

# Determination of a Semi-Empiric Formula to Estimate Reference Evapotranspiration at the Puno Weather Station, Perú

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**Abstract:** The objective of the investigation was to determine a semi-empirical formula that relates the reference evapotranspiration with the vapor pressure deficit, wind speed, temperature range and extraterrestrial radiation using data from the Puno meteorological station. Multiple linear regression was applied in the calibration and then the formula was validated by comparing its results with the class A tank method, Penman-Monteith, Hargreaves, and Serruto. The multiple linear regression coefficients are statistically significant ( $p < 0.05$ ), a correlation coefficient of 84% and determination of 71% was determined. The formula has a physical basis because the terms wind speed and vapor pressure deficit represent the aerodynamic factor; the temperature and extraterrestrial radiation terms include the energy balance factor. The formula better estimates the reference evapotranspiration than the methods with which it was compared and has a lower mean square error with respect to values obtained with a class A tank. The variation of values obtained with the formula is statistically equal to the variation obtained with a class A tank. The formula slightly overestimates the mean because the reference evapotranspiration values in the calibration are higher than those in the validation.

**Keywords:** Formula, semi-empirical, evapotranspiration, reference, regression, Puno.

## I. INTRODUCTION

Agricultural production must be more efficient in the use of water. Determination of crop water needs is important for integrated water management, water use efficiency, design and operation of irrigation systems, and in hydrological studies. A fundamental parameter for this determination is the reference evapotranspiration.

Global climate change alters the hydrological cycle, directly affecting evapotranspiration, requiring studies in this regard. In the Puno region, there are few studies related to the determination of reference evapotranspiration and do not recommend adequate methodologies, nor do they show the influence of climate factors and elements; Therefore, it is proposed to semi-empirically determine a formula adapted to the area.

Doorenbos and Pruitt in 1977 have defined the reference crop evapotranspiration (Doorenbos & Pruitt, 1977) as the rate of evapotranspiration from a large area covered by green grass of uniform height between 8 and 15 cm that grows normally, covers completely the soil with its shade and does not lack water (Chow et al., 1988).

Hargreaves and Samani proposed a reference evapotranspiration estimation method based only on temperature and extraterrestrial radiation data, their results being acceptable under different climatic conditions (H. Hargreaves & A. Samani, 1985). Serruto proposed a formula for calculating evapotranspiration based on extraterrestrial solar radiation and average monthly temperature (Serruto, 1993). The FAO 56 manual (Allen et al., 1998) recommends the use of the Penman-Monteith equation as the only standard method for estimating reference evapotranspiration, however, it requires records of hours of sunshine, which they are scarce in the Puno region. García, Sánchez and Paredes developed methods for estimating potential evapotranspiration based on the daytime range of temperature and extraterrestrial solar radiation, for Puno, based on multiple linear and non-linear regressions, obtaining satisfactory results (García V. et al., 2000).

The relationship between the reference evapotranspiration (ET<sub>o</sub>) with climatic factors and elements in conditions of the Peruvian highlands has been determined. Using multiple linear regression, ET<sub>o</sub> was modeled using two terms as independent variables: one energy balance and the other aerodynamic (Flores-quispe, 2013).

To model potential evapotranspiration, Fourier series have been used using the temperature range as an independent variable, in addition multiple linear regression was used with parameters estimated by maximum likelihood, in addition it has been regionalized, obtaining models for each homogeneous region (Flores-Quispe et al. , 2015).

Reference evapotranspiration (ET<sub>o</sub>) plays a fundamental role in irrigated agriculture. Algorithms have even been used to simulate the ET<sub>o</sub> at a monthly level. Maximum temperature, minimum temperature, relative humidity, wind speed, and sunshine hours have been selected as inputs for the simulation models (Ehteram et al., 2019).

Baseline evapotranspiration is a major component of the hydrological cycle linking irrigation water requirement and water resources planning and management (Malik et al., 2019).

Reference evapotranspiration has been modeled with machine learning techniques such as artificial neural networks, within neural networks there are different variants with optimization algorithms (Tikhmarine et al., 2019).

Reference evapotranspiration is a basic variable in hydrology and in general in the management of water resources. FAO manual 56 recommends the Penman-Monteith method as the standard and most robust model for calculating reference evapotranspiration (Salam et al., 2020).

The evapotranspiration of the reference crop has been evaluated even with climate data sources gridded at regional scales, that is, climate information distributed in space with high-resolution information has been used (Pelosi et al., 2020).

