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Analysis of Spread of Crime: A Socio-Mathematical Study

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Abstract: For the analysis of spreading of the crime, a non-linear model has been suggested as well as stability analysis is performed in this paper. Under normal situations crime is modelled as a contagion spread of infectious disease. Here, the modelling is done by dividing the total population into non-susceptible, susceptible, criminal and jailed populations and the connection between them is given by simultaneous ordinary nonlinear differential equations. The interaction between them plays an important role and hence care has been taken while modelling. The threshold parameter R_0 for the elimination of criminal activity is enumerated using the approach of the next generation matrix. Analysis indicates that when $R_0 < 1$, criminal activity vanishes, while the reverse $R_0 > 1$ indicates the endurance of criminal activity in the society. Numerical simulation along with the stability analysis is performed. However, various investigators have already worked in this field, but still there are vital possibilities for further research in this arena in the future.

Key Words: Crime, Equilibria, Mathematical Model, Numerical Simulation, Stability Analysis.

I.INTRODUCTION

Our society is going through numerous troubles in the modern-day and crime is one of the major troubles amongst them. Gross violation of law, such as armed robbery, rapes, killings, burning of houses, theft etc., is the action of crime, done by few individuals known as criminals [1]. Because of numerous reasons, crime is increasing constantly worldwide as well as has occupied the transcontinental form [2]. The interaction between non-criminal and criminal active population can lead to crime. These interactions provoke non-criminals to participate in criminal activities [3], [4]. Furthermore, criminal activities have been intact to numerous situations such as unemployment, inequality and payments [5]. Involvement that is constantly related to crime is punishment. Punishments such as imprisonment or sending to rehabilitation centres reduce the criminals to perpetrate crime [6]. Human behaviour is inherently unpredictable, so both crime and criminal behaviour may be represented by a nonlinear system suitably [7]. The best described model for crime is a socially infectious disease model.

Using ordinary differential equations [8]–[12] and partial differential equations [13]–[16] many mathematical model for crime have been formulated and analysed over the last few years as cited above. In addition, other mathematical techniques like [17], [18] has been proposed. Same Susceptible-Infected-Recovery (SIR) model used in this paper have been used to study many social problem like [19]–[22]. In most of the crime studies cited above, neither the interaction between non-susceptible and criminals is considered nor counselling the criminals has been taken into account in the process of modelling.

Therefore in present article a non-linear model has been proposed through dividing the total population into: (i) non-susceptible population; (ii) susceptible population; (iii) criminal population and (iv) jailed population. Further the following sections have been provided in this paper for detailed study - Mathematical Model of the system, different equilibria, stability analysis and numerical simulation.

II.MATHEMATICAL MODEL

Mathematical model has been developed by considering a region with total population (T) and it is further divided into four categories (N, S, C & J) who vary on the basis of their lifestyle. N represents non-susceptible to a life of crime, S denotes susceptible, individuals

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who haven't steadfast the crimes till date, however are vulnerable towards criminal activity, C denotes active criminals, who have been active in numerous illegal activities. J denotes prisoners, who are basically criminals residing in the jails as a punishment to the crimes committed.

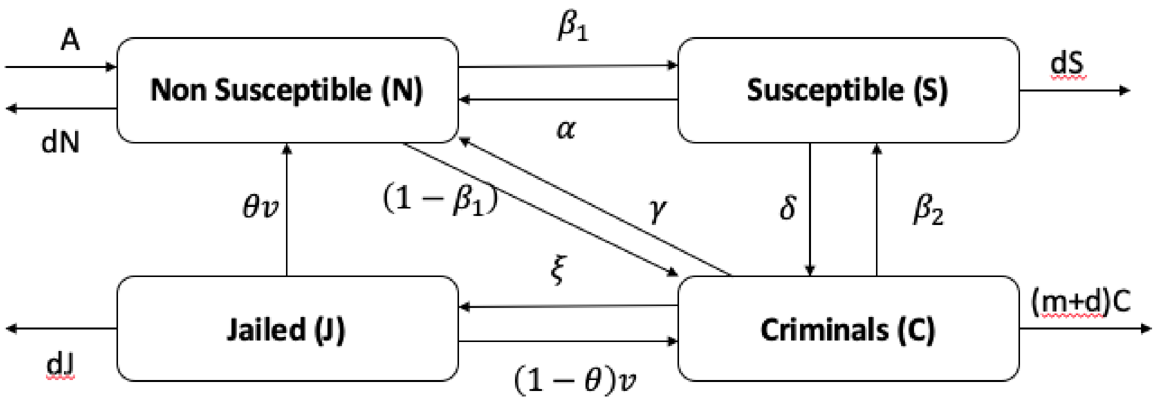

Figure 1: Illustrates schematic diagram for crime dynamics

The flow of parameters considered in modelling of crime is shown in figure 1 and defined through subsequent system of non-linear differential equations:

