IARJSET ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

ISO 3297:2007 Certified ⋇ Impact Factor 7.105 ⋇ Vol. 9, Issue 6, June 2022

DOI: 10.17148/IARJSET.2022.96118

Applications of Double General Integral Transform for Solving Boundary Value Problems in Partial Differential Equations

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Abstract: In this paper we use double general integral transform to solve boundary value problems in partial differential equations.

Key words: Boundary value problems, double general integral transform, Partial Differential Equations.

1. INTRODUCTION:

Recently, integral transforms are one of the mostly used simple mathematical tools to obtain the solutions of advance problems of space, science, technology, engineering, commerce and economics. The important feature of these integral transform is to provide exact solution of the problem without lengthy calculations.

Recently, S.R Kushare and D. P. Patil [1] introduce Kushare transform in September 2021.In October 2021, S. S. Khakale and D. P. Patil [2] introduce Soham transform. Emad Kuffi et al [19] developed Emad Sara transform. As researchers are going introducing new integral transforms at the same time many researchers are interested to apply these transforms to various types of problems. In January 2022, R .S. Sanap and D. P. Patil [3] used Kushare transform to solve the problems based on Newton's law of cooling. In April 2022 D. P. Patil et al. [4] use Kushare transform to solve the problems on growth and decay. In October 2021 D. P. Patil [5] used Sawi transform in Bessel function. D. P. Patil [6] used Sawi transform of error function for evaluating improper integral further, Lpalce and Shenu transforms are used in chemical science by D. P. Patil [7]. Dr. D. P. Patil [8] solved the wave equation by Sawi transform and its convolution theorem. Further Patil [9] also used Mahgoub transform for solving parabolic boundary value problems.

Dr. Patil [10] obtains solution of the wave equation by using double Laplace and double Sumudu transform. Dualities between double integral transforms are derived by D. P. Patil [11]. Laplace, Elzaki, and Mahgoub transforms are used for solving system of first order and first degree differential equations by Kushare and Patil [12].Boundary value problems of the system of ordinary differentiable equations are by using Aboodh and Mahgoub transform by D. P. Patil [13]. D. P. Patil [14] study Laplace, Sumudu, Elzaki and Mahgoub transforms comparatively and apply them in Boundary value problems. Parabolic Boundary value problems are also solved by Dinkar Patil [15]. Soham transform is used to obtain the solution of system of differential equations by D. P. Patil et al [16]. D. P. Patil et al also used Soham transform for solving Volterra integral equations of first kind [17]. D. P. Patil et al [18] used Anuj transform to solve Volterra integral equations of first kind. Emad Sara transform is used to solve telegraph equation by D. P. Patil et al [19]. D. P. Patil, Sonal Borse and Darshana Kapadi, Applications of Emad-Falih transform for general solution of telegraph equation [20]. Recently, Zankar, Kandekar and D. P. Patil used general integral transform of error function for evaluating improper integrals [21]. Recently, Dinkar Patil, Prerana Thakare and Prajakta Patil [22] used double general integral transform for obtaining the solution of parabolic boundary value problems. Snehal Patil, Komal patil and Dinkar Patil [23] used Emad Falih transform for Newton’s law of Cooling. D. P. Patil et al [24] used Soham transform for solving Newton’s law of cooling. Further, HY transform is used to handling exponential growth and decay problems by Areen Shaikh, Neha More, Jaweria Shaikh and Dinkar Patil [25]. Bachhav, Gangurde, Wagh and Patil [26] used HY transform for solving problems based on Newton’s law of cooling.

In this paper we use Emad Sara transform to solve the boundary value problems in partial differential equations. This paper is organized as follows. Second section is used for preliminaries. Double general integral transform is used to solve boundary value problems in partial differential equations in third section. Fourth section is devoted for conclusion

International Advanced Research Journal in Science, Engineering and Technology

ISO 3297:2007 Certified ⁘ Impact Factor 7.105 ⁘ Vol. 9, Issue 6, June 2022

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2. PRELIMINARY:

2.1. Definition of the Double New General Integral Transform [27] :

Let $f(x,y)$ be an intrgrable function defined for the variables x and y in the first quadrant with $p_1(s) \neq 0$, $p_1(s) \neq 0$ and $q_1(s), q_2(s)$are positive real functions; the Double general integral transform $T_2\{f(x,y)\}$ is defined as follows

$T_2\{f(x,y)\} = \tau(s) = p_1(s)p_2(s) \int_0^\infty \int_0^\infty e^{-(q_1(s)x+q_2(s)y} f(x,y)\, dx\, dy$

provided that the integral exists for some $q_1(s), q_2(s)$.