The timely and accurate estimate of the reference evapotranspiration (ET<sub>o</sub>) is essential for water management in agriculture for its efficient use. Studies have targeted the amount of ET<sub>o</sub> with machine learning methods using minimum meteorological parameters in arid and semi-arid climates (Sattari et al., 2021).

To estimate reference evapotranspiration, empirical methods have been developed that require rigorous local calibrations and all of them have shown limited global validity; additionally, achieving their accuracy requires a lot of time and resources (Flores-quispe et al., 2021).

A new semi-empirical formula is proposed to determine the reference evapotranspiration in the absence of insolation data. It can be used in experiments to determine the water needs of agricultural crops in the study area. The following question was formulated: What relationship exists between reference evapotranspiration, vapor pressure deficit, wind speed, temperature range, and extraterrestrial radiation at the Puno Meteorological Station?

The objective of this research was to determine a semi-empirical formula that relates the reference evapotranspiration with the vapor pressure deficit, wind speed, temperature range and extraterrestrial radiation at the Puno weather station.

## **II. MATERIALS AND METHODS**

Data recorded by the National Meteorology and Hydrology Service (SENAMHI) was used, at the Puno CP-708 meteorological station, of the main climatological type, whose geographical location is 15°48'03" South Latitude, 70°02'00" West Longitude and 3812 m.s.n.m. altitude. The record used was 26 years (1981 to 2006) of mean maximum temperature, mean minimum temperature, mean relative humidity, monthly evaporation from class A tanks, mean daily insolation, and mean wind speed. For the calculations, Microsoft Office Excel, Cropwat 8.0 and Minitab 15 were used. The reference evapotranspiration value was determined by the class A tank method, considering it close to the real value, because many investigations affirm that the tank reflects the combined effect of climatic variables that affect the reference evapotranspiration (Allen et al., 1998). The value of the coefficient of the tank was taken as 0.75, since it has coefficient variations from 0.6 to 0.8; these values are greatly influenced by wind, humidity and the vegetation cover surrounding the tank. The wind has great variation, the humidity is measured indirectly and the surrounding vegetation in the Puno station does not conform to tabular values.

Statistical modeling was performed using multiple linear regression. The reference evapotranspiration was taken as that obtained by the class A tank method, according to the expression.

$$ET_0 = E_0 \times K$$

Where  $ET_0$  = reference evapotranspiration (mm/month),  $E_0$  = evaporation from class “A” tank (mm/month),  $K$  = tank coefficient. The general regression model with dependent variable “y” is

$$y = f(x_1, x_2, \dots, x_n, \theta_1, \theta_2, \dots, \theta_n)$$

Where  $x_1, x_2, \dots, x_n$  are the independent variables,  $\theta_i$ , with  $i=1,2,\dots,n$ , are model parameters.

Calibration was performed by least square estimation of regression coefficients for the period 1981-1995. Hypothesis tests such as regression analysis of variance and t-test for each coefficient were performed.

In the calibration, the parameters of the statistical relationship between the reference evapotranspiration (dependent variable) and the vapor pressure deficit, the average wind speed, the temperature range and extraterrestrial radiation (independent variables) were estimated.

The validation was carried out in the period 1996-2005, comparing the results of the formula with the tank class A, Penman-Monteith, Hargreaves and Serruto methods. The mean square error (EMC) was determined with respect to the value obtained with a class A tank in the validation period.

To test whether the formula produces reference evapotranspiration values statistically equal to those obtained by class A tanks, a mean difference t-test, a variance difference F-test, and a one-factor analysis of variance were performed using the full design. randomly, considering treatments to the formula and to the class A tank. These tests were carried out for the validation period.

### III. RESULTS AND DISCUSSION

The following table 1 shows the results of the estimation of the multiple linear regression parameters. In table 1 the variable  $X_1$  is the product of the wind speed and the vapor pressure deficit, the variable  $X_2$  is the square root of the temperature range and the variable  $X_3$  is the extraterrestrial radiation.

