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Some Generalized Normed Spaces Characteristics

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Abstract: In this paper, some characteristics of generalized metric spaces are presented, which will help many researchers to prove the results when working within groups volume.

Key words: normed space, inner product space, characteristic.

MSC2010: 46A40; 46J10, 60E10.

1. INTRODUCTION

In this work, we will focus on n-normed and n -inner generated spaces, two topics that are central to the field of functional analysis, which was first developed at the turn of the 19th century and finally established in the 1920s and 1930s.

Gahler offered a fascinating n-norm theory on linear spaces in [4]. Numerous authors, including Kim et al. [9], Malceski [11], Misiak [12], and Gunawan [5], have developed linear n-normed spaces systematically. As of late, Kritiantoo et al. [10] have researched the equivalence of n-norms in n-normed spaces. In a linear n-Banach space, n-norms must satisfy certain conditions in order to be fully equivalent, and that is what this work aims to achieve. current research on the functional analysis parts we're referring to ([1,2], [6-8], [13,15]).

According to Chen et al. [3], the extended parallelogram law is a necessary and sufficient condition for an n-normed space (n-Ns) to be an n-inner product space as follows:

$$\frac{1}{2}\|u+v,\alpha_2,\dots,\alpha_n\|^2+\|u-v,\alpha_2,\dots,\alpha_n\|^2=(\|u,\alpha_2,\dots,\alpha_n\|^2+\|v,\alpha_2,\dots,\alpha_n\|^2) \quad (1.1)$$

such that the n−inner product (n-Es) space for all $u,v,\alpha_2,\dots,\alpha_n$ is introduced by

$$4\langle u,v|\alpha_2,\dots,\alpha_n\rangle=(\|u+v,\alpha_2,\dots,\alpha_n\|-\|u-v,\alpha_2,\dots,\alpha_n\|). \quad (1.2)$$

Soenjaya [16] consider a relation (1.2) as the attribution of n-Es

We require the following definitions for this work:

Definition 1.1. [2] A real-valued function $\langle\cdot,\cdot\,|\,\cdot,\dots,\cdot\rangle$ on U^{n+1} satisfied the following properties:

nI 1: $\langle u_1,u_1|u_2,\dots,u_n\rangle\geq 0$ and $\langle u_1,u_1|u_2,\dots,u_n\rangle=0$,

if and only if $u_1,u_2\dots,u_n$ are linearly dependent.

nI 2: $\langle u_1,u_1|u_2,\dots,u_n\rangle=\langle u_{i_1},u_{i_1}|u_{i_2},\dots,u_{i_n}\rangle$, for any permutation $(i_1,\dots,i_n)$ of $(1,\dots,n)$.

nI 3: $\langle \acute{u}_1,u_1|u_2,\dots,u_n\rangle=\langle u_1,\acute{u}_1|u_2,\dots,u_n\rangle$,

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nI 4: $\langle \sigma u_1, u_1 | u_2, \dots, u_n\rangle = \sigma\langle u_1, u_1 | u_2, \dots, u_n\rangle$, for every $\sigma \in \mathbb{R}$.

nI 5: $\langle u_0 + \acute{u}_0, u_1 | u_2, \dots, u_n\rangle = \langle u_0, u_1 | u_2, \dots, u_n\rangle + \langle \acute{u}_0, u_1 | u_2, \dots, u_n\rangle$.

is called n-E on a vector space U. The pair $(U, \langle \cdot,\cdot \mid \cdot, \dots, \cdot\rangle)$ is called an n- Es

Definition 1.2. [2] Let X be a real vector space of $\dim \geq n$. An n–norm on U is a mapping $\|\cdot, \dots, \cdot\| : U^n \to \mathbb{R}$, which satisfies the following four conditions:

nN 1: $\|u_1, \dots, u_n\| = 0$, if and only if $u_1, \dots, u_n$ are linearly dependent,

nN 2: $\|u_1, \dots, u_n\| = \|u_{i_1}, \dots, u_{i_n}\|$, for every permutation $(i_1, \dots i_n)$ of $(1, \dots, n)$,

nN 3: $\|\sigma u_1, \dots, u_n\| = |\sigma| \|u_1, \dots, u_n\|$ for $\sigma \in \mathbb{R}$,

nN 4: $\|u_1 + \acute{u}_1, u_2, \dots, u_n\| \leq \|u_1, u_2, \dots, u_n\| + \|\acute{u}_1, u_2, \dots, u_n\|$,

for all $u_1, \acute{u}_1, u_2, \dots, u_n \in U$. The pair $(U, \|\cdot, \dots, \cdot\|)$ is called an n–normed space (n-N).

2. MAIN RESULTS

Our main result states as:

Theorem 2.1. A characterization of n- Es using n-Ns on $\mathbb{C}$, for every $u, v, u_2, \dots, u_n \in U$ are:

i. $$2(\|u, u_2, \dots, u_n\|^2 - \|v, u_2, \dots, u_n\|^2) = \|x + iy, u_2, \dots, u_n\|^2 + \|x - iy, u_2, \dots, u_n\|^2, \quad (2.1)$$

ii. $$8\langle u, v | u_2, \dots, u_n\rangle = \begin{pmatrix} \|u + v, u_2, \dots, u_n\|^2 - \|u - v, u_2, \dots, u_n\|^2 \\ + \\ i(\|u + iv, u_2, \dots, u_n\|^2 - \|u - iv, u_2, \dots, u_n\|^2) \end{pmatrix}. \quad (2.2)$$

Proof.

To prove equation (2.1) of (i),

$$\|u + iv, u_2, \dots, u_n\|^2 + \|u - iv, u_2, \dots, u_n\|^2$$

$$= \langle u + iv, x + iy | u_2, \dots, u_n\rangle + \langle u - iv, u - iv | u_2, \dots, u_n\rangle$$

$$= \langle u, u | u_2, \dots, u_n\rangle - 2i\langle u, v | u_2, \dots, u_n\rangle - \langle v, v | u_2, \dots, u_n\rangle$$
$$+$$
$$\langle u, u | u_2, \dots, u_n\rangle + 2i\langle u, v | u_2, \dots, u_n\rangle - \langle y, y | u_2, \dots, u_n\rangle$$

$$= 2\langle u, u | u_2, \dots, u_n\rangle - 2\langle v, v | u_2, \dots, u_n\rangle$$

$$= 2(\|u, u_2, \dots, u_n\|^2 - \|v, u_2, \dots, u_n\|^2).$$

To prove equation (2.2) of (ii),

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$$\frac{1}{8}\begin{pmatrix} \|u+v,u_2,\dots,u_n\|^2 - \|u-v,u_2,\dots,u_n\|^2 \\ + \\ i(\|u+iv,u_2,\dots,u_n\|^2 - \|u-iv,u_2,\dots,u_n\|^2) \end{pmatrix}$$

$$= \frac{1}{8}\big(4\langle u,v|u_2,\dots,u_n\rangle + i(-4i\langle u,v|u_2,\dots,u_n\rangle)\big)$$

$$= \langle u,v|u_2,\dots,u_n\rangle.$$

CONCLUSION

In order to make it easier for many researchers to demonstrate the results while operating inside the volume of the spaces, a few characteristics of generalized metric regions are described in this study.

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