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Applications of Emad-Sara Transform for General Solution of Telegraph Equation

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Abstract: Recently, integral transform are of very much useful for solving variety of problems. Many researchers are now engaged in developing various integral transforms. This paper is devoted to obtain the general solution of one dimensional hyperbolic telegraph equation of second order using Emad-Sara transform.

Keywords: Telegraph equation, Emad Sara transform, Integral transforms, second order one dimensional hyperbolic equation.

1. INTRODUCTION:

One dimensional second order hyperbolic telegraph equation was formulated by using Ohm's law. This equation was solved by a recent and reliable semi analytic method. The telegraph equation had been introduced by Oliver Heaviside in 1876. Telegraph equation is linear partial differential equation which describes relationship between the transmitted voltage and current with time and distance. This equation describes the reflection of the electromagnetic waves in the transmission medium. Many researchers solved this equation by using various integral transforms. Till now many researchers have developed several integral transforms Laplace, Kamal, Sadik, Elzaki, Aboodh, Mohand, Rishi, Tarig , Mahgoub Emad-Sara, Emad-Falih transforms. Integral transforms are very much useful to solve many advanced problems of science and engineering such as Radioactive decay problems, Heat conduction problems, problems of motion of the particle under gravity, Vibration of beam, problems in electric circuits, etc. Integral transforms are best tool to solve the ordinary, partial as well as fractional differential equations.

Recently, S.R Kushare and D. P. Patil [1] introduce Kushare transform in September 2021.In October 2021, S.S.Khakale and D. P. Patil [2] introduce Soham transform. As researchers are going introducing new integral transforms at the same time many researchers are interested to apply these transforms to various types of problems. In January 2022,R.S.Sanap and D. P. Patil [3] used Kushare transform to solve the problems based on Newton's law of cooling. In April 2022 D. P. Patil etc. [4] use Kushare transform to solve the problems on growth and decay. In October 2021 D. P. Patil [5] used Sawi transform in Bessel function. D. P. Patil [6] used Sawi transform of error function for evaluating improper integral further, Lpalce and Shenu transforms are used in chemical science by D. P. Patil [7]. Dr. Patil [8] solved the wave equation by Sawi transform and its convolution theorem. Further Patil [9] also used Mahgoub transform for solving parabolic boundary value problems.

Dr. Patil [10] obtains solution of the wave equation by using double Laplace and double Sumudu transform. Dualities between double integral transforms are derived by D. P. Patil [11]. Laplace, Elzaki, and Mahgoub transforms are used for solving system of first order and first degree differential equations by Kushare and Patil [12].Boundary value problems of the system of ordinary differentiable equations are by using Aboodh and Mahgoub transform by D. P. Patil [13]. D. P. Patil [14] study Laplace, Sumudu, Elzaki and Mahgoub transforms comparatively and apply them in Boundary value problems. Parabolic Boundary value problems are also solved by Dinkar Patil [15]. For that he used double Mahgoub transform.
Soham transform is used to obtain the solution of system of differential equations by D. P. Patil et al [16]. D. P. Patil et al also used Soham transform for solving Volterra integral equations of first kind [17]. D. P. Patil et al [20] used Anuj transform to solve Volterra integral equations of first kind. Soham transform is used to solve same equations by D. P. Patil et al [21] . Rathi sisters and D. P. Patil used Soham transform for system of differential equations [22]. Recently Zankar , Kandekar and D. P. Patil used general integral transform of error function for evaluating improper integrals[23]. Recently, Dinkar Patil, Prerana Thakare and Prajakta Patil [24] used double general integral transform for obtaining the solution of parabolic boundary value problems
This paper is organized as follows. Second section is used for preliminaries. Emad- Sara transform is used to solve telegraph equations in third section. Fourth section is devoted for conclusion.

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2. PRELIMINARIES:

In this section we state some preliminary concepts which are required for solving telegraph equation, which contains definition of telegraph equation, Emad Sara transform, Emad Sara transform of some fundamental functions, Emad Sara transform of derivative of functions.

2.1. The Telegraph Equation: [18]

The general form of Telegraph Equation is:

$$\frac{\partial^2 u}{\partial t^2} + (\alpha + \beta)\frac{\partial u}{\partial t} + \alpha\beta u = c^2 \frac{\partial^2 u}{\partial x^2}.$$

Where $u(x,t)$ can be voltage or current through the wire at position x and time $t, \alpha = \frac{G}{C}, \beta = \frac{R}{L}$ and $c^2 = \frac{1}{LC}$, where G is conductance of resistor, R is resistance of resistor, L is inductance of coil, and C is capacitance of capacitor.