TABLE I ESTIMATED PARAMETERS OF MULTIPLE LINEAR REGRESSION

Variables	Coefficients	Typical error	t-statistic	Probability	Inferior 95%	Superior 95%
Interception	-110.6230	20.6450	-5.3584	2.60E-07	-151.3666	-69.8795
Variable $X_1$	23.4193	2.1362	10.9629	1.20E-21	19.2034	27.6352
Variable $X_2$	28.0464	4.0791	6.8756	1.03E-10	19.9961	36.0967
Variable $X_3$	6.7557	0.6580	10.2667	1.13E-19	5.4571	8.0544

Below is the equations that have the variables.

$$\text{Variable } X_1 = u_2 \times \text{Def}$$

$$\text{Variable } X_2 = \sqrt{\Delta T}$$

$$\text{Variable } X_3 = Ra$$

So the formula obtained is

$$ET_0 = -110.62 + 23.42(u_2 \times \text{Def}) + 28.05\sqrt{\Delta T} + 6.76Ra$$

Where  $ET_0$  = reference evapotranspiration (mm/month),  $u_2$  = wind speed measured at a height of 2 meters (m/s),  $\text{Def}$  = vapor pressure deficit (kPa),  $\Delta T$  = temperature range ( $^{\circ}\text{C}$ ) or difference between average maximum and minimum temperature,  $Ra$  = extraterrestrial radiation (mm/month),  $\text{Def}$  is determined by

$$\text{Def} = e_s - e_a$$

Where  $e_s$  = saturation vapor pressure (kPa),  $e_a$  = actual vapor pressure (kPa).

According to the t-test, the coefficient of all the variables is significantly different from zero at the 5% level of significance. A correlation coefficient of 84% was determined, which shows a high degree of linear relationship between the regression variables. The coefficient of determination of 70% is the variation explained by the independent variables. The regression analysis of variance concludes that calculated F (141.94) is greater than theoretical F, with the probability of exceedance ( $p = 9.21 \times 10^{-47}$ ) being much lower than the 5% significance level, concluding that R<sup>2</sup> is significantly different from zero.

The residuals comply with the hypotheses of normality and constant variance (homoscedasticity), which is shown in the following figures 1 and 2.