$$\frac{dN}{dt} = A - \frac{\beta_1 NS}{T} + \alpha S - \frac{(1-\beta_1)NC}{T} + \gamma C + \theta v J - dN$$

$$\frac{dS}{dt} = \frac{\beta_1 NS}{T} - \frac{\beta_2 SC}{T} + \delta C - (\alpha + d)S$$

$$\frac{dC}{dt} = \frac{\beta_2 SC}{T} + \frac{(1-\beta_1)NC}{T} + (1-\theta)vJ - (\xi + \gamma + \delta + m + d)C$$

$$\frac{dJ}{dt} = \xi C - (v + d)J$$

$$\frac{dT}{dt} = A - (dT + mC)$$

$$T = N + S + C + J \qquad (1)$$

where, β_1 is the rate at which the non-susceptible are converted into susceptible, α is interference/deterrence parameter for susceptible, β_2 is the conversion rate of susceptible to criminal, δ represents the people who decide to leave criminal activities and return to susceptible class by counselling or their own decision. ξ is the imprisonment rate, γ is the rate at which criminals leave criminal activities and return back to non-susceptible class through counselling or their own decision, $(1 - \beta_1)$ is the rate at which non-susceptibles come in contact with criminals and get converted to criminals. $(1 - \theta)$ is the backsliding rate. v^{-1} gives the average duration of the prison. A is the number of immigrants, d is the natural death rate and m is crime related death rate.

Non-susceptible member (N) by contact $\left(\frac{\beta_1 NS}{T}\right)$ with susceptible become susceptible member (S). In spite of these deterrence act, interaction among the susceptible and criminals $\left(\frac{\beta_2 SC}{T}\right)$ may result in additional criminals (C). We assume that $\beta_1 \geq \beta_2$, thus it's easy to persuade anyone to be susceptible than it is to convince someone who is a susceptible to take the last step and perpetrate crime. These criminals (C) are sent to prison with an incarceration rate of ξ. Two possibilities are considered after completion of prison sentence. One, jailed participants may recover and join the non-susceptible (N) due to rehabilitation or they may setback to emerge as criminals (C). Once prisoners complete their sentence, part of them move back to C which is defined as $(1 - \theta)$, hence $(1 - \theta)v$ gives the setback rate while θv gives the recovery rate. Non-susceptible member (N) by contact $\frac{(1-\beta_1)NC}{T}$ with criminals get converted to criminals (C). Criminals (C) through counselling relapse to non-susceptible class at the rate of γ and back to susceptible class at the rate of δ. This model considers a voluntary flow out of the criminal class which is unique from other studies. These parameters taken into consideration as positive since the fact that non-positive solution will not have any epidemiological importance. We consider

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$$N(0) \geq 0, S(0) \geq 0, C(0) \geq 0, J(0) \geq 0.$$

Initial conditions and values of parameter are chosen from previous studies. The actions of the model are inspected through calculation of R_0, finding equilibrium points, stability of equilibria and numerical simulation. The model is mathematically and realistically well-posed within the domain

$$D = \{(N,S,C,J)^T \in \mathbb{R}_+^4 : N(t), S(t), C(t), J(t) \geq 0, t \geq 0\}$$

Thus, the domain, D is positively invariant.

III.ANALYSIS

Calculation of R_0

The basic reproductive number R_0 is obtained by using next generation matrix. This method was first introduced by [23] but was further elaborated by [24]. In employing this method, S and C are taken into consideration as the contagious classes, N indicates the non-contagious class and J represents individuals who are contagious but do not transfer the disease. The Jacobian matrix (J) for the contagious compartments is decomposed into sum of two matrices; the transmission matrix (F) and the transition matrix (V). F accounts for the number of new crime (infection), while V is used to characterize movement to and fro compartments.

