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Kushare transform in solving Faltung Type Volterra Integro-Differential Equation of first kind

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Abstract: In this paper we use Kushare integral transform for solving the Faltung Type Volterra Integro-Differential Equation of the first kind. The results obtained by applying this integral transform from the numerical problems demonstrate the efficiency and the capability of the integral transform in solving the integro-differential equations of first kind.

Keywords: Integral Transform, Ordinary Differential Equation, KUSHARE Transform, Faltung Type Volterra Integro-Differential Equation.

AMS 2020 Classification: 44AXX, 31A10, 44J05, 32G34

1. INTRODUCTION

Volterra integro-differential equations appears in various fields like Astronomy, Biology, Biotechnology, Engineering, Physics, Radiology and many other scientific fields. These equations are used in very interesting applications like heat and mass transfer, diffusion process and growth of cells. When Volterra was examining a population growth model for the study of hereditary influence he came across where both differential and integral operators appeared together in the same equation. This new type of equation was named as Volterra integro-differential equation. These equations are of the form

$$u^{(n)}(x) = f(x) + \lambda \int_0^{(x)} K(x,t)u(t)dt$$

where $u^{(n)}(x) = \frac{d^n u(x)}{dx^n}$

As this equation is combination of differential and integral operators, to determine the particular solution $u(x)$, it is necessary to define initial conditions $u(0), u'(0), u''(0), \dots, u^{(n-1)}(0)$.

Recently, Integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.
Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.
In April 2022 D. P. Patil, et al [4] solved the problems on growth and decay by using Kushare transform. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Dinkar Patil [8]

solved wave equation by using Sawi transform and its convolution theorem. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9].
D .P. Patil used double Laplace and double Sumudu transforms to obtain the solution of wave equation [10]. Dr. Patil [11] also obtained dualities between double integral transforms. Kandalkar, Gatkal and Patil [12] used Kushare transform to solve the system of differential equations. D. P. Patil [13] solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplac, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. D. P. Patil et al [16] solved Volterra Integral equations of first kind by using Emad-Sara transform. Futher Patil with Tile and Shinde [17] used Anuj transform and solved Volterra integral equations for first kind. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil[21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil used Emad- Falih transform for solving problems based on Newton's law of cooling [22]. D. P. Patil et al [23] used Soham transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [24] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling [25]. D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [27] introduced double kushare transform. Recently, D. P. Patil et al [28] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by patil et al [29]. Thete et al [30] used Emad Falih transform for handling growth and decay problems. Nikam, Patil et al [31] used , Kushare transform of error functions in evaluating improper integrals. Wagh sisters and Patil used Kushare [32] and Soham [33] transform in chemical Sciences. Malpani, Shinde and Patil[34] used Convolution theorem for Kushare transform and applications in convolution type Volterra integral equations of first kind. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [36] generalized Double rangaig integral transform. Emad A. Kuffi et al [37][38] developed and used SEE transform to solve Volterra integro-differential equation.

2. BASIC CONCEPTS OF KUSHARE TRANSFORM

A new integral transform said to be KUSHARE change characterized for capacity of outstanding request. KUSHARE transform defined on the set A

$$A = \{f(t)/\exists\, M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau j}}, \text{ if } t \in (-1)^j \times [0,\infty)\}$$

For a given function in the set A, the constant M must be finite number, τ_1, τ_2 may be finite or infinite. Kushare Transform denoted by function f. The purpose of this study is to show the applicability of this interesting transform and operator s(v) defined by the integral equations.

$$K[f(t)] = S(v) = v\int_0^\infty f(t)e^{-tv^\alpha}\,dt, \qquad t \geq 0, \qquad \tau_1 \leq v \leq \tau_2$$

2.1 Basic Concepts of Kushare Transform:

Kushare transform of some required functions are stated in following table.

Sr. No.	Function	Kushare Transform
1	1	$\frac{1}{v^{\alpha-1}}$
2	t	$\frac{1}{v^{2\alpha-1}}$
3	t^n	$\frac{\Gamma(n+1)}{v^{\alpha(n+1)-1}}$

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4	e^{at}	$\frac{v}{v^{\alpha} - a}$
5	$\sin(at)$	$\frac{av}{v^{2\alpha} + a^2}$
6	$\cos(at)$	$\frac{v^{\alpha+1}}{v^{2\alpha} + a^2}$

2.2 Inverse Kushare Transform:

Following table is for inverse Kushare transform of some required functions

Sr. No.	Function	Inverse Kushare Transform
1	$\frac{1}{v^{\alpha-1}}$	1
2	$\frac{1}{v^{2\alpha-1}}$	t
3	$\frac{\Gamma(n+1)}{v^{\alpha(n+1)-1}}$	t^n
4	$\frac{v}{v^{\alpha} - a}$	e^{at}
5	$\frac{av}{v^{2\alpha} + a^2}$	$\sin(at)$
6	$\frac{v^{\alpha+1}}{v^{2\alpha} + a^2}$	$\cos(at)$

2.3 Properties of Kushare Transform:

In this section we state some required properties of Kushare transform.