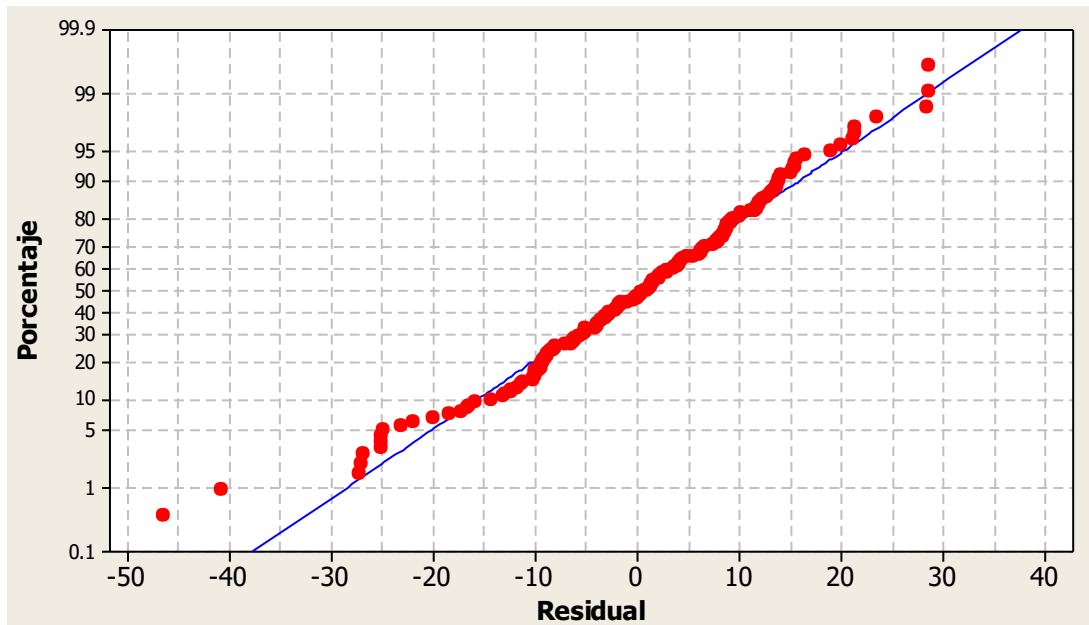


Fig. 1 Normal probability plot of the residuals

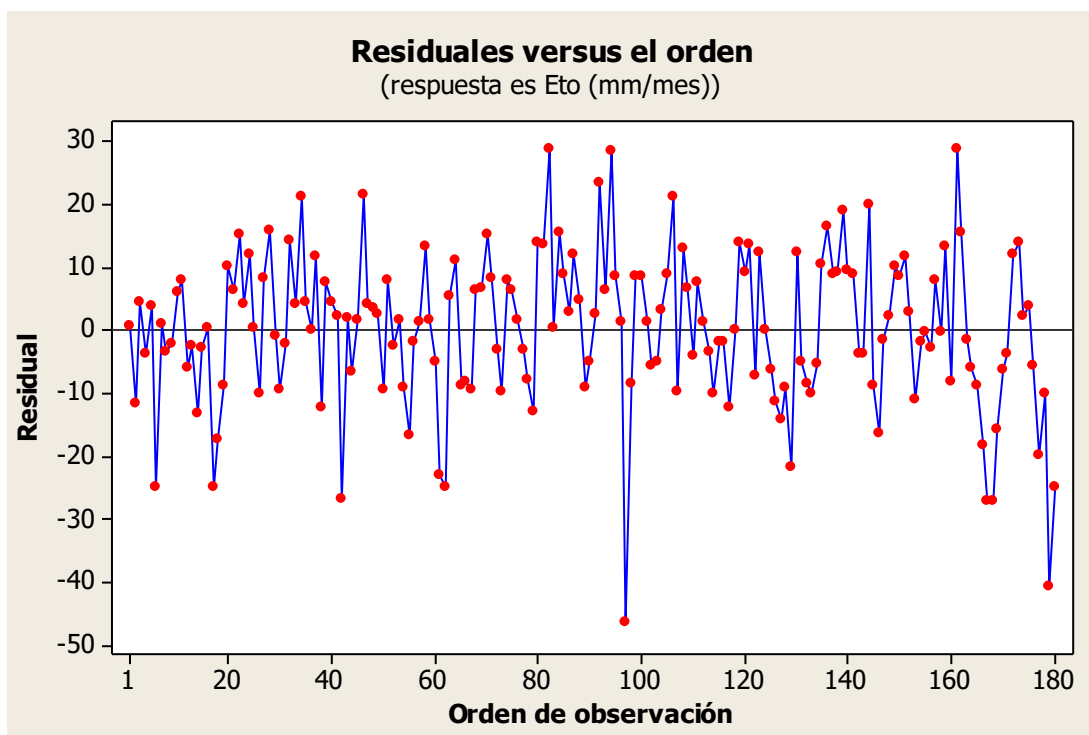


Fig. 2 Variation of the residuals.

The following figure 3 shows the validation by comparing the results of applying the formula, with the results with the class A tank method, Penman-Monteith, Hargreaves, and Serruto in the validation period 1996-2005.