At (N, S, C, J) = $\left(\frac{A}{d}, 0, 0, 0\right)$, the Jacobian matrix is given by

$$J_{(S,C)} = \begin{bmatrix} \beta_1 - (\alpha + d) & 0 \\ 0 & (1-\beta_1) + \frac{(1-\theta)v\xi}{v+d} - (\xi + \gamma + \delta + m + d) \end{bmatrix}$$

R_0 is the dominant eigenvalue of FV^{-1}. Hence

$$R_0 = max\left(\frac{\beta_1}{\alpha + d}, \frac{(1-\beta_1)(v+d)}{(\xi + \gamma + \delta + m + d)(v+d) - (1-\theta)v\xi}\right)$$

where,

$$\frac{(1-\beta_1)(v+d)}{(\xi + \gamma + \delta + m + d)(v+d) - (1-\theta)v\xi} < 1.$$

Equilibrium Analysis

The equilibrium points are the steady state of the system. These steady states are established by equating the system of equations in the model equal to zero. The model exhibits no-crime equilibrium and crime-persistent equilibrium. The values of N^*, S^*, C^* and J^* can be prevailed by solving the following set of algebraic equations

$$A - \frac{\beta_1 NS}{T} + \alpha S - \frac{(1-\beta_1)NC}{T} + \gamma C + \theta v J - dN = 0 \quad (2)$$

$$\frac{\beta_1 NS}{T} - \frac{\beta_2 SC}{T} + \delta C - (\alpha + d)S = 0 \quad (3)$$

$$\frac{\beta_2 SC}{T} + \frac{(1-\beta_1)NC}{T} + (1-\theta)vJ - (\xi + \gamma + \delta + m + d)C = 0 \quad (4)$$

$$\xi C - (v+d)J = 0 \quad (5)$$

$$A - (dT + mC) = 0 \quad (6)$$

On solving the above equations (2) - (6), we get following equilibrium points

Case 3.2.1: Crime-free equilibrium, when $C^* = 0$, we get two cases

- $E_0 = (T^*, N^*, S^*, C^*, J^*) = \left(\frac{A}{d}, \frac{A}{d}, 0, 0, 0\right)$
- $E_1 = (T^*, N^*, S^*, C^*, J^*) = \left(\frac{A}{d}, \left(\frac{\alpha+d}{\beta_1}\right)\frac{A}{d}, \left(1 - \frac{\alpha+d}{\beta_1}\right)\frac{A}{d}, 0, 0\right)$

Case 3.2.2: Co-existence Equilibrium, when $C^* \neq 0$

$$E_2 = (N^*, S^*, C^*, J^*) = \left(MT^* - L, KT^* - G_1 N^*, \frac{A - dT^*}{m}, X\left(\frac{A}{m} - \frac{dT^*}{m}\right)\right)$$

where,

$$X = \frac{\xi}{v+d}, Y = (\xi + \gamma + \delta + m + d), K = \frac{Y - (1-\theta)vX}{\beta_2},$$

$$M = \frac{(1 - K + dZ)}{1 - G_1}, L = \frac{AZ}{1 - G_1}, G_1 = \frac{(1-\beta_1)}{\beta_2},$$

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$$Z = \frac{1+X}{m} \text{ and } (1 - K + dZ) > 0.$$

Stability Analysis

In this section, the stability behaviour of the equilibria is discussed in particular. The stability is obtained by defining Jacobian matrix (J) for model (1) which is constructed as follows:

$$= \begin{bmatrix} \frac{-\beta_1 S}{T} - \frac{(1-\beta_1)C}{T} - d & \frac{-\beta_1 N}{T} + \alpha & \frac{(1-\beta_1)N}{T} + \gamma & \theta v \\ \frac{\beta_1 S}{T} & \frac{\beta_1 N}{T} - \frac{\beta_2 C}{T} - (\alpha + d) & \frac{-\beta_2 S}{T} + \delta & 0 \\ \frac{(1-\beta_1)C}{T} & \frac{\beta_2 C}{T} & \frac{\beta_2 S}{T} + \frac{(1-\beta_1)N}{T} - Y & (1-\theta)v \\ 0 & 0 & \xi & -(v+d) \end{bmatrix}$$

Case 3.3.1 Let us consider first $E_0 = (T^*, N^*, S^*, C^*, J^*) = \left(\frac{A}{d}, \frac{A}{d}, 0, 0, 0\right)$

On substituting E_0 to Jacobian matrix and solving, we get one of the eigenvalue as –d, which is clearly negative and rest of the eigenvalues are given by

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

Where,

$a_1 = (\alpha + d) + [Y - (1 - \beta_1) + (v + d)] - \beta_1$

$a_2 = (\alpha + d)[Y - (1 - \beta_1) + (v + d)] + [(v + d)Y - (v + d)(1 - \beta_1) - (1 - \theta)v\xi] - \beta_1[Y - (1 - \beta_1) + (v + d)]$

$a_3 = (\alpha + d)[(v + d)Y - (v + d)(1 - \beta_1) - (1 - \theta)v\xi] - \beta_1[(v + d)Y - (v + d)(1 - \beta_1) - (1 - \theta)v\xi]$

On solving, we get eigen values as both positive and negative. Hence, E_0 is unstable.

Case 3.3.2 $E_1 = (T^*, N^*, S^*, C^*, J^*) = \left(\frac{A}{d}, \left(\frac{\alpha+d}{\beta_1}\right)\frac{A}{d}, \left(1 - \frac{\alpha+d}{\beta_1}\right)\frac{A}{d}, 0, 0\right)$

substituting in Jacobian matrix, we get the characteristic polynomial which is of the form:

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

Where,

$a_1 = -(\Gamma + \Delta + \Lambda)$

$a_2 = [\Lambda(\Gamma + \Delta) + \Gamma\Delta - (1 - \theta)v\phi + \Sigma d]$

$a_3 = [(1 - \theta)v\phi\Lambda - \Gamma\Delta\Lambda - \Gamma\Sigma - \Delta\Sigma d]$

$a_4 = \Gamma\Delta\Sigma d - (1 - \theta)v\phi d\Sigma$

and

$$P = \frac{-\beta_2(\beta_1 - \alpha - d)}{\beta_1} + \delta$$

$$\Gamma = \frac{\beta_2(\beta_1 - \alpha - d)}{\beta_1} + \frac{(1 - \beta_1)(\alpha + d)}{\beta_1} - Y$$

$$\Delta = -(v + d); \quad \Sigma = \beta_1 - (\alpha + d); \quad \Lambda = (\alpha - \beta_1);$$

Solving the characteristic polynomial, we get positive and negative eigen values. Hence, E_1 is unstable.

Case 3.3.1 and Case 3.3.2 are unstable, interpreting this situation to crime, this means that it is not possible for the society to be crime-free.

Case 3.3.3 here the coexistence equilibrium is considered. Into the Jacobian matrix $E_2 = (N^*, S^*, C^*, J^*) = \left(MT^* - L, KT^* - G_1N^*, \frac{A - dT^*}{m}, X\left(\frac{A}{m} - \frac{dT^*}{m}\right)\right)$ is substituted and simplified, characteristic polynomial is obtained which is of the form:

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

The Routh-Hurwitz criteria predict stability if and only if $a_1, a_2, a_3, a_4 > 0$ and $(a_1a_2 - a_3)a_3 > a_1^2a_4$. Due to the complex nature of the polynomial obtained, this evaluation will be done using the parameter values considered.

IV.NUMERICAL SIMULATION

MATLAB has been used to carry out numerical simulation for proposed model (1) by considering parameters given as in Table 1 [25], [26].

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Table 1: Illustrates the multifarious values of the parameters of the proposed model.

Parameter	A	β_1	β_2	α	ξ	θ	v	d	m	δ	γ
Value	750	0.71	0.21	0.1265	0.115	0.44	0.2	0.00258	0.0503	0.02	0.0057

Considering these values of parameter case 3.3.1 and case 3.3.2 are found to be unstable. This indicates the model (1) will not generally approach no criminal state ($C^* \neq 0$). Rather crime-persistent equilibrium is attained. The crime-persistent equilibrium values are obtained as:

$$T^* = 42307, N^* = 10208, S^* = 12082, C^* = 12768, J^* = 7249.$$

All the four eigenvalues obtained by the Jacobian matrix have negative real parts and hence, the crime-persistent equilibrium is locally asymptotically stable. This implies that system tends to rise in the number of criminals. As well, $R_0 = \frac{\beta_1}{\alpha+d} > 1$.

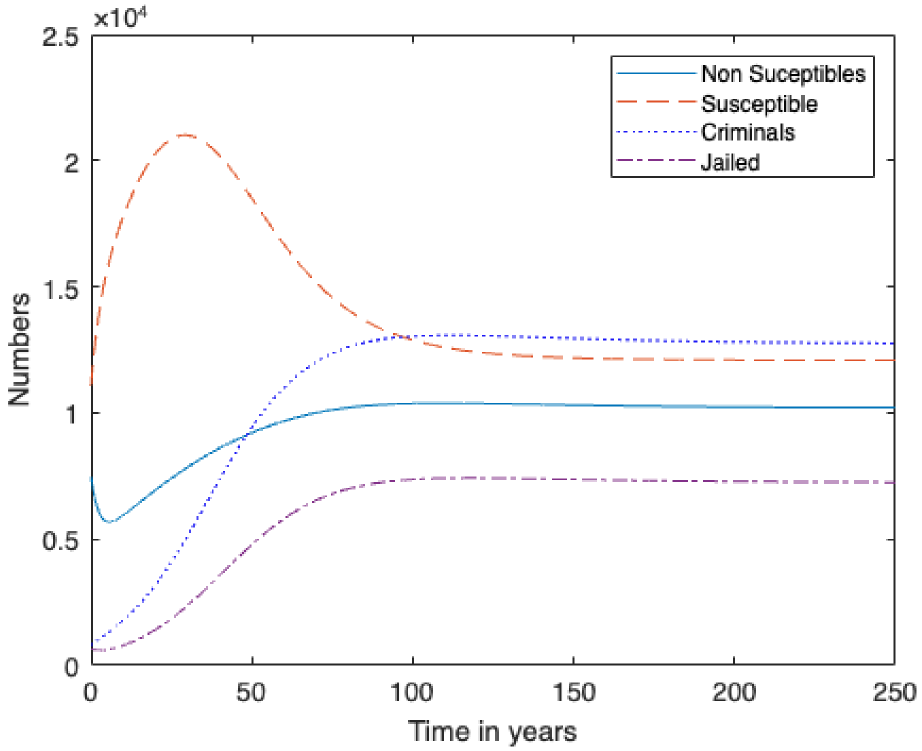

Figure 2: Numerical simulation with respect to time for non-susceptible, susceptible, criminals and jailed population.

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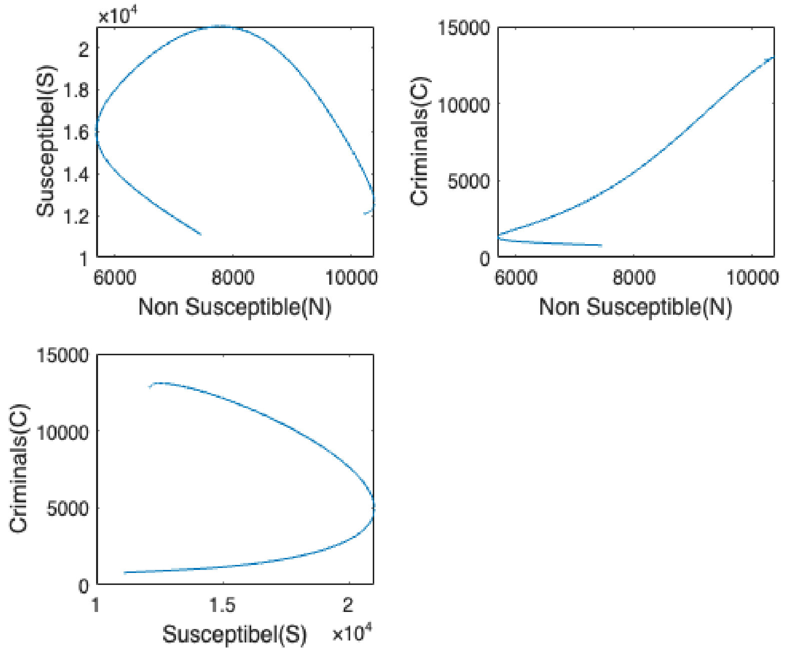

Figure 3: Interaction between groups (non-susceptible, susceptible, criminals) for model developed for this study of spread of crime

Model (1) with the set of parameter values as considered in Table 1 is simulated numerically. Figure 2 shows the coexistence persistent equilibrium and also shows an increase in all the subpopulations for the parameter values considered. People committing crime is being increased continuously from year to year can be perceived in the figure 2. Figure 3 clearly depicts interaction between groups. In figure 3 sub figure 1 shows the interaction between non susceptible and susceptible population. Sub figure 2 gives interaction between non susceptible and criminal population and last, sub figure 3 is the interaction between susceptible and criminal population. Crime is an act, harmful not only to some individual but also to a community or society. Crime and criminal behaviour are described by a nonlinear system as human behaviour is inherently unpredictable. It is noticed that criminal behaviour is infectious like epidemics and spreads through contact. The below mentioned factors are not considered in the other modelling process before, whereas they are considered in this model:

- Flow out of criminal class by counselling.
- Contact between the non-susceptible and criminal population.

V.CONCLUSION

A Non-Linear model to study the spread of crime in a society is proposed and analysed. Criminals in the society increase by coming in contact with criminals and people prone to criminality. R_0 for the elimination of criminality is determined by employing the method of Next Generation Matrix. Three equilibrium states are obtained by the analysis conducted, two of which contains no criminals. From the stability analysis result, crime free equilibrium is unstable and crime-persistent equilibrium is proved to be locally asymptotically stable.
Unstable crime equilibrium means that it is not possible to diminish the crime in society. The stable crime equilibrium means that the crime will continuously persist in the society. Numerical simulations show that that the crime would still exist in the society. Further research can include illegal immigrants, government invention rate to reduce crime.

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