

A Review of Inerter Based Damper and its Application in Civil-Structural Engineering

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Abstract: The fluid inerter damper is a new type of vibration control device that has recently gained popularity in the field of civil-structural engineering. It consists of a fluid-filled chamber that is connected to the structure being protected. The device is designed to reduce the effects of dynamic loads such as earthquakes, wind, and traffic on buildings and other structures. In this review, we summarize the current state of research on fluid inerter dampers, including their working principle, design considerations, and application in civil-structural engineering. We also discuss the advantages and disadvantages of this technology compared to other vibration control devices such as tuned mass dampers and viscous dampers. Overall, the fluid inerter damper offers several advantages, including low mass, low energy consumption, and high durability. It can effectively reduce the amplitude of vibrations in structures, improving their performance during extreme loading events. However, further research is needed to optimize the design of these devices for different structural applications and to explore their potential for energy harvesting and other innovative uses. In conclusion, fluid inerter dampers have significant potential for use in civil-structural engineering and offer an alternative to traditional vibration control devices. They represent a promising technology for reducing the risk of damage to structures during extreme loading events, and their use is expected to increase in the coming years as they continue to gain recognition within the engineering community.

Keywords: Fluid inerter damper, vibration control, civil-structural engineering, dynamic loads, earthquakes, wind, optimization, developing technology.

I. INTRODUCTION

A. General

Structural control using fluid inerter dampers is a relatively new and effective technique for improving the performance of structures subjected to dynamic loads. The fluid inerter damper is a passive device that utilizes the inertial properties of a fluid to counteract the motion of a structure. By integrating fluid inerter dampers into a structural system, it is possible to control the response of the structure to a wide range of dynamic loads, including wind, earthquakes, and vibrations. This technique has been studied extensively in recent years, and researchers have demonstrated its effectiveness in a variety of applications. For example, fluid inerter dampers have been used to mitigate the effects of seismic loads on bridges and buildings, control wind-induced vibrations in tall structures, and reduce the impact of dynamic loads on offshore structures such as oil platforms. Using fluid inerter dampers in structural control offers several advantages over traditional approaches, such as passive-tuned mass dampers or active control systems. Fluid inerter dampers are relatively inexpensive and easy to install, require little or no maintenance, and are highly effective in various applications. Additionally, they do not require external power or control systems, making them a highly attractive option for many engineering applications.

Vibration Control is one of the most important topics in Structural Engineering. Buildings are usually subjected to various types of vibrations and all of them are termed External Excitations. It is very important to mitigate the disastrous effects of such vibrations and over the years many devices are studied for reliable and effective use. All the Vibration Control Devices are categorized under three main categories: active, semi-active, and passive. These categories are based on whether the energy supply is required or not. Active and semi-active devices usually bring the best vibration suppression performance but Passive devices have an advantage in Stability, reliability, and economy. One such Passive Vibration Control Device is Fluid Inerter Damper. Inerters were first introduced in 2002 and since then, it has become a popular research topic. "The inerter is a novel element for vibration control in a mechanical system, which has the property that the force through the device is proportional to the relative acceleration across the terminals, and the proportional coefficient is defined as the Inertance." The inertance in Fluid-based inerters is generated by the acceleration of mass fluid flowing through the external tube channel which is connected with the piston cylinder mechanism on both ends.

There are multiple designs available for the fluid-based inerters but this research study focuses on the use of Helical tube fluid inerters designs. Following is the design of the studied device and it is called a Fluid Inerter Damper (FID). Some literature also states the name as Fluid Inerter Integrated Damping Device. Both names are aimed at the same design. In Fluid Inerter Damper (FID) there is a piston with a through-rod within a cylinder with hydraulic fluid. The fluid is on both sides of the piston and a (helical) tube connects to the two sides of the piston. The movement of the piston relative to the cylinder causes fluid to flow along the tube, and energy is dissipated by internal fluid motion.

B. Physical Realization of Inerter

There have been numerous inerter prototypes constructed in practice using various mechanisms such as ball-screw, rack-and-pinion, hydraulic, helical fluid, electromagnetic, and living-hinge. This subsection provides a summary of these inerter devices, with a particular focus on their working principles.

a) Ball-Screw Inerter

The schematic drawing of a typical ball-screw inerter is shown in Fig. (1.2). The inerter is comprised of a ball screw, a ball nut, a radial bearing, a housing, and a flywheel that is connected to the ball nut. In the ball-screw inerter, the ball screw is the most critical component, as it converts the linear motion between its two ends into the rotation of the ball

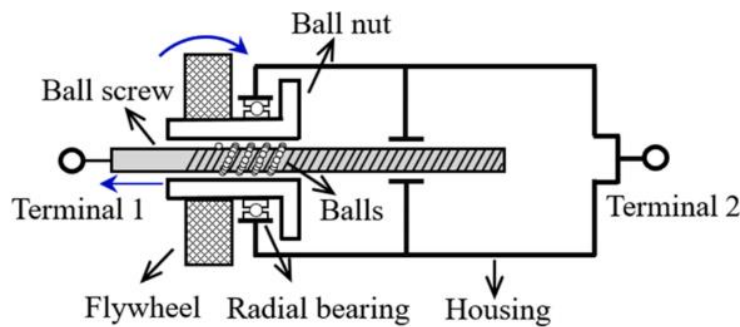


Figure I.1 Schematic Drawing of Ball-Screw Inerter [15]

nut, and consequently, the rotation of the flywheel. This design amplifies the physical mass of the flywheel, resulting in a substantial apparent mass, or inertance. The ideal inertance of the ball-screw inerter can be calculated using the following equation:

$$b = \left(\frac{2\pi}{l}\right)^2 \cdot I$$

The equation for ideal inertance of the ball-screw inerter includes the moment of inertia of the flywheel (I) and the lead of the ball screw (l), which refers to the linear distance covered by the screw for each full revolution.