2.2. Emad – Sara Integral Transform:[19]

The Emad – Sara integral transform is defined for an exponential order function in the set B as:

$B = \{f(t): \exists\, K, m_1, m_2 > 0, |f(t)| < Ke^{m_j|t|}\ if\ t \in (-1)^j X[0,\infty)\}(1)$

Where, $f(t)$ is a function in the B set, K is a finite constant number, m_1 and m_2 may or may not be finite.

The kernel function of Emad-Sara integral transform symbolized by ES(.) is defined by the integral equation:

$ES\{f(t)\} = T(\alpha) = \frac{1}{\alpha^2}\int_0^\infty e^{-\alpha t} f(t)dt(2)$

Where $t \geq 0, m_1 \leq \alpha \leq m_2$ and α is a variable that is used as a factor to the variable t in the function f.

In Emad – Sara integral transform, t in which is an argument of the function f is factored by the transform variable.

2.3. Emad-Sara integral transform for some fundamental functions:[19]

Sr. No.	Functions $f(t) = ES^{-1}\{T(\alpha)\}$	Emad-Sara Transform $T(\alpha) = ES\{f(t)\}$
1	1	$\frac{1}{\alpha^3}$
2	k (a constant)	$\frac{k}{\alpha^3}$
3	t	$\frac{1}{\alpha^4}$
4	t^n	$\frac{n!}{\alpha^{n+3}}$
5	e^{at}	$\frac{1}{\alpha^2(\alpha - a)}$ $(a\ is\ constant)$
6	$sinat$	$\frac{a}{\alpha^2(\alpha^2 + a^2)}$
7	$cosat$	$\frac{1}{\alpha(\alpha^2 + a^2)}$
8	$sinhat$	$\frac{a}{\alpha^2(\alpha^2 - a^2)}$
9	$coshat$	$\frac{1}{\alpha(\alpha^2 - a^2)}$

2.4. Emad-Sara Transform of derivative functions:[19]

$Let\ T(\alpha)\ is\ the\ Emad - Sara\ integral\ transform, where\ ES\{f(t)\} = T(\alpha)$

- $ES\{f'(t)\} = \frac{-f(0)}{\alpha^2} + \alpha T(\alpha)$
- $ES\{f''(t)\} = \frac{-f'(0)}{\alpha^2} + \alpha ES\{f'(t)\}$
- $ES\{f^{(n)}(t)\} = \frac{-f^{(n-1)}(0)}{\alpha^2} + \alpha ES\{f^{(n-1)}(t)\}$

2.5. Emad-Sara Transform of Partial Derivative functions:[19]

$Let\ T(x,\alpha)\ be\ the\ Emad - Sara\ transform\ where\ ES\{f(x,t)\} = T(x,\alpha), then:$

- $ES\left\{\frac{\partial f}{\partial t}(x,t)\right\} = \alpha T(x,\alpha) - \frac{f(x,0)}{\alpha^2}$

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- $ES\left\{\frac{\partial^2 f}{\partial t^2}(x,t)\right\} = \alpha^2 T(x,\alpha) - \frac{1}{\alpha^2}\frac{\partial f}{\partial t}(x,0) - \frac{1}{\alpha} f(x,0)$
- $ES\left\{\frac{\partial f}{\partial x}\right\} = \frac{\partial}{\partial x}\{T(x,\alpha)\}$
- $ES\left\{\frac{\partial^2 f}{\partial x^2}\right\} = \frac{\partial^2}{\partial x^2}\{T(x,\alpha)\}$
- $ES\left\{\frac{\partial^2 f}{\partial x \partial t}\right\} = \frac{\partial}{\partial x}\left[ES\left\{\frac{\partial f}{\partial t}\right\}\right]$

3. SOLVING TELEGRAPH EQUATIONS USING EMAD-SARA TRANSFORM:

In this section we solve telegraph equations by using Emad-Sara transform.

Telegraph Equation (1): Consider telegraph equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} + u\,,$$

with initial conditions,

$$u(x,0) = e^x\,, u_t(x,0) = -2e^x$$

To solve this telegraph equation, we use Emad-Sara transform as follows:

$$ES\left\{\frac{\partial^2 u}{\partial x^2}\right\} = ES\left\{\frac{\partial^2 u}{\partial t^2}\right\} + 2ES\left\{\frac{\partial u}{\partial t}\right\} + ES\{u(x,t)\}.$$

$$T''(x,\alpha) - \left[\alpha^2 T(x,\alpha) - \frac{1}{\alpha^2}u_t(x,0) - \frac{1}{\alpha}u(x,0)\right] - 2\left[\alpha T(x,\alpha) - \frac{1}{\alpha^2}u(x,0)\right] - T(x,\alpha) = 0\,.$$