A. **Linearity:** If $f_1(t)$ and $f_2(t)$ are two functions and a and b are constants then

$$K[a\, f_1(t) + b\, f_2(t)] = a\, K[f_1(t)] + bK[f_2(t)]$$

B. **Change of scale:** If $f(t)$ is a function and a is a constant then,

$$K[f(at)] = a^{\alpha^{-1}-1} s\left(\frac{v}{\sqrt[\alpha]{a}}\right)$$

C. Shifting:

$$K[e^{at} f(t)] = \frac{v}{(v^{\alpha} - a)^{1/\alpha}} s\left(\sqrt[\alpha]{(v^{\alpha} - a)}\right)$$

D. First derivative: If $f(t)$ is a function then,

$$K[f'(t)] = v^{\alpha} s(v) - v f(0)$$

E. Second derivative: If $f(t)$ is a function then,

$$K[f''(t)] = v^{2\alpha} s(v) - v^{\alpha+1} f(0) - v f'(0)$$

F. n^th derivative: If $f(t)$ is a function then,

$$K[f^n(t)] = v^{n\alpha} s(v) - v \sum_{k=0}^{n-1} v^{\alpha(n-k-1)} f^k(0)$$

G. Convolution theorem:

$$K[f(t) * g(t)] = \frac{1}{v} f(v).g(v)$$

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H. KUSHARE transform of function $tf(t)$:

$$K[tf(t)] = \left(\frac{-1}{\alpha v^{\alpha-1}}\right)\left[\frac{d}{dv} - \frac{1}{v}\right] s(v)$$

3. USING KUSHARE TRANSFORM TO SOLVE FALTUNG TYPE VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF FIRST KIND

In this section, Kushare transform has been applied to find the solution of Faltung Type Volterra Integro-Differential Equation of first kind.

Faltung Type Volterra Integro Differential Equation of first kind could be written as:

$$\int_{u=0}^{t} k_1(t-u)w(u)du + \int_{u=0}^{t} k_2(t-u)w^{(n)}(u)du = f(t)\ ;\ k_2(t-u) \neq 0 \qquad ..(1)$$

With $w(0) = \delta_0, w'(0) = \delta_1, w''(0) = \delta_2, \dots\dots\dots, w^{(n-1)}(0) = \delta_{n-1}$ … (2)

Where, $k_1(t-u)$ and $k_2(t-u)$are the Faltung Type Kernel of Integral Equation

$w(t) \equiv$ Unknown Function

$w^{(n)}(t) \equiv$ Derivative of Unknown Function

$f(t) \equiv$ Known Function

$\delta_0, \delta_1, \delta_2, \dots\dots\dots, \delta_{n-1}$ denotes the real numbers

Taking Kushare transform on both side of the equation (1)

$$K\left[\int_{u=0}^{t} k_1(t-u)w(u)du + \int_{u=0}^{t} k_2(t-u)w^{(n)}(u)du = f(t)\right] = K[f(t)]$$

Applying linearity property of Kushare transform on above equation

$$\Rightarrow\ K\left[\int_{u=0}^{t} k_1(t-u)w(u)du\right] + K\left[\int_{u=0}^{t} k_2(t-u)w^{n}(u)du\right] = K[f(t)] \qquad \dots(3)$$

Applying convolution property of Kushare transform on equation (3)

$$\Rightarrow\ \frac{1}{v} K[k_1(t)] . K[w(t)] + \frac{1}{v} K[k_2(t)] . K[w^n(t)] = K[f(t)] \qquad \dots(4)$$

Applying property of Kushare transform of derivative of functions on equation (4)

$$\Rightarrow \frac{1}{v} K\,[k_1(t)] .\ K\,[w(t)] + \frac{1}{v} K\,[k_2(t)]\ .$$

$$\left[v^{n\alpha} K\,[w(t)] - v\left(v^{\alpha(n-1)}w(0) + v^{\alpha(n-2)}w'(0) + \dots + w^{(n-1)}(0)\right)\right] = K[f(t)] \qquad \dots(5)$$