b) Rack and Pinion Inerter

The schematic drawing of a rack-and-pinion inerter is depicted in Fig. (1.3). This inerter comprises a rack, a flywheel, a housing, as well as pinions and gears. The rack is capable of sliding within the housing and driving the flywheel's rotation

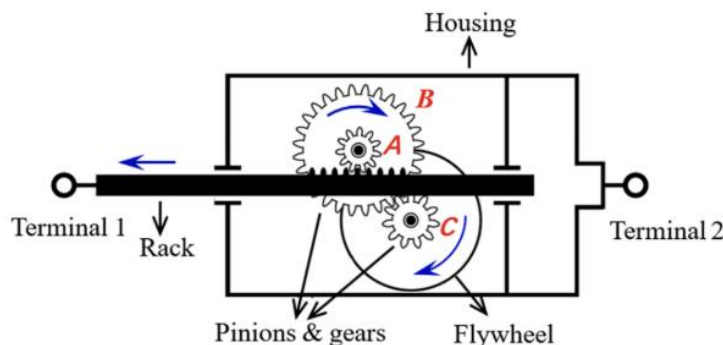


Figure I.2 Schematic of Rack-and-Pinion Inerter [15]

via the pinions and gears. The equation for the ideal inertance of a rack-and-pinion inerter is dependent on two variables: the moment of inertia of the flywheel (I), and the gear ratio (η). r_1 , r_2 and r_3 are the radius of gears A, B and C respectively. It can be mathematically expressed as follows:

$$b = \eta^2 \cdot I$$

$$\eta = \frac{r_2}{r_1 r_3}$$

c) Hydraulic Inerter

Fig. (1.4) depicts the schematic drawing of the hydraulic inerter, which employs a hydraulic mechanism to drive the flywheel. This inerter includes a hydraulic motor, a cylinder, a piston, a flywheel, and connecting pipes. When the piston moves, it creates a pressure difference between the two sides of the hydraulic motor (i.e., P3 and P4), causing fluid to flow through the pipes. The flowing fluid drives the rotation of the hydraulic motor, and ultimately the rotation of the

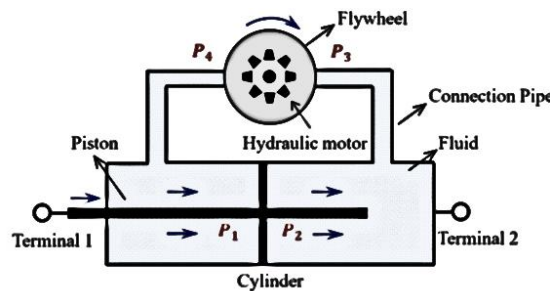


Figure I.3 Schematic of Hydraulic Inerter [15]

flywheel. The equation for ideal inertance of the hydraulic inerter can be derived using the following formula:

$$b = \left(\frac{A}{D}\right)^2 \cdot I$$

Where, A is the working area of the hydraulic cylinder, D is the pitch of the hydraulic motor, and I is the moment of inertia of the flywheel.

d) Helical Fluid Inerter

The helical fluid inerter employs the flow of fluid through an external channel to produce inertance. Fig. (1.5) illustrates the schematic drawing of a helical fluid inerter, which comprises a piston, a cylinder, and a helical channel. As the piston

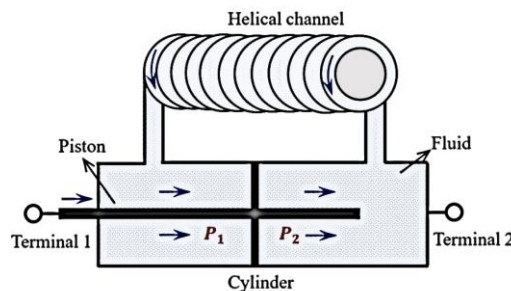


Figure I.4 Schematic of Helical Fluid Inerter [15]

moves relative to the cylinder, a pressure difference is created between the left and right chambers, causing the fluid to flow through the external helical channel. The fluid velocity in the helical channel is scaled up by the ratio of the cylinder area to the helical channel area. The equation for the ideal inertance of the helical fluid inerter can be calculated using the following formula:

$$b = \rho l_h \frac{A_1^2}{A_2}$$

The symbols used represent the following variables: ρ for the density of the fluid, l_h for the length of the helical channel, A_1 for the annular area of the cylinder, and A_2 for the cross-sectional area of the helical channel.

e) Electromagnetic Inerter

The diagram in Figure (1.6) depicts an electromagnetic inerter comprising a coil winding, magnet rod, and capacitor. As the magnet rod moves, it causes a change in magnetic flux which, in turn, generates a voltage across the coil. When