By applying the Emad-Sara integral transform to the initial conditions of the equation:

$$T''(x,\alpha) - \alpha^2 T(x,\alpha) + \frac{e^x}{\alpha} - \frac{2e^x}{\alpha^2} - 2\alpha T(x,\alpha) + \frac{2e^x}{\alpha^2} - T(x,\alpha) = 0\,.$$

$$T''(x,\alpha) - (\alpha^2 + 2\alpha + 1)T(x,\alpha) + \frac{e^x}{\alpha} = 0\,.$$

$$T''(x,\alpha) - (\alpha+1)^2 T(x,\alpha) = \frac{-e^x}{\alpha}$$

By using Differential Operator $D = \frac{\partial}{\partial x}$;we get,

$$[D^2 - (\alpha+1)^2]T(x,\alpha) = \frac{-e^x}{\alpha}$$

$$T(x,\alpha) = \frac{-e^x}{\alpha(D^2 - (\alpha+1)^2)}$$

By using the following known theorem,

$If\ D = \frac{d}{dx}\ and\ f(D) is\ polynomial\ in\ D\ with\ constant\ coefficient\ then\ \frac{1}{f(D)}e^{ax} = \frac{e^{ax}}{f(a)}$
$if\ f(a) \neq 0.$

$$\therefore T(x,\alpha) = \frac{-e^x}{\alpha\big((1 - (\alpha^2 + 2\alpha + 1)\big)}$$

$$T(x,\alpha) = \frac{-e^x}{\alpha - \alpha^3 - 2\alpha^2 - \alpha}$$

$$= \frac{-e^x}{-\alpha^3 - 2\alpha^2} = \frac{-e^x}{-(\alpha^2(\alpha+2))}$$

Therefore, $T(x,\alpha) = \frac{e^x}{\alpha^2(\alpha+2)}$

The function $u(x,t)$ could be obtained using inverse Emad-Sara integral transform as:

$$ES^{-1}\{T(x,\alpha)\} = e^x ES^{-1}\left\{\frac{1}{\alpha^2(\alpha+2)}\right\}$$

By Emad-Sara integral transform, $ES\left\{\frac{1}{\alpha^2(\alpha+2)}\right\} = e^{-2t}$, $as\ ES\{e^{at}\} = \frac{1}{\alpha^2(\alpha-a)}$

$$\therefore u(x,t) = e^x . e^{-2t} = e^{x-2t}$$

which is required Solution.

Telegraph equation (2): Consider

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}+4\frac{\partial u}{\partial t}+4u\,,$$

with initial conditions:

$$u(x,0)=1+e^{2x},\qquad u_t(x,0)=-2.$$

To solve this telegraph equation, we apply the Emad-Sara integral transform as follows:

$$ES\left\{\frac{\partial^2 u}{\partial x^2}\right\}=ES\left\{\frac{\partial^2 u}{\partial t^2}\right\}+4ES\left\{\frac{\partial u}{\partial t}\right\}+4ES\{u(x,t)\}$$

$$T''(x,\alpha)-\left[\alpha^2T(x,\alpha)-\frac{1}{\alpha^2}u_t(x,0)-\frac{1}{\alpha}u(x,0)\right]-4\left[\alpha T(x,\alpha)-\frac{1}{\alpha^2}u(x,0)\right]-4T(x,\alpha)=0$$

By applying the Emad-Sara integral transform to the initial conditions of the equation:

$$\mathrm{T}''(\mathrm{x},\alpha)-\alpha^2\mathrm{T}(\mathrm{x},\alpha)+\frac{1}{\alpha}(1+\mathrm{e}^{2\mathrm{x}})-\frac{2}{\alpha^2}-4\alpha\mathrm{T}(\mathrm{x},\alpha)+\frac{4}{\alpha^2}(1+\mathrm{e}^{2\mathrm{x}})-4\mathrm{T}(\mathrm{x},\alpha)=0$$

$$\mathrm{T}''(\mathrm{x},\alpha)-(\alpha^2+4\alpha+4)\mathrm{T}(\mathrm{x},\alpha)=\frac{-2-\alpha-\alpha\mathrm{e}^{2\mathrm{x}}-4\mathrm{e}^{2\mathrm{x}}}{\alpha^2}$$

$$\mathrm{T}''(\mathrm{x},\alpha)-(\alpha+2)^2\mathrm{T}(\mathrm{x},\alpha)=\frac{-2}{\alpha^2}-\frac{1}{\alpha}-\left(\frac{\alpha+4}{\alpha^2}\right)\mathrm{e}^{2\mathrm{x}}$$

By using differential operator, $\frac{\partial}{\partial x}=D$,

$$[\mathrm{D}^2-(\alpha+2)^2]\mathrm{T}(\mathrm{x},\alpha)=\frac{-2}{\alpha^2}-\frac{1}{\alpha}-\left(\frac{\alpha+4}{\alpha^2}\right)\mathrm{e}^{2\mathrm{x}}$$

$$\mathrm{T}(\mathrm{x},\alpha)=\frac{1}{[\mathrm{D}^2-(\alpha+2)^2]}\left(\frac{-2}{\alpha^2}\right)-\frac{1}{[\mathrm{D}^2-(\alpha+2)^2]}\left(\frac{1}{\alpha}\right)-\frac{1}{[\mathrm{D}^2-(\alpha+2)^2]}\left(\left(\frac{\alpha+4}{\alpha^2}\right)\mathrm{e}^{2\mathrm{x}}\right)$$

$$T(x,\alpha)=\frac{1}{-(\alpha+2)^2\left[1+\frac{D^2}{(\alpha+2)^2}\right]}\left(\frac{-2}{\alpha^2}\right)-\frac{1}{-(\alpha+2)^2\left[1+\frac{D^2}{(\alpha+2)^2}\right]}\left(\frac{1}{\alpha}\right)-\left(\frac{\alpha+4}{\alpha^2}\right)\frac{\mathrm{e}^{2\mathrm{x}}}{[\mathrm{D}^2-(\alpha+2)^2]}$$

By using, expansion of $\frac{1}{1+x}=1-x+x^2-x^3+\cdots$

We can write,

$$\frac{1}{\left[1+\frac{D^2}{(\alpha+2)^2}\right]}=1-\frac{D^2}{(\alpha+2)^2}+\left[\frac{D^2}{(\alpha+2)^2}\right]^2-\cdots$$

Then operating differential operator D on given function, where $=\frac{d}{dx}$, We get,

$$T(x,\alpha)=\frac{1}{-(\alpha+2)^2}\left[\left(1-\frac{D^2}{(\alpha+2)^2}\right)\left(-\frac{2}{\alpha^2}\right)\right]+\frac{1}{(\alpha+2)^2}\left[\left(1-\frac{D^2}{(\alpha+2)^2}\right)\left(\frac{1}{\alpha}\right)\right]-\frac{(\alpha+4)e^{2x}}{\alpha^2(4-(\alpha^2+4\alpha+4))}$$

$$\therefore T(x,\alpha)=\frac{1}{-(\alpha+2)^2}\left(-\frac{2}{\alpha^2}\right)+\frac{1}{(\alpha+2)^2}\left(\frac{1}{\alpha}\right)-\frac{(\alpha+4)e^{2x}}{\alpha^2(4-\alpha^2-4\alpha-4)}$$

$$\therefore T(x,\alpha)=\left[\frac{2}{\alpha^2(\alpha+2)^2}+\frac{1}{\alpha(\alpha+2)^2}\right]-\frac{(\alpha+4)e^{2x}}{-\alpha^3(\alpha+4)}=\frac{2+\alpha}{\alpha^2(\alpha+2)^2}+\frac{(\alpha+4)e^{2x}}{\alpha^3(\alpha+4)}$$

$$T(x,\alpha)=\frac{(\alpha+2)}{\alpha^2(\alpha+2)^2}+\frac{(\alpha+4)e^{2x}}{\alpha^3(\alpha+4)}$$

$$T(x,\alpha)=\frac{1}{\alpha^2(\alpha+2)}+\frac{e^{2x}}{\alpha^3}$$

The function $u(x,t)$ could be obtained using inverse Emad-Sara integral transform as:

$$ES^{-1}\{T(x,\alpha)\}=ES^{-1}\left\{\frac{1}{\alpha^2(\alpha+2)}\right\}+e^{2x}ES^{-1}\left\{\frac{1}{\alpha^3}\right\}$$

$$As\ we\ know,ES\left\{\frac{1}{\alpha^2(\alpha+2)}\right\}=e^{-2t}\ and\ ES\left\{\frac{1}{\alpha^3}\right\}=1\quad \dots By\ Emad-Sara\ integral\ transform$$

$$u(x,t)=e^{-2t}+e^{2x}$$

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4. CONCLUSION:

Thus Emad-Sara transform is successfully used to obtain the solution of Telegraph equation easily. The solution is correct as compared to the solution obtained by using different types of integral transforms.

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