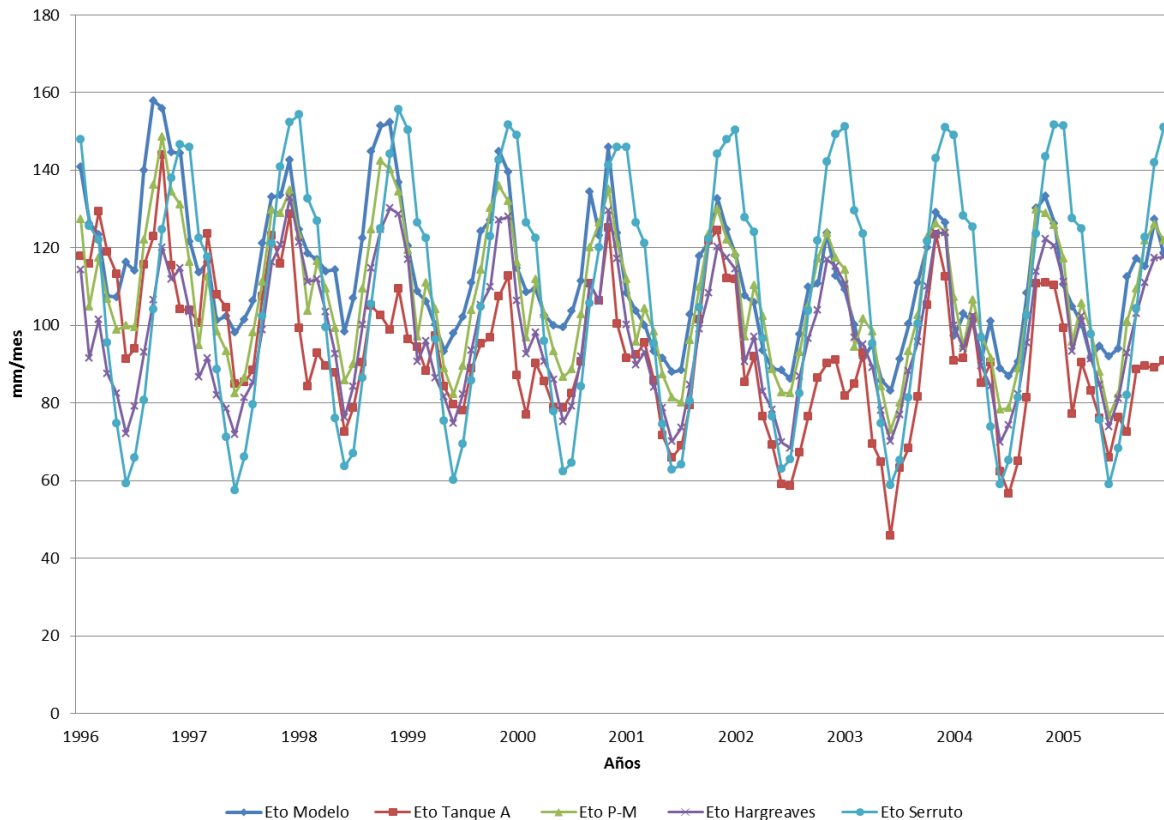


Fig. 3 Comparison of methods for model validation.

The formula is very close to the reference evapotranspiration obtained by class A tank. The root mean square error (EMC) with respect to the reference evapotranspiration of the class A tank, for the formula is 327.49 mm<sup>2</sup>/month, for Penman-Monteith 406.47 mm<sup>2</sup>/month, for Hargreaves it is 681.87 mm<sup>2</sup>/month and for Serruto 1046.92 mm<sup>2</sup>/month. The formula obtained has the lowest EMC, so it better estimates the reference evapotranspiration, in the validation period.

The t-test for difference in means of the values obtained by the formula and the class A tank rejects the hypothesis of equality of means (calculated t of 8.66 and theoretical t of 1.97) at 5% significance, the mean obtained with the formula (112.68 mm/month) to the average with tank A (92.72 mm/month). The model slightly overestimates the reference evapotranspiration value.

The F test concludes that the hypothesis of equality of variances is accepted at 5% significance (calculated F of 0.91 and theoretical F of 1.35). So, the model well represents the reference evapotranspiration variability equal to the class A tank. The result of the analysis of variance of a factor considering the treatments to the methods of tank A and the formula, concludes that the means are not equal to the level of significance of 5% (F calculated 74.96 and F theoretical 3.88). The formula slightly overestimates the reference evapotranspiration, because the data used in the calibration is greater than the validation data. The reference evapotranspiration in the validation decreased, probably due to some climatic change. The model obtained from multiple linear regression, as has been done by Flores-Quispe (Flores-Quispe, 2013), uses the temperature range as a good predictor of reference evapotranspiration, which has a precedent in several models such as García, Sánchez and Paredes (García V. et al., 2000), the Flores-Quispe model with Fourier series (Flores-Quispe et al., 2015) and the models for different homogeneous regions in the altiplano (Flores-Quispe et al., 2021). So the temperature range is a good predictor of the reference evapotranspiration, for weather conditions of the Puno station. The use of multiple linear regression is more practical and easier than using algorithms (Ehteram et al., 2019), machine learning techniques, optimized neural networks (Tikhmarine et al., 2019), the neuro-fuzzy inference system co-active (Malik et al., 2019) or methods based on Kernel functions (Sattari et al., 2021) as other investigations have done, because an empirical model is a formula that is easy to apply.

The empirical model obtained, like others in the world, has been compared with other empirical models, as has been done in background research where, for example, the Penman-Monteith formula has been compared with thirteen empirical models (Salam et al., 2020). . The advantage of using empirical models lies in their practicality, simplicity and reduced number of variables required, which is suitable for areas with a lack of meteorological data such as the southern highlands of Peru. Although the lack of data in stations can be supplied with the use of gridded climate data sources that can be a high-resolution reanalysis data set as performed in previous research (Pelosi et al., 2020).

#### IV. CONCLUSIONS

The reference evapotranspiration formula obtained for the climatic conditions of the Puno station is linear, its multiple linear regression coefficients are statistically significant, for all the independent variables (product of the wind by the vapor pressure deficit, the temperature range and the extraterrestrial radiation). A correlation coefficient of 84% and a determination coefficient of 71% were determined.

The obtained formula has physical bases, because the terms of wind speed and vapor pressure deficit represent the aerodynamic factor, and the terms of temperature and extraterrestrial radiation represent the energy balance factor.

The formula better estimates the reference evapotranspiration than the Penman-Monteith, Hargreaves and Serruto methods, due to its lower mean square error with respect to the reference evapotranspiration obtained with a class A tank. The formula adequately estimates the variation, while it slightly overestimates the average obtained with a class A tank, due to the fact that the reference evapotranspiration values in the calibration are higher than those of the validation, having probably decreased due to climate change.

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