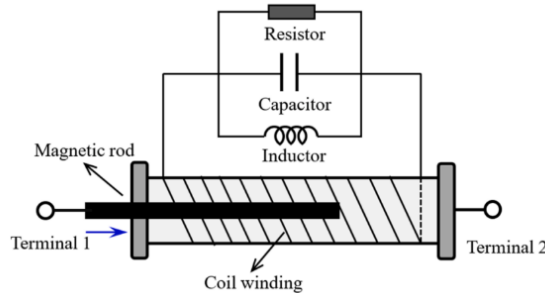


Figure I.5 Schematic of Electromagnetic Inerter [15]

current flows through the coil, it creates an opposing magnetic field resulting in a resistant force. The total inductance of the electromagnetic inerter can be determined using the following formula:

$$b = (Bl)^2 C_a$$

Where, Bl is the coupling coefficient, and C_a is the capacitance.

f) Living-Hinge Inerter

Figure (1.7) illustrates a living-hinge inerter that comprises two connecting rods, one flywheel, and two living hinges. The flywheel has one living hinge positioned at its centre, while the other living hinge is offset from the centre by a

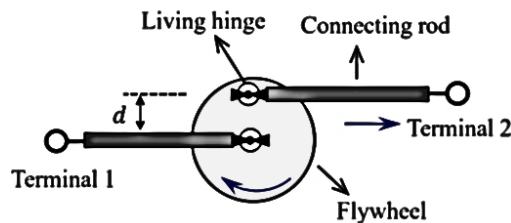


Figure I.6 Schematic of Living-Hinge Inerter [15]

distance d . This design enables the transformation of linear motion into rotational motion of the flywheel, thereby achieving the lever arm effect. The ideal inductance of the living-hinge inerter can be determined using the following formula:

$$b = \frac{1}{d^2} \cdot I$$

C. Advantages of Inerter Dampers over Mass Dampers

Inerter dampers are becoming a popular solution for reducing vibrations in buildings and structures due to several advantages they offer over mass dampers. One of the most significant advantages is their size and weight. Inerter dampers are smaller and lighter, making them more practical to integrate into buildings, particularly in high-rise buildings where space is at a premium. This feature makes it easier to install the dampers and minimize any disruption to building occupants during the installation process. Additionally, it makes it possible to install them in buildings where installing a mass damper would not be possible due to structural constraints.

Another significant advantage of inerter dampers is their broadband performance. They can effectively reduce vibrations across a wide range of frequencies, which is not possible with mass dampers. Inerter dampers produce a force that is proportional to acceleration rather than displacement. This unique characteristic enables them to work effectively over a

broader frequency range, making them more efficient in damping vibrations than mass dampers. As a result, buildings equipped with inerter dampers can have reduced vibrations across a wide range of frequencies, providing a more comfortable living or working environment for occupants.

Inerter dampers also require less maintenance than mass dampers, making them a more cost-effective solution. Mass dampers can be costly and time-consuming to inspect and maintain due to their complex support structures and installation procedures. In contrast, inerter dampers have fewer moving parts and are less prone to wear and tear, making them less costly and less time-consuming to maintain. This feature is particularly important in large buildings, where maintenance costs can add up quickly over time.

Furthermore, inerter dampers are simpler to design and install than mass dampers. As a result, they can be installed more quickly and with less disruption to building occupants. They require fewer materials and have fewer installation procedures, making them a more straightforward and cost-effective solution for reducing vibrations in buildings and structures. Additionally, since they are simpler to design, it is easier to optimize their performance for specific applications, leading to more efficient solutions and lower costs.

Lastly, inerter dampers are often less expensive than mass dampers, making them a more cost-effective solution overall. This is because they require fewer materials and are easier to install, reducing the overall cost of the system. Inerter dampers are also becoming more widely available, and advancements in technology are making them even more efficient and cost-effective.

In conclusion, inerter dampers offer several advantages over mass dampers, including their size and weight, broadband performance, low maintenance requirements, simplicity in design and installation, and cost-effectiveness. These benefits make them a practical solution for reducing vibrations in buildings and structures, and their increasing popularity is expected to continue in the coming years.

D. Applications of Inerters

The first application of the inerter was in the suspension system of a Formula One racing car in 2005, contributing to McLaren's triumph in the Spanish Grand Prix. However, the public only became aware of the inerter's existence as the "J-damper" after a few articles were published in Autosport in 2008. Since then, the potential advantages of integrating inerter(s) into vibration suppression systems have been demonstrated in various applications spanning railway engineering, civil engineering, aerospace engineering, and automotive engineering. This section reviews selected literature from these fields.

a) Railway Engineering

- Several studies have explored the potential benefits of incorporating inerter dampers in train suspension systems. One study by Wang et al. focused on a one-wheel train model and applied a higher-order controller to investigate the advantages of inerter dampers for improving passenger comfort and dynamic wheel load. The study concluded that optimum admittance could be achieved using Bott-Duffin methods.

- Wang and Minkai also conducted a study to verify the benefits of inerter dampers for lateral stability in train suspension systems using 12-DOF and 16-DOF train models. They found that incorporating inerter dampers could improve lateral stability, which is a critical aspect of train performance and passenger comfort.