Now, substituting equation (2) in equation (5)

$$\Rightarrow\ \frac{1}{v} K\,[k_1(t)] .\ K\,[w(t)] + \frac{1}{v} K\,[k_2(t)] .$$

$$\left[v^{n\alpha} K\,[w(t)] - v^{\alpha(n-1)+1}w(0) - v^{\alpha(n-2)+1}w'(0) - \dots\dots\dots - v w^{(n-1)}(0)\right] = K[f(t)]$$

$$\Rightarrow \frac{1}{v} K[k_1(t)]\,.\,K[w(t)] + \frac{1}{v} K[k_2(t)]\,.\, v^{n\alpha} K[w(t)] =$$

$$K[f(t)] + \frac{1}{v} K[k_2(t)]\,.\left[\delta_0 v^{\alpha(n-1)+1} + \delta_1 v^{\alpha(n-2)+1} + \dots\dots\dots + \delta_{n-1} v\right]$$

$$\Rightarrow \left[\frac{1}{v} K[k_1(t)] + \frac{v^{n\alpha}}{v} K[k_2(t)]\right] K[w(t)]$$

$$= K[f(t)] + \frac{1}{v} K[k_2(t)]\left[\delta_0 v^{\alpha(n-1)+1} + \delta_1 v^{\alpha(n-2)+1} \dots\dots\dots \dots + \delta_{n-1} v\right]$$

$$K[w(t)] = \frac{K[f(t)] + \frac{1}{v} K[k_2(t)]\left[\delta_0 v^{\alpha(n-1)+1} + \delta_1 v^{\alpha(n-2)+1} \dots\dots\dots \dots + \delta_{n-1} v\right]}{\frac{1}{v}\left[K[k_1(t)] + v^{n\alpha} K[k_2(t)]\right]} \quad \dots(6)$$

And $K[k_1(t)] + v^{n\alpha} K[k_2(t)] \neq 0$

Taking the inverse Kushare transform of equation (6) gives required exact solution of Faltung Type Volterra Integro-Differential Equation of first kind.

4. NUMERICAL APPLICATIONS

In this section we solve some problems.

Application (I): Consider the Faltung Type of Volterra Integro -Differential Equation of first kind

$$\int_0^t (t-u)w(u)du + \int_0^t (t-u)^2 w'(u)du = 3(t - \sin t) \quad \dots(7)$$

With $w(0) = 0$ … (8)

Applying Kushare transform on equation (7)

$$\Rightarrow K\left[\int_0^t (t-u)w(u)du + \int_0^t (t-u)^2 w'(u)du\right] = K[3(t-\sin t)]$$

Applying linearity property of Kushare transform on above equation

$$\Rightarrow K\left[\int_0^t (t-u)w(u)du\right] + K\left[\int_0^t (t-u)^2 w'(u)du\right] = 3K[t] - 3K[\sin t] \quad \dots(9)$$

Applying convolution property of Kushare transform on equation (9)

$$\Rightarrow \frac{1}{v} K[t]\,.\,K[w(t)] + \frac{1}{v} K[t^2]\,.\,K[w'(t)] = 3K[t] - 3K[\sin t]$$

Applying property of Kushare transform of derivative of functions

$$\Rightarrow \frac{1}{v} K[t]\,.\,K[w(t)] + \frac{1}{v} K[t^2].[v^\alpha K[w(t)] - v\,w(0)] = 3K[t] - 3K[\sin t]$$

$$\Rightarrow \frac{1}{v}\left(\frac{1}{v^{2\alpha-1}}\right) K[w(t)] + \frac{1}{v}\left(\frac{2}{v^{3\alpha-1}}\right).[v^\alpha K[w(t)] - v\,w(0)] = 3\left(\frac{1}{v^{2\alpha-1}}\right) - 3\left(\frac{v}{v^{2\alpha}+1}\right) \quad \dots(10)$$

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Substituting equation (8) in equation (10)

$$\Rightarrow \frac{1}{v^{2\alpha}} K[w(t)] + \frac{2}{v^{3\alpha}} (v^{\alpha} K[w(t)]) = \frac{3}{v^{2\alpha-1}} - \frac{3v}{v^{2\alpha}+1} \qquad \ldots(11)$$

$$\Rightarrow \left[\frac{1}{v^{2\alpha}} + \frac{2}{v^{2\alpha}}\right] K[w(t)] = \frac{3}{v^{2\alpha-1}} - \frac{3v}{v^{2\alpha+1}} \Rightarrow \left(\frac{3}{v^{2\alpha}}\right) K[w(t)] = \frac{3}{v^{2\alpha-1}} - \frac{3v}{v^{2\alpha+1}}$$

$$\Rightarrow K[w(t)] = v - \frac{v^{2\alpha+1}}{v^{2\alpha+1}} \Rightarrow K[w(t)] = \frac{v}{v^{2\alpha+1}} \qquad \ldots(12)$$

Taking Inverse Kushare transform of equation (12)

$$\Rightarrow w(t) = K^{-1}\left[\frac{v}{v^{2\alpha+1}}\right] \Rightarrow w(t) = \sin t$$

This is required exact solution of equation (7).

Application (II): Consider the Faltung Type of Volterra Integro -Differential Equation of first kind

$$\int_0^t sin(t-u)w(u)du - \frac{1}{2}\int_0^t (t-u)w''(u)du = \frac{1}{2}(t - t\cos t) \qquad \ldots(13)$$

With $w(0) = 0 \;\&\; w'(0) = 1$ … (14)

Applying Kushare transform on equation (13)

$$\Rightarrow K\left[\int_0^t sin(t-u)w(u)du - \frac{1}{2}\int_0^t (t-u)w''(u)du\right] = K\left[\frac{1}{2}(t - t\cos t)\right]$$

Applying linearity property of KUSHARE transform on above equation

$$\Rightarrow K\left[\int_0^t sin(t-u)w(u)du\right] - \frac{1}{2}K\left[\int_0^t (t-u)w''(u)du\right] = \frac{1}{2}K[t] - \frac{1}{2}K[t\cos t] \qquad \ldots(15)$$

Applying convolution property of KUSHARE transform on equation (15)

$$\frac{1}{v} K[\sin t].K[w(t)] - \frac{1}{2}\left(\frac{1}{v}\right)K[t].K[w''(t)] = \frac{1}{2}K[t] - \frac{1}{2}K[t\cos t] \qquad \ldots(16)$$

$$\Rightarrow \frac{1}{v}\left(\frac{v}{v^{2\alpha}+1}\right)K[w(t)] - \frac{1}{2}\left(\frac{1}{v}\right)\left(\frac{1}{v^{2\alpha-1}}\right)[v^{2\alpha}K[w(t)] - v^{\alpha+1}w(0) - v\,w'(0)]$$

$$= \frac{1}{2}\left(\frac{1}{v^{2\alpha-1}}\right) - \frac{1}{2}\left(\frac{v(v^{2\alpha}-1)}{(v^{2\alpha}+1)^2}\right) \qquad \ldots(17)$$

Substituting equation (14) in equation (17)

$$\Rightarrow \left(\frac{1}{v^{2\alpha}+1}\right)K[w(t)] - \frac{1}{2v}\left(\frac{1}{v^{2\alpha-1}}\right)[v^{2\alpha}K[w(t)] - v] = \frac{1}{2}\left(\frac{1}{v^{2\alpha-1}}\right) - \frac{1}{2}\left(\frac{v(v^{2\alpha}-1)}{(v^{2\alpha}+1)^2}\right)$$

$$\Rightarrow \left(\frac{1}{v^{2\alpha}+1}\right)K[w(t)] - \frac{1}{2}\left(\frac{1}{v^{2\alpha}}\right)[v^{2\alpha}K[w(t)] - v] = \frac{1}{2}\left(\frac{1}{v^{2\alpha-1}}\right) - \frac{1}{2}\left(\frac{v(v^{2\alpha}-1)}{(v^{2\alpha}+1)^2}\right)$$

$$\Rightarrow \left(\frac{1}{v^{2\alpha}+1}\right)K[w(t)] - \frac{1}{2}K[w(t)] + \frac{1}{2}\left(\frac{1}{v^{2\alpha-1}}\right) = \frac{1}{2}\left(\frac{1}{v^{2\alpha-1}}\right) - \frac{1}{2}\left(\frac{v(v^{2\alpha}-1)}{(v^{2\alpha}+1)^2}\right)$$

$$\Rightarrow \left[\frac{1}{v^{2\alpha}+1} - \frac{1}{2}\right]K[w(t)] = \frac{-1}{2}\left(\frac{v(v^{2\alpha}-1)}{(v^{2\alpha}+1)^2}\right)$$

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$$\Rightarrow \left[\frac{(1-v^{2\alpha})}{2(v^{2\alpha}+1)}\right] K[w(t)] = \frac{-1}{2}\left(\frac{v(v^{2\alpha}-1)}{(v^{2\alpha}+1)^2}\right)$$

$$\Rightarrow \left[\frac{(1-v^{2\alpha})}{2(v^{2\alpha}+1)}\right] K[w(t)] = \frac{1}{2}\left(\frac{v(1-v^{2\alpha})}{(v^{2\alpha}+1)^2}\right)$$

$$\Rightarrow K[w(t)] = \frac{v}{v^{2\alpha}+1} \qquad \dots(18)$$

Taking Inverse Kushare transform of equation (18)

$$\therefore w(t) = K^{-1}\left[\frac{v}{v^{2\alpha}+1}\right]$$

$$\therefore w(t) = \sin t$$

This is required exact solution of equation (13).

5. CONCLUSION

We have successfully used Kushare transform to solve Faltung type Volterra Integro differential equations of first kind.

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