2.2. Properties of double general integral transform:

a) **Linearity Property:[27]**

$T_2\{af(x,y) + bg(x,y)\} = aT_2\{f(x,y)\} + bT_2\{g(x,y)\}$

b) **Shifiting Property** :[27]

If $T_2\{f(x,y)\} = \mathcal{T}(s)$ then $T_2\{e^{-(ax+by)} f(x,y) = \mathcal{T}(s,a,b)$

c) **Change Of Scale Property**:[27]

If $T_2\{f(x,y) = \mathcal{T}(s)$ then $T_2\{f(ax,by)\} = \frac{1}{ab}\mathcal{T}(s,a,b)$

d) Let $F_D(s,r)$ be the general double transform of the function $f(x,y)$ and let $F_G(0,r)$be the general transform of the function$f(x,y)$ .Then ,

(i) $T_2\left\{\frac{\partial f(x,y)}{\partial}\right\} = \varphi(s)F_D(s,r) - p(s)F_G(0,r)_{\varphi(s)}$

(ii) $T_2\left\{\frac{\partial^2 f(x,y)}{\partial x^2}\right\} = \varphi^2(s)F_D(s,r) - \varphi(s)p(s)F_G(0,r) - p(s)\frac{\partial F_G(0,r)}{\partial x}$,

(iii) $T_2\left\{\frac{\partial^2 f(x,y)}{\partial x^2}\right\} = \varphi^2(s)F_G(s,r) - p(s)\sum_{k=0}^{n-1} \varphi^{n-1-k}(s)\frac{\partial^k F_G(0,r)}{\partial x^k}$

e) Let $F_D(s,r)$ and $F_G(s,0)$ be the general double transforms of the functions $f(x,y)$ and $f(x,0)$ respectively then

(i) $\mathcal{T}_2\left\{\frac{\partial f(x,y)}{\partial y}\right\} = \psi(r)F_D(s,r) - q(r)F_G(s,0)$

(ii) $\mathcal{T}_2\left\{\frac{\partial^2 f(x,y)}{\partial y^2}\right\} = \psi^2(r)F_D(s,r) - \psi(r)q(r)F_G(s,0) - q(r)\frac{\partial F_G(s,0)}{\partial y}$,

(iii) $\mathcal{T}_2\left\{\frac{\partial^2 f(x,y)}{\partial y^n}\right\} = \psi^n F_D(s,r) - q(r)\sum_{k=0}^{n-1} \psi^{n-1-k}(r)\frac{\partial^k F_G(s,0)}{\partial y^k}$.

2.3. **Double Integral Transformation Formulas For Elementary Functions: [27]**

Function $f(x,y)$	Double General Integral Transform $T_2\{f(x,y)\}$
1	$\frac{p_1(s)p_2(s)}{q_1(s)q_2(s)}$
$\exp(ax+by)$	$\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)}$
$\exp(i(ax+by))$	$\frac{p_1(s)p_1(s)}{(q_1(s)-ia)(q_2(s)-ib)}$
$\cosh(ax+by)$	$\frac{1}{2}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)} + \frac{p_1(s)p_2(s)}{(q_1(s)+a)(2(s)+b)}\right)$
$\sinh(ax+by)$	$\frac{1}{2}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)} - \frac{p_1(s)p_2(s)}{(q_1(s)+a)(q_2(s)+b)}\right)$

IARJSET ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

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DOI: 10.17148/IARJSET.2022.96118

$\cos(ax+by)$	$\frac{1}{2}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)}+\frac{p_1(s)p_2(s)}{(q_1(s)+ia)(q_2(s)+ib)}\right)$
$\sin(ax+by)$	$\frac{1}{2i}\left(\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)}-\frac{p_1(s)p_2(s)}{(q_1(s)+ia)(q_2(s)+ib)}\right)$
$(xy)^n \quad ,n>0$	$\frac{(\Gamma(n+1))^2 p_1(s)p_2(s)}{(q_1(s)q_2(s))^2}$
$x^m y^n \quad ,m>0,n>0$	$\frac{\Gamma(m+1)\Gamma(n+1)p_1(s)p_2(s)}{(q_1(s))^{m+1}(q_2(s)^{n+1}}$

3. APPLICATION:

In this section we solve some boundary value problems by using double general integral transform.