- Furthermore, Wang et al. conducted a study on a 28-DOF full-train model and concluded that using inerter dampers in the train suspension layout could improve critical speed, settling time, and passenger comfort compared to the conventional suspension layout.

- In another study by Jiang et al., a 7-DOF plain-view train model was used to investigate the benefits of inerter dampers in the secondary train suspension systems. The study found that inerter dampers could effectively reduce lateral body movement and acceleration, leading to improved passenger comfort.

- Moreover, incorporating inerter dampers in both lateral and vertical suspensions of two-axle railway vehicle models could potentially improve lateral and vertical ride comfort by up to 39% and 12.2%, respectively.

- Finally, a recent study by Lewis et al. found that incorporating inerter dampers in the lateral suspension of trains could improve passenger comfort by up to 43% and reduce track wear simultaneously.

- Overall, these studies suggest that incorporating inerter dampers in train suspension systems can improve passenger comfort, reduce dynamic wheel load and lateral body movement, and improve train performance. The use of inerter dampers has the potential to enhance the ride quality for passengers, reduce wear and tear on the tracks, and provide a more comfortable and enjoyable travel experience.

b) Civil Engineering

- The use of inerters or similar devices in vibration suppression systems of civil engineering structures has been the subject of various research studies. These studies have aimed to investigate the potential benefits of using inerters in reducing vibrations caused by different sources such as traffic, earthquakes, and support-excited chain-like structural systems.
- One study examined the effectiveness of using an inerter in reducing vibrations on 1-DOF and 2-DOF building models subjected to traffic and earthquakes. The results demonstrated that the inerter has potential for improving vibration suppression in these structures.
- Another study introduced an advanced tuned mass-damper-inerter (TMDI) that could reduce vibration in chain-like structural systems with a lighter weight or improve the performance with the same total weight. This approach could be beneficial for structures where weight reduction is important.
- The Tuned-Inerter-Damper (TID) was proposed in another study to reduce vibrations caused by base excitation inputs, such as earthquakes, in a multi-storey building model. The study also assessed the influence of attachment stiffness by connecting an additional viscous damper and inerter to the TMD.
- The Fixed-Size-Inerter (FSI) concept was proposed in yet another study to diminish the maximum relative displacement of a 3-storey building. This concept was validated using earthquake input signals and showed promising results for reducing the displacement of the structure.
- The possibility of using the TID to control the vibration of bridge cables was investigated in another study. The results showed that the TID has advantages over the viscous damper, including the ability to overcome the maximum level modal damping limitation provided by the viscous damper when connected to a specified position.
- Finally, a systematic approach was presented to identify the optimal vibration controller layout for cable vibration suppression. Using this method, several optimum inerter-based layouts were identified and verified to improve the cable critical damping ratio compared to the traditional viscous damper. These studies suggest that inerters have potential benefits in reducing vibrations in civil engineering structures and can offer improved performance over traditional vibration suppression systems.

c) Aerospace Engineering

- The use of inerter in landing gear suspension systems has been a topic of research in recent years, and several studies have shown its effectiveness in improving shimmy performance. One advantage of the inerter is its ability to reduce natural frequency and settling time within a specific range of inertances, as demonstrated in one study that examined torsional vibration models with and without lateral flexibility.
- Another study compared inerter-involved layouts to optimised spring-damper layouts and found significant improvements in torsional-yaw motion, including a 16.7% decrease in peak amplitude and up to 57.3% improvement in settling time. This suggests that inerter-based layouts have the potential to outperform traditional spring-damper systems in certain landing gear applications.
- Further research has focused on the benefits of inerter-based layouts for shimmy-suppression devices on main-landing-gear systems. These studies have shown that the use of inerter can lead to a reduction in the lower bound on the maximum zero-shimmy region and an increase in the maximum width in damping of the zero-shimmy region. These improvements in shimmy performance can lead to a smoother and safer landing for aircraft.
- Incorporating an additional inerter into the landing gear shock strut was also found to have several benefits. This included improved touchdown performance, efficiency, maximum strut load, and maximum strut stroke. These improvements can lead to a safer and more efficient landing, reducing wear and tear on the aircraft and improving overall performance.
- Overall, the research suggests that inerter-based layouts have significant potential for improving the performance of landing gear suspension systems, particularly in reducing shimmy and improving landing efficiency and safety. Further research is needed to fully explore the range of benefits that inerter can provide in this application.

d) Automotive Engineering

- In recent years, researchers have conducted several studies on semi-active vehicle suspension systems that incorporate inerter technology. These studies have shown that inerter-based semi-active suspension systems offer numerous benefits over other types of suspension systems.
- One such study compared inerter-based semi-active suspension to ordinary semi-active suspension, and found that the former provided improved ride comfort, suspension deflection, and tyre grip. The study also found that the inerter-based system was more effective at reducing body roll during cornering, which led to better vehicle stability and increased safety.
- Another study compared inerter-based semi-active suspension to passive suspension, and found that the former provided significant improvements in ride comfort, suspension deflection, and tyre grip. The study also found that the

inertor-based system was able to adapt to changing road conditions more quickly than the passive system, which led to improved safety and better overall vehicle performance.

- While many of these studies have focused on passenger vehicles, there has also been research into the benefits of inertor-based suspension for heavy vehicles, such as trucks. One study investigated the dynamic tyre load of trucks and found that inertor-based suspension was able to reduce the load on the tyres, which led to improved tyre wear and longer tyre life.
- In addition to the benefits of inertor-based suspension for passenger and heavy vehicles, there is also the issue of road damage to consider. Road damage not only affects the economic benefits of road transportation, but also poses a safety risk to vehicles and passengers. Inertor-based suspension systems have been found to reduce the amount of road damage caused by heavy vehicles, which can lead to significant cost savings and increased safety.
- Overall, these studies have demonstrated that inertor-based suspension systems offer numerous benefits over other types of suspension systems for both passenger and heavy vehicles.

II. LITERATURE REVIEW

Smith, ^[15] (2002) presents a study on the synthesis of mechanical networks using a new passive mechanical element called the inertor. In this study, a new passive mechanical element called the inertor is introduced, and its potential applications in solving mechanical design problems using modern control methods are discussed. The paper begins by drawing an analogy between mechanical and electrical networks, where force and velocity correspond to current and voltage, respectively. The authors point out the limitation in the correspondence between mass and capacitor due to the force-velocity relationship satisfied by the mass, which makes it difficult to apply classical theorems on electrical networks to the mechanical context. To overcome this limitation, the paper proposes the inertor, a new mechanical element that can be used to solve various mechanical design problems. The inertor is the true network dual of the mechanical spring and can be realized easily. By adding the inertor to a mechanical system, it is possible to obtain a new degree of freedom that can be controlled independently of the other degrees of freedom. This property of the inertor makes it a powerful tool for solving mechanical design problems. The authors provide examples of how the inertor can be used to solve various mechanical design problems, such as vibration control of structures, vehicle suspension design, and robotics. The paper concludes that the ideal inertor is a new mechanical element that can be used to solve mechanical design problems using modern control methods. The potential applications of the inertor in various mechanical systems are discussed, and it is suggested that the inertor has the potential to revolutionize the field of mechanical design. The paper suggests that future work can explore the possibility of using the inertor to simulate a mass element with one of its terminals connected to ground. This could have potential benefits in certain situations.

Swift et al., ^[17] (2013) present a study on the design and modelling of a fluid inertor, a passive mechanical device that can reduce vibrations in structures subjected to dynamic loads. The paper introduces a new implementation of an ideal mechanical modelling element called the inertor, which has the property that the applied force at the terminals is directly proportional to the relative acceleration between them. The new implementation uses the mass of a fluid flowing through a helical channel to provide the inertance. The paper also discusses how general passive mechanical impedances can be realized using mechanical circuits comprising springs, dampers, and inertors only. The inertor has been successfully used in Formula 1 race cars and has been given the name 'J-damper'. The paper presents a new implementation of an ideal mechanical modelling element called the inertor, which uses the mass of a fluid flowing through a helical channel to provide the inertance. The paper also discusses how general passive mechanical impedances can be realized using mechanical circuits comprising springs, dampers, and inertors only. The authors have presented a mathematical model of the fluid inertor and compared it with experimental data from prototypes to investigate parasitic damping effects of the fluid. The paper presents a new implementation of the inertor using a fluid flowing through a helical channel to provide the inertance. The authors have presented a mathematical model of the fluid inertor and compared it with experimental data from prototypes to investigate parasitic damping effects of the fluid. The results show that the fluid inertor has advantages over mechanical ball screw devices in terms of simplicity of design and can be readily adapted to implement various passive network layouts. The authors have also demonstrated that variable orifices and valves can be included to provide series or parallel damping. The paper concludes that the fluid inertor implementation introduced in the paper is robust and durable due to its simple design and the crossover with existing damper construction. The device size is comparable to ball screw implementations with the additional advantage that variable valves and flow restrictions may be included to compactly realize more suspension networks. The authors have shown that the device can be modelled as an ideal inertor in parallel with a nonlinear parasitic damping component. This parasitic damping can be calculated from the device geometry and fluid properties. The inertance and device compliance have been estimated from the experimental data by optimizing these model parameters to fit simulation results to the observed time series.

Lazar et al., ^[6] (2014) present a study on the use of an inerter-based device for structural vibration suppression. The paper introduces a new type of passive control system for suppressing unwanted vibrations in civil engineering structures using an inerter device. The inerter is a two-terminal flywheel device that can generate high apparent mass and was initially developed for Formula 1 racing cars suspension systems. The paper presents an analytical tuning procedure for inerter-based systems inspired by traditional tuning rules for damped vibration absorbers. The performance of the inerter-based system is compared to traditional tuned mass dampers and is shown to be effective in suppressing the response of all modes. The methods used in this paper include the development of a new type of passive control system for suppressing unwanted vibrations in civil engineering structures using an inerter device. The paper also presents an analytical tuning procedure for inerter-based systems inspired by traditional tuning rules for damped vibration absorbers. The performance of the inerter-based system is verified through numerical analysis of a multiple-degree-of-freedom structure subjected to a range of excitation inputs, including wind and earthquake loads. The paper presents a new type of passive control system for suppressing unwanted vibrations in civil engineering structures using an inerter device. The inerter-based system is shown to be effective in suppressing the response of all modes and is comparable or superior to traditional tuned mass dampers. The paper also presents an analytical tuning procedure for inerter-based systems inspired by traditional tuning rules for damped vibration absorbers. The new system is most effective when located at ground storey level, which is advantageous for its installation. The performance of the inerter-based system is verified through numerical analysis of a multiple-degree-of-freedom structure subjected to a range of excitation inputs, including wind and earthquake loads. The paper concludes that the inerter-based system is a promising method for suppressing unwanted vibrations in civil engineering structures. The new system is shown to be effective in suppressing the response of all modes and is comparable or superior to traditional tuned mass dampers. The analytical tuning procedure for inerter-based systems is inspired by traditional tuning rules for damped vibration absorbers. The new system is most effective when located at ground storey level, which is advantageous for its installation. The numerical analysis of a multiple-degree-of-freedom structure subjected to a range of excitation inputs, including wind and earthquake loads, verifies the performance of the inerter-based system. There are a few limitations of this paper, which include: The numerical analysis is limited to a multiple-degree-of-freedom structure, and the performance of the inerter-based system may vary for more complex structures. The paper does not consider the cost-effectiveness of the inerter-based system compared to traditional tuned mass dampers. The paper does not consider the long-term durability and maintenance requirements of the inerter-based system. The paper does not consider the effect of temperature and other environmental factors on the performance of the inerter-based system.

Liu et al., ^[9] (2019) present a study on the development of a generalizable model for fluid-inerter integrated damping devices (FIIDDs). This paper is about a novel element called the inerter, which is used for vibration control in mechanical systems. The paper focuses on fluid-based inerters, which are durable, structurally simple, and easy to integrate with existing damper constructions. The authors developed a generalizable model for an example fluid-inerter integrated damping (FID) design, using hydraulic and mechanical networks to represent the device and tailored experimental testing to characterize each network element. The paper highlights the benefits of incorporating inerters into vibration suppression systems in various areas, such as road vehicles, railway vehicles, tall buildings, cables of suspension bridges, and airplane landing gears. The paper uses a methodology that involves developing a generalizable model for a fluid-inerter integrated damping (FID) design. The approach uses hydraulic and mechanical networks to represent the device and tailored experimental testing to characterize each network element. The authors designed different valve close/open status settings to study the elements separately and achieve more accurate results. The effective schematic diagram, corresponding hydraulic network, and effective mechanical network for each prototype test configuration are plotted in the paper. The paper presents a generalizable model for a fluid-inerter integrated damping (FID) design, which can be used to accurately characterize the terminal behaviour of the device while allowing its design parameters to vary within their pre-defined ranges. The authors also demonstrate the effectiveness of the FID design in reducing vibration in a mechanical system. The results show that the FID design can provide significant vibration reduction compared to traditional damping systems, especially in the low-frequency range. The paper also highlights the potential applications of the FID design in various areas, such as road vehicles, railway vehicles, tall buildings, cables of suspension bridges, and airplane landing gears. The paper presents a methodology for developing a generalizable model for a fluid-inerter integrated damping (FID) design, which can be used to accurately characterize the terminal behaviour of the device while allowing its design parameters to vary within their pre-defined ranges. The results show that the FID design can provide significant vibration reduction compared to traditional damping systems, especially in the low-frequency range. The paper highlights the potential applications of the FID design in various areas, such as road vehicles, railway vehicles, tall buildings, cables of suspension bridges, and airplane landing gears.

Wen et al., ^[18] (2017) present a study on the design and evaluation of tuned inerter-based dampers for the seismic control of multi-degree-of-freedom (MDOF) structures. This paper investigates the performance of two types of tuned inerter-based dampers (TIBDs) called the tuned viscous mass damper (TVMD) and tuned inerter damper (TID). The contributions of this paper are: The paper proposes a new type of TIBD called the TVMD, which combines the benefits

of a tuned mass damper (TMD) and an inerter damper. The paper also investigates the performance of the TID, which is another type of TIBD that utilizes the amplified apparent mass and tuning effects of an inerter. The paper provides a detailed analysis of the TVMD and TID, including their mathematical models, design procedures, and performance evaluation. The paper compares the TVMD and TID with other types of TIBDs and shows that they can provide better vibration control performance in certain scenarios. The paper also discusses the practical implementation of the TVMD and TID and provides recommendations for their design and installation. In this paper, the authors used the following methods: They proposed a new type of tuned inerter-based damper (TIBD) called the tuned viscous mass damper (TVMD) and investigated its performance. They also analysed the performance of another type of TIBD called the tuned inerter damper (TID). They developed mathematical models for the TVMD and TID and used them to evaluate their performance in reducing structural vibrations. They compared the TVMD and TID with other types of TIBDs and showed their advantages in certain scenarios. They provided recommendations for the design and installation of the TVMD and TID in practical applications. The paper presents the results of the investigation of the performance of two types of tuned inerter-based dampers (TIBDs) called the tuned viscous mass damper (TVMD) and tuned inerter damper (TID). The results show that both TVMD and TID can provide effective vibration control in certain scenarios. The TVMD is found to be more effective in reducing the vibration of structures with low damping ratios, while the TID is more effective in reducing the vibration of structures with high damping ratios. The paper also shows that the TVMD and TID can outperform other types of TIBDs in certain scenarios. The authors provide recommendations for the design and installation of the TVMD and TID in practical applications. The conclusions from the paper are that the tuned viscous mass damper (TVMD) and tuned inerter damper (TID) are effective types of tuned inerter-based dampers (TIBDs) that can provide better vibration control performance in certain scenarios. The TVMD is more effective in reducing the vibration of structures with low damping ratios, while the TID is more effective in reducing the vibration of structures with high damping ratios. The paper also provides recommendations for the design and installation of the TVMD and TID in practical applications.

Liu et al., ^[8] (2018) present a study on the model identification methodology for fluid-based inerters, which are passive mechanical devices used for vibration control in engineering structures. This paper is about the inerter, a mechanical device used for passive vibration control in mechanical systems. The inerter has the property that the force is proportional to the relative acceleration across the two terminals. The paper focuses on the fluid-based inerter, which has advantages over the flywheel-based inerter in terms of durability, damping, and simplicity of design. The paper proposes a comprehensive model identification methodology to improve the understanding of the physical behaviour of the fluid-based inerter, especially caused by the hydraulic resistance and inertial effects in the external tube. The methodology involves a modelling procedure and an experimental sequence to identify friction, stiffness, and various damping effects. The paper presents theoretical models with improved confidence for a helical-tube fluid inerter prototype. The paper proposes a comprehensive model identification methodology to improve the understanding of the physical behaviour of the fluid-based inerter. The methodology involves a modelling procedure and an experimental sequence to identify friction, stiffness, and various damping effects. The theoretical models with improved confidence are obtained using the proposed methodology for a helical-tube fluid inerter prototype. The paper presents theoretical models with improved confidence for a helical-tube fluid inerter prototype. The sources of remaining discrepancies are further analysed. The proposed methodology can be used to improve the understanding of the physical behaviour of the fluid-based inerter, especially caused by the hydraulic resistance and inertial effects in the external tube. The paper presents a lumped parameter hydraulic model for fluid-based inerters, which allows the equivalent mechanical model to be established. The damping, inertance, and stiffness elements in the mechanical model are mapped directly to the hydraulic resistance, inertance, and compliance effects in the hydraulic model. The paper proposes a general model identification procedure to enhance the modelling accuracy, which enables different damping terms to be identified separately. The refined network of the helical-tube fluid inerter shows that the leakage damping is negligible, and the theoretical formula for the helical tube damping is chosen by exploring the fitting error between multiple candidate models.

Zhang et al., ^[19] (2017) present a study on the optimal configurations for a linear vibration suppression device (LVSD) in a multi-storey building. This paper is about finding ways to reduce the vibrations of buildings during earthquakes. The paper discusses the use of tuned mass dampers (TMDs) as a way to reduce vibrations, but notes that TMDs have downsides such as their weight and the need to place them on top of the building. The paper proposes the use of a two-terminal vibration suppression device called an inerter, which has been used in Formula 1 racing cars and other systems. The paper studies the use of inerter-spring-damper configurations for a multi-storey building structure and identifies several devices that incorporate inerter(s) for vibration suppression. The paper presents four optimum absorber layouts for minimising the maximum relative displacements of the building, taking into account the inerter's size and the brace stiffness. The paper proposes the use of a two-terminal vibration suppression device called an inerter for reducing the vibrations of buildings during earthquakes. The paper studies the use of inerter-spring-damper configurations for a multi-storey building structure and identifies several devices that incorporate inerter(s) for vibration suppression. The paper

presents four optimum absorber layouts for minimising the maximum relative displacements of the building, taking into account the inerter's size and the brace stiffness. The paper also uses a real-life earthquake data to show the advantage of proposed absorber configurations. The paper presents four optimum absorber layouts for minimising the maximum relative displacements of the building, taking into account the inerter's size and the brace stiffness. The corresponding parameter values for the optimum absorber layouts are also presented. Furthermore, a real-life earthquake data is used to show the advantage of proposed absorber configurations. The results show that the proposed inerter-based absorber configurations can effectively reduce the vibrations of buildings during earthquakes. The paper proposes the use of inerter-based absorber configurations for reducing the vibrations of buildings during earthquakes. The paper presents four optimum absorber layouts for minimising the maximum relative displacements of the building, taking into account the inerter's size and the brace stiffness. The results show that the proposed absorber configurations can effectively reduce the vibrations of buildings during earthquakes.

Patel and Jangid, ^[13] (2022) explores the use of the modal multiplicity criteria for the optimum tuning of mass dampers. This paper discusses the use of tuned mass dampers (TMD) in high-rise structural systems to control vibrations induced by wind and seismic activity. TMD is a passive control system that consists of a small mass connected to the structural system through an elastic spring and a dashpot. The paper highlights the importance of tuning the TMD properly to absorb undesirable energy and suppress unwanted oscillatory responses of the system. The paper also mentions that while there have been many studies on tuning TMD with undamped and damped systems, a closed-form solution for the damped system is currently unavailable with zero tolerance. The paper uses a modal multiplicity criterion to derive closed-form solutions for the optimum parameters of a damped TMD system. The derived parameters depend solely on the structural damping and mass ratio. The paper also investigates the effect of parametric uncertainty on the TMD system's response under harmonic and earthquake excitations. Finally, the paper uses the energy concept to evaluate the effectiveness of the optimum TMD parameters in reducing kinetic energy, damping energy, strain energy, and input energy. The paper presents closed-form solutions for the optimum parameters of a damped TMD system based on the modal multiplicity criterion. The derived parameters depend solely on the structural damping and mass ratio. The paper also investigates the effect of parametric uncertainty on the TMD system's response under harmonic and earthquake excitations. The results show that the derived optimum parameters are effective in reducing the kinetic energy, damping energy, strain energy, and input energy of the TMD system. The paper also compares the derived parameters with existing parameters in the field. The conclusions drawn from the paper are: The modal multiplicity criterion can be used to derive closed-form optimum parameters for a damped TMD system. The derived optimum parameters depend solely on the structural damping and mass ratio and are independent of the excitation frequency of the input force. The derived parameters result in a relatively lower optimum tuning frequency ratio but a higher optimum TMD damping ratio, making TMD less frequency-sensitive devices. The derived parameters are unique for the displacement response and the acceleration response. The derived optimum parameters are effective in reducing the kinetic energy, damping energy, strain energy, and input energy of the TMD system.

Ikago et al., ^[14] (2012) present a study on the seismic control of a single-degree-of-freedom structure using a tuned viscous mass damper (TVMD). The paper discusses the use of supplemental energy dissipation devices for reducing the response of building systems to wind-induced or earthquake-induced excitation. The tuned mass damper is one such device that achieves substantial energy dissipation by applying large deformation and resistance force of the viscous material in the device by tuning the fundamental period of the supplemental oscillator to that of the primary system. The paper proposes a new seismic control device, tuned viscous mass damper (TVMD), for building systems and compares its performance with conventional viscous damper and viscous mass damper systems. The effectiveness of the TVMD for seismic excitation is verified by analyses and shake table tests with a small-scale TVMD. The paper proposes a new seismic control device, tuned viscous mass damper (TVMD), for building systems. The paper gives a detailed description of an apparent mass amplifier using a ball-screw mechanism, which is one of the most important components for realizing the new device. The paper also derives a closed-form solution of an optimum seismic control design for a single-degree-of-freedom structure subjected to harmonic excitation. The performance of the new device is compared with those of the conventional viscous damper and viscous mass damper systems. The effectiveness of the TVMD for seismic excitation is verified by analyses and shake table tests with a small-scale TVMD. The paper shows that the vibration control system using the TVMD is the most effective for linear structural systems with dampers having the same additional damping coefficient. The TVMD is shown to be effective in reducing the response of building systems to seismic excitation. The paper also provides a closed-form solution of an optimum seismic control design for a single-degree-of-freedom structure subjected to harmonic excitation. The effectiveness of the TVMD for seismic excitation is verified by analyses and shake table tests with a small-scale TVMD. The paper suggests that future work could involve the development of a practical design method for the TVMD system for multi-degree-of-freedom structures. Additionally, the paper suggests that further experimental studies could be conducted to verify the effectiveness of the TVMD system for seismic excitation in full-scale structures.

Gonzalez-Buelga et al., ^[3] (2017) investigates the performance of a tuned inerter damper in the presence of nonlinearities. This paper discusses the use of a tuned inerter damper (TID) as a vibration absorber in civil engineering applications. Inerters are mechanical devices that complete the analogy between mechanical and electrical networks, and were developed in the 2000s. The paper explains that the ideal inerter is a mechanical device with the property that the equal and opposite force applied at the nodes is proportional to the relative acceleration between the nodes. The paper also discusses the challenges of using real inerter devices, which have nonlinear dynamics, in civil engineering applications. The paper proposes a new approach for testing inerter devices using real-time dynamic sub-structuring (RTDS) or hybrid testing. The practical implications of this paper are that it provides a method for testing the performance of a tuned inerter damper (TID) as a vibration absorber in civil engineering applications. The paper proposes a new approach for testing TID devices using real-time dynamic sub-structuring (RTDS) or hybrid testing, which allows for the optimization of TID parameters without altering the experimental setup. The paper also discusses the challenges of using real inerter devices, which have nonlinear dynamics, in civil engineering applications. The results of the study show that with appropriate retuning of the components in the TID device, the performance of the TID incorporating the real inerter device is close to the ideal inerter device. This research could lead to the development of more effective vibration absorbers for civil engineering structures. The paper proposes a new approach for testing the performance of a tuned inerter damper (TID) using real-time dynamic sub-structuring (RTDS) or hybrid testing. The inerter was tested physically, while the remaining components of the TID device, the spring and damper, together with the host structure, were simulated numerically. Displacements and forces at the interface between numerical and physical components are updated in real time. The paper also proposes a new sub-structuring algorithm for physical inertial substructures, which consists of feeding back the measured force deviation from the ideal inerter instead of the actual force at the interface. The results of the paper show that with appropriate retuning of the components in the TID device, the performance of the TID incorporating the real inerter device is close to the ideal inerter device. The proposed sub-structuring algorithm for physical inertial substructures, which consists of feeding back the measured force deviation from the ideal inerter instead of the actual force at the interface, was found to be effective in dealing with the delays arising at the interface between the experimental and the numerical substructures. The paper concludes that the performance of a tuned inerter damper (TID) used as a vibration absorber was studied analytically and experimentally. The study included nonlinearities to study their effect on the TID performance. It was shown that the optimisation of a linear TID when the external forcing is acting on the host structure mass is equivalent to that of tuned mass dampers (TMDs), proposