, y)\} + q_1(s) T_2\{f_x(x, y)\} \text{...........\{from above theorem 1\}}$$

$$\therefore T_2\{\frac{\partial^2}{\partial x^2} f(x, y)\} = -p_1(s)\, T\{f_x(0, y)\} + q_1(s)[-p_1(s)\, T\{f(0, y) + q_1(s) T_2\{f(x, y)\}] \text{....\{ from theorem 1\}}$$

$$\therefore\ T_2\{\frac{\partial^2}{\partial x^2} f(x, y)\} = -p_1(s)\, [T\{f_x(0, y)\} + q_1(s) T\{f(0, y)\}] + q_1(s)^2 T_2\{f(x, y)\}$$

Theorem 4: Let (x, y) be a function of two variables. If the first and second ordered partial derivative $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ are exists and f(x,0), f_x(0, y) be given.

p_1(s), $p_2(s)$, $q_1(s) and q_2(s)$ are positive real functions then

$T_2\{\frac{\partial^2}{\partial y^2} f(x, y)\} = -p_2(s)\, [T\{f_y(x,0)\} + q_2(s) T\{f(x,0)\}] + q_2(s)^2 T_2\{f(x, y)\}$

where, $T\{f_y(x,0)\}$, $T\{f(x,0)\}$ is the new general integral transform of the f_y(x,0), f (x,0) respectively

Proof: Applying transform on second order partial derivative with respect to y

$$T_2\{\frac{\partial^2}{\partial y^2} f(x, y)\} = T_2\{\frac{\partial}{\partial y} f_y(x, y)\}$$

$$\therefore\ T_2\{\frac{\partial^2}{\partial y^2} f(x, y)\} = -p_2(s)\, T\{f_y(x,0)\} + q_2(s) T_2\{f_y(x, y)\} \text{...........\{from above theorem 2\}}$$

$$\therefore T_2\{\frac{\partial^2}{\partial x^2} f(x, y)\} = -p_2(s)\, T\{f_y(x,0)\} + q_2(s)[-p_2(s)\, T\{f(x,0) + q_2(s) T_2\{f(x, y)\}] \text{....\{ from theorem 2\}}$$

$$\therefore\ T_2\{\frac{\partial^2}{\partial x^2} f(x, y)\} = -p_2(s)\, [T\{f_y(x,0)\} + q_2(s) T\{f(x,0)\}] + q_2(s)^2 T_2\{f(x, y)\}$$

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3.1. Some useful formule: Following table contain formulae of transforms of elementary functions

Function $f(t)$	New integral transform $T(f(t)) = \mathcal{T}(s)$
e^{-t}	$\frac{p(s)}{q(s)+1}$
$e^{-\propto t}$	$\frac{p(s)}{q(s)+\propto}$
$e^{\propto t}$	$\frac{p(s)}{q(s)-\propto}$, $q(s) > \propto$

4. Application of transform on boundary value problems

In this section we will solve some boundary value problems by using transform

Example 1. Consider the heat equation, $u_t = u_{xx} + \sin x$, $t > 0$

With conditions $u(0,t) = e^{-t}$; $u(x,0) = \cos x$; $u_x(0,t) = 1 - e^{-t}$

Solution: *Let* $u_t = u_{xx} + \sin x$, $t > 0$

Applying double general integral transform ,we get

$T_2\{u_t\} = T_2\{u_{xx} + \sin x\}$

$\therefore \quad T_2\{u_t\} = T_2\{u_{xx}\} + T_2\{\sin x\}$

$\therefore \quad T_2\{\frac{\partial u}{\partial t}(x,t)\} = T_2\{\frac{\partial}{\partial x}u_x(x,t)\} + T_2\{\sin x\}$

$\therefore \quad -p_2(s)\, T\{u(x,0)\} + q_2(s)T_2\{u(x,t)\} = -p_1(s)\, T\{u_x(0,t)\} + q_1(s)T_2\{u_x(x,t)\} + T_2\{\sin x\}$

$\therefore \quad -p_2(s)\, T\{u(x,0)\} + q_2(s)T_2\{u(x,t)\} = -p_1(s)\, T\{u_x(0,t)\} + q_1(s)[-p_1(s)\, T\{u(0,t)\}$
$+ q_1(s)T_2\{u(x,t)\}] + T_2\{\sin x\}$

$\therefore \quad q_2(s)T_2\{u(x,t)\} - [q_1(s)]^2 T_2\{u(x,t)\} = -p_1(s)\, T\{u_x(0,t)\} + p_2(s)\, T\{u(x,0)\}$
$-p_1(s)q_1(s)T\{u(0,t)\} + T_2\{\sin x\}$

$\therefore \quad T_2\{u(x,t)\}[q_2(s) - (q_1(s))^2] = -p_1(s)\, T\{1 - e^{-t}\} + p_2(s)\, T\{\cos x\}$
$+T_2\{\sin x\} - p_1(s)q_1(s)T\{e^{-t}\}$

$\therefore \quad T_2\{u(x,t)\}[q_2(s) - (q_1(s))^2] = -p_1(s)[T\{1\} - T\{e^{-t}\}] + p_2(s)\, T\{\cos x\}$
$+T_2\{\sin x\} - p_1(s)q_1(s)T\{e^{-t}\}$ ....(1)

Now, $T\{1\} = \frac{p_2(s)}{q_2(s)}$, $T\{e^{-t}\} = \frac{p_2(s)}{1+q_2(s)}$, $T\{\cos x\} = \frac{p_1(s)q_1(s)}{(q_1(s))^2+1}$,

$T_2\{\sin x\} = \frac{p_1(s)p_2(s)}{q_2(s)[(q_1(s))^2+1]}$

Equation (1) implies,

$$\therefore \quad T_2\{u(x,t)\}[q_2(s) - (q_1(s))^2] = -p_1(s)[\frac{p_2(s)}{q_2(s)} - \frac{p_2(s)}{1+q_2(s)}] + p_2(s)[\frac{p_1(s)q_1(s)}{(q_1(s))^2+1}] + \frac{1}{q_2(s)}\left[\frac{p_1(s)p_2(s)}{(q_1(s))^2+1}\right] - p_1(s)q_1(s)\frac{p_2(s)}{1+q_2(s)}$$

$$\therefore \quad T_2\{u(x,t)\}[q_2(s) - (q_1(s))^2] = -p_1(s)[\frac{p_2(s)}{q_2(s)+(q_2(s))^2}] + p_2(s)[\frac{p_1(s)q_1(s)}{(q_1(s))^2+1}] + \frac{1}{q_2(s)}\left[\frac{p_1(s)p_2(s)}{[(q_1(s))^2+1]}\right] - p_1(s)q_1(s)[\frac{p_2(s)}{1+q_2(s)}]$$

$$\therefore \quad T_2\{u(x,t)\}[q_2(s) - (q_1(s))^2] = \frac{-p_1(s)p_2(s)}{q_2(s)+(q_2(s))^2} + \frac{p_1(s)p_2(s)q_1(s)}{(q_1(s))^2+1} + \frac{p_1(s)p_2(s)}{q_2(s)[(q_1(s))^2+1]} - p_1(s)q_1(s)[\frac{p_2(s)}{1+q_2(s)}]$$

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$$\therefore \quad = p_1(s)p_2(s)\left[\frac{1}{q_2(s)\left[(q_1(s))^2+1\right]} - \frac{1}{q_2(s)+(q_2(s))^2}\right] + p_1(s)p_2(s)q_1(s)\left[\frac{1}{(q_1(s))^2+1} - \frac{1}{1+q_2(s)}\right]$$

$$= \frac{p_1(s)p_2(s)}{q_2(s)}\left[\frac{q_2(s)-q_1(s)^2}{(q_2(s)+1)\left[(q_1(s))^2+1\right]}\right] + p_1(s)p_2(s)q_1(s)\left[\frac{q_2(s)-q_1(s)^2}{(q_2(s)+1)\left[(q_1(s))^2+1\right]}\right]$$

$T_2\{u(x,t)\}\,[\,q_2(s) - (q_1(s))^2] =$

$$[q_2(s) - q_1(s)^2]\{[\frac{p_1(s)p_2(s)}{q_2(s)(q_2(s)+1)\left[(q_1(s))^2+1\right]}]+[\frac{p_1(s)p_2(s)q_1(s)}{(q_2(s)+1)\left[(q_1(s))^2+1\right]}]\}$$

$$T_2\{u(x,t)\} = [\frac{p_1(s)p_2(s)}{q_2(s)(q_2(s)+1)\left[(q_1(s))^2+1\right]}]+[\frac{p_1(s)p_2(s)q_1(s)}{(q_2(s)+1)\left[(q_1(s))^2+1\right]}]$$

$$T_2\{u(x,t)\} = [\frac{p_1(s)p_2(s)}{\left[(q_1(s))^2+1\right]}][\frac{1}{1+q_2(s)}]\frac{1}{q_2(s)}+[\frac{p_1(s)q_1(s)}{\left[(q_1(s))^2+1\right]}][\frac{p_2(s)}{1+q_2(s)}] \;\text{.......(2)}$$

By partial fraction,

$$\frac{1}{[1+q_2(s)]}\frac{1}{q_2(s)} = \frac{1}{q_2(s)} - \frac{1}{1+q_2(s)}$$

Equation (2) becomes

$$T_2\{u(x,t)\} = [\frac{p_1(s)}{\left[(q_1(s))^2+1\right]}][\frac{p_2(s)}{q_2(s)} - \frac{p_2(s)}{1+q_2(s)}]+[\frac{p_1(s)q_1(s)}{\left[(q_1(s))^2+1\right]}][\frac{p_2(s)}{1+q_2(s)}]$$

$$\Rightarrow \quad \boxed{u(x,t)=[\sin x][1 - e^{-t}] + [\cos x]e^{-t}}$$

Example2. Consider the heat equation, $u_t = u_{xx}$, t > 0

with conditions u(0,t) =0 ; u(x,0)=sin x ; $u_x(0,t)=e^{-t}$

Solution: *Let* $u_t = u_{xx}$, t > 0

Applying double general integral transform, we get

$T_2\{u_t\} = T_2\{u_{xx}\}$

$T_2\{\frac{\partial u}{\partial t}(x,t)\} = T_2\{\frac{\partial}{\partial x}u_x(x,t)\}$

$-p_2(s)\,T\{u(x,0)\} + q_2(s)T_2\{u(x,t)\} = -p_1(s)\,T\{u_x(0,t)\} + q_1(s)T_2\{u_x(x,t)\}$

$-p_2(s)\,T\{u(x,0)\} + q_2(s)T_2\{u(x,t)\} = -p_1(s)\,T\{u_x(0,t)\} + q_1(s)[-p_1(s)\,T\{u(0,t) + \quad q_1(s)T_2\{u(x,t)\}]$

$\therefore \quad T_2\{u(x,t)\}[q_2(s)-q_1(s)^2] = -p_1(s)\,T\{u_x(0,t)\}+p_2(s)\,T\{u(x,0)\} - p_1(s)q_1(s)T\{u(0,t)\}$

$\therefore \quad T_2\{u(x,t)\}[q_2(s)-q_1(s)^2] = -p_1(s)\,T\{e^{-t}\}+p_2(s)\,T\{\sin x\} - p_1(s)q_1(s)T\{0\}\text{.........(1)}$

Now we know that,

$\therefore \quad T\{e^{-t}\} = \frac{p_2(s)}{1+q_2(s)},\; T\{\sin x\} = \frac{p_1(s)}{q_1(s)^2+1} \;, T\{0\} = 0$

Equation (1) becomes,

$\therefore \quad T_2\{u(x,t)\}[q_2(s)-q_1(s)^2] = -p_1(s)\,(\frac{p_2(s)}{1+q_2(s)})+p_2(s)\,(\frac{p_1(s)}{q_1(s)^2+1})$

$\therefore \quad T_2\{u(x,t)\}[q_2(s)-q_1(s)^2] = p_1(s)p_2(s)\,[(\frac{1}{1+q_2(s)})-(\frac{1}{q_1(s)^2+1})]$

$\therefore \quad T_2\{u(x,t)\}[q_2(s)-q_1(s)^2] = p_1(s)p_2(s)\,[\frac{q_2(s)-q_1(s)^2}{(q_1(s)^2+1)(1+q_2(s))}]$

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$\therefore \quad T_2\{u(x,t)\} = [\frac{p_1(s)p_2(s)}{(q_1(s)^2+1)(1+q_2(s))}]$

$\therefore \quad T_2\{u(x,t)\} = [\frac{p_1(s)}{q_1(s)^2+1}][\frac{p_2(s)}{1+q_2(s)}]$

$\Rightarrow \quad \boxed{u(x,t) = e^{-t}\sin x}$

Example 3. Consider the heat equation, $u_t = u_{xx} - 3u + 3$, t > 0

with conditions u(0,t) =1 ; u(x,0)=1+ sin x ; $u_x(0,t) = e^{-4t}$

Solution: *Let* $u_t = u_{xx} - 3u + 3$, t > 0

Applying double general integral transform, we get

$T_2\{u_t\} = T_2\{u_{xx} - 3u + 3\}$

$T_2\{u_t\} = T_2\{u_{xx}\} - 3T_2\{u\} + T_2\{3\}$ ……{by linearity property}

$T_2\{\frac{\partial u}{\partial t}(x,t)\} = T_2\{\frac{\partial}{\partial x}u_x(x,t)\} - 3T_2\{u(x,t)\} + 3T_2\{1\}$

$-p_2(s)\,T\{u(x,0)\} + q_2(s)T_2\{u(x,t)\} =$

$-p_1(s)\,T\{u_x(0,t)\} + q_1(s)T_2\{u_x(x,t)\} - 3T_2\{u(x,t)\} + 3T_2\{1\}$

$-p_2(s)\,T\{u(x,0)\} + q_2(s)T_2\{u(x,t)\} =$

$-p_1(s)\,T\{u_x(0,t)\} + q_1(s)[-p_1(s)\,T\{u(0,t)\} + q_1(s)T_2\{u(x,t)\}] - 3T_2\{u(x,t)\} + 3T_2\{1\}$

$\therefore \quad T_2\{u(x,t)\}[q_2(s) - q_1(s)^2 + 3] = -p_1(s)\,T\{u_x(0,t)\} + p_2(s)\,T\{u(x,0)\} - p_1(s)q_1(s)T\{u(0,t)\} + 3T_2\{1\}$

$\therefore \quad T_2\{u(x,t)\}[q_2(s) - q_1(s)^2 + 3] = -p_1(s)\,T\{e^{-4t}\} + p_2(s)\,T\{1+\sin x\} - p_1(s)q_1(s)T\{1\} + 3T_2\{1\}$…..(1)

Now we know that , $T\{e^{-4t}\} = \frac{p_2(s)}{4+q_2(s)}$, $T_2\{1\} = \frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}$,

For u (0, t)=1 $\Rightarrow T\{1\} = \frac{p_2(s)}{q_2(s)}$

$\{1+\sin x\} = \{1\} + T\{\sin x\}$ …….{by linearity property}

$\{1+\sin x\} = \frac{p_1(s)}{q_1(s)} + \frac{p_1(s)}{q_1(s)^2+1}$

Equation (1) becomes,

$\therefore \quad T_2\{u(x,t)\}[q_2(s) - q_1(s)^2 + 3] =$

$-p_1(s)(\frac{p_2(s)}{4+q_2(s)}) + p_2(s)(\frac{p_1(s)}{q_1(s)} + \frac{p_1(s)}{q_1(s)^2+1}) - p_1(s)q_1(s)(\frac{p_2(s)}{q_2(s)}) + 3\frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}$

$\therefore \quad T_2\{u(x,t)\}[q_2(s) - q_1(s)^2 + 3] =$

$-\frac{p_2(s)\,p_1(s)}{4+q_2(s)} + \frac{p_1(s)p_2(s)}{q_1(s)} + \frac{p_2(s)p_1(s)}{q_1(s)^2+1} - \frac{p_2(s)p_1(s)q_1(s)}{q_2(s)} + 3\frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}$

$\therefore \quad T_2\{u(x,t)\}[q_2(s) - q_1(s)^2 + 3] =$

$p_2(s)\,p_1(s)[\frac{-1}{4+q_2(s)} + \frac{1}{q_1(s)^2+1}] + \frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}[\,q_2(s) - q_1(s)^2 + 3]$

$\therefore \quad T_2\{u(x,t)\}[q_2(s) - q_1(s)^2 + 3] =$

$p_2(s)\,p_1(s)[\frac{q_2(s) - q_1(s)^2 + 3}{(q_1(s)^2+1)(4+q_2(s))}] + \frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}[\,q_2(s) - q_1(s)^2 + 3]$

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$\therefore \quad T_2\{u(x,t)\}[q_2(s)-q_1(s)^2+3]=$

$\left[\frac{p_2(s)\,p_1(s)}{(q_1(s)^2+1)(4+q_2(s))}+\frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}\right][\,q_2(s)-q_1(s)^2+3]$

$\therefore \quad T_2\{u(x,t)\}=\frac{p_2(s)\,p_1(s)}{(q_1(s)^2+1)(4+q_2(s))}+\frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}$

$\therefore \quad T_2\{u(x,t)\}=\frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}+\left(\frac{p_2(s)}{4+q_2(s)}\right)\left(\frac{p_1(s)}{q_1(s)^2+1}\right)$

$\Rightarrow \quad \boxed{u(x,t)=1+e^{-4t}(\sin x)}$

CONCLUSION

We succesfully used **A double general integral transform** for the solution of parabolic boundary value problems. To obtained the solution of parabolic boundary value problems of heat equations this shows that such that is useful and effective in solving the parabolic boundary value problems.

REFERNCES

1) S.R. Kushare, D. P. Patil and A.M.Takate, The new integral transform “Kushare Transform”, International Journal of Advances in Engineering and Management ,Vol.3, Issue- 9 ,Sept- 2021,PP .1589-1592
2) D. P. Patil and S. S. Khakale , The new integral transform ,”Soham Transform”, International Journal of Advances in Engineering and Management ,Vol.3, Issue- 10 , Oct- 2021,PP. 126-132
3) D. G. Kaklij, A double new general integral transform, International Journal of Advances in Engineering and Management ,Vol.4, Issue-3 ,Mar- 2022,PP. 1249-1258
4) R.S. Sanap and D. P. Patil , Kushare integral Transform for Newton’s law of cooling , International Journal of Advances in Engineering and Management ,Vol.4, Issue- 1 , Jan- 2022,PP .166-170
5) D. P. Patil , P. S. Nikam , S. D. Shirsath and A. T. Aher , Kushare Transform for solving problems on growth and decay ,Journal of Engineering technologies and Innovate Research ,Vol.9, Issue- 4 ,April- 2022,PP .h317-h323
6) D. P. Patil, Application of Sawi Transform in Bessel functions ,Ayushi International Interdisciplinary Research Journal, Special issue no.86,PP. 171-175
7) D. P. Patil, Application of Sawi Transform of error function for evaluating Improper integral , Journal of Research and Development ,Vol.11, Issue- 20 ,June- 2021,PP. 41-45
8) D. P. Patil, Application of integral Transform (Laplace and Shehu) in Chemical Sciences, Ayushi International Interdisciplinary Research Journal, Special issue no.88,PP. 437-441
9) D. P. Patil, Sawi Transform and Convolution Theorem for initial boundary value problem (wave equation), Journal of Research and Development ,Vol.11, Issue- 14 ,June- 2021,PP .133-136
10) D. P. Patil, Solution of Wave equation by double Lapalce and double Sumudu Transform,Vidyabharati International Interdisciplinary Research Journal, Special issue IVCIMS 2021,Aug-2021,PP. 135-138
11) D. P. Patil, Dualities between double integral transform, International Advanced Journal in Science, Engineering and Technology, Vol.7, Issue- 14 ,June- 2021,PP. 133-136
12) S. R. Kushare , D. P. Patil and A. M. Takate ,Comparison between Laplace ,Elzaki and Mahagoub Transform for solvind system of first order and first degree differential equations , Vidyabharati International Interdisciplinary Research Journal, Special issue IVCIMS 2021,Aug-2021,PP. 139-144
13) D. P. Patil, Aboodh and Mahagoub transform in boundary value problems of system of ordinary differential equations , International Journal of Advanced Research In science, Communication and Technology, Vol.6, Issue- 1 ,Jan- 2021,PP .67-75
14) D. P. Patil, Comparative study of Laplace ,Sumudu, Aboodh,Elzaki and Mahagoub transfoms and Applications in Boundary value problems, International Journal of Research and Analytical Reviews, Vol.5, Issue- 4 ,Dec- 2018
15) D. P. Patil, Double mahagoub transform for the solution of parabolic boundary value problems, Journal of Engineering mathematics and statistics , Vol.4, Issue- 2 ,May- 2020
16) D. P. Patil, Application of Mahagoub Transform in Parabolic boundary value problems, International Journal of Current Advanced Research, Vol.9, Issue- 4(c) , April- 2020,PP. 21941-21951

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DOI: 10.17148/IARJSET.2022.9612

17) D. P. Patil, Classification of second order Partial differential equations using maple, International Journal of Latest Technology in Engineering , Management and Applied Sciences, , Vol.7, Issue- 1 ,Jan- 2018,PP. 135-143.
18) D. P. Patil, P. D. Shinde and G. K. Tile, Volterra integral equations of first kind by using Anuj transform, International Journal of Advences in Engineering and Management, Vol. 4, Issue 5 , May 2022, pp. 917-920.
19) D. P. Patil, Y. S. Suryawanshi and M. D. Nehete, Application of Soham transform for solving Volterra integral equations of first kind, International Advanced Research Journal in Science , Engineering and Technology, Vol. 9, Issue 4, April2022.
20) Dinkar P. Patil, Shweta D. Rathi and Shrutika D. Rathi, The new integral transform " Soham transform" for system of differential equations, International Journal of Advences in Engineering and Management, Vol. 4, Issue 5, May 2022, pp. 1675-1678.
21) D. P. Patil, K. S. Kandekar and T. V. Zankar, Application of general integral transform of error function for evaluating improper integrals, International Journal of Advences in Engineering and Management, Vol. 4, Issue 6, June 2022.