Problem (1) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

with conditions,

$u(x,b)=0\,,\quad u(a,y)=0,\quad u(0,y)=0$ and $u(x,0)=f(x)$

Solution:

We apply double general integral transform to both sides of the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

$$T_2\left\{\frac{\partial^2 u}{\partial x^2}\right\}+T_2\left\{\frac{\partial^2 u}{\partial y^2}\right\}=0$$

$$\therefore T_2\left\{\frac{\partial^2 u}{\partial x^2}\right\}+T_2\left\{\frac{\partial^2 u}{\partial y^2}\right\}$$

$$=\varphi^2(s)v_D(s,r)-(s).p(s)v_G(0,r)-p(s)\frac{\partial}{\partial x}v_G(0,r)+\psi^2 v_D(s,r)-\psi(r).q(r)v_G(s,0)-\quad q(r)\frac{\partial}{\partial x}v_G(s,0)$$

$$=v_D(s,r)[\varphi^2(s)-\psi^2(r)]$$

$$=\varphi(s).p(s)v_G(0,r)-p(s)\frac{\partial}{\partial x}v_G+\psi(r)q(r)v_G(s,0)-q(r)\frac{\partial}{\partial x}v_G(s,0)=0$$

$$v_D(s,r)=\frac{\left[\varphi(s).pS)v_G(0,r)-p(s)\frac{\partial}{\partial x}v_G(0,r)+\psi(r)q(r)v_G(s,0)-q(r)\frac{\partial}{\partial x}v_G(s,0)\right]}{\varphi^2(s)-\psi^2(r)} \qquad \text{....(1)}$$

By using general transformation to initial transformation

$$\boldsymbol{v_G}(s,0)=T_X(f(x))$$

$$\boldsymbol{v_G}(a,r)=0$$

$$\boldsymbol{v}_G(s,b)=0$$

$$\boldsymbol{v_G}(0,r)=0$$

Putting these values in equation (1) we get,

$$\boldsymbol{v}_D(s,r)=\frac{\psi(r)q(r)T_x\left[f_0(x)-q(r)T_x[f(x)]\right]}{\varphi^2(s)-\psi^2(r)}$$

$$\boldsymbol{u}(x,y)=T_2^{-1}\left\{\frac{\psi(s)q(r)T_x(f_0(x)-q(r)T_x[T_xf(x)])}{\varphi^2(s)-\psi^2(r)}\right\}$$

Problem (2) $\nabla^2\boldsymbol{u}=\mathbf{0}$

with boundary Conditions: $u(0,y)=0\,,\ u(a,y)=0,\,u(x,b)=0$ and $u(x,0)=T_x(x-a)$

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Solution:

We solve the equation $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \mathbf{0}$

Applying double general integral transform to above equation,

$$T_2\left\{\frac{\partial^2 u}{\partial x^2}\right\} + T_2\left\{\frac{\partial^2 u}{\partial y^2}\right\} = 0$$

$$\therefore T_2\left\{\frac{\partial^2 u}{\partial x^2}\right\} + T_2\left\{\frac{\partial^2 u}{\partial y^2}\right\}$$

$$= \varphi^2(s)v_D(s,r) - (s).p(s)v_G(0,r) - p(s)\frac{\partial}{\partial x}v_G(0,r) + \psi^2 v_D(s,r) - \psi(r).q(r)v_G(s,0) - q(r)\frac{\partial}{\partial x}v_G(s,0) = 0$$

$$\therefore v_D(s,r)[\varphi^2(s) - \psi^2(r)]$$

$$= \varphi(s).p(s)v_G(0,r) - p(s)\frac{\partial}{\partial x}v_G + \psi(r)q(r)v_G(s,0) - q(r)\frac{\partial}{\partial x}v_G(s,0)$$

$$\therefore v_D(s,r) = \frac{\left[\varphi(s).pS)v_G(0,r) - p(s)\frac{\partial}{\partial x}v_G(0,r) + \psi(r)q(r)v_G(s,0) - q(r)\frac{\partial}{\partial x}v_G(s,0)\right]}{\varphi^2(s) - \psi^2(r)} \quad \ldots\ldots \quad (1)$$

Using general transform to the initial conditions and the boundary conditions, we get

$$v_G(s,o) = T_x(x-a)$$
$$v_G(a,r) = 0$$
$$v_G(s,b) = 0$$
$$v_G(0,r) = 0$$

Using these values in equation (1), we get

$$v_G(s,r) = \frac{\psi(r)q(r)T_x(x-a) - q(r)T_x(1)}{\varphi^2(s) - \psi^2(r)}$$

$$u(x,y) = T_2^{1-}\left\{\frac{\psi(r)q(r)T_x(x-a) - q(r)T_x(1)}{\varphi^2(s) - \psi^2(r)}\right\}$$

4. CONCLUSION:

Double general integral transform is successfully applied to boundary value problems in partial differential equations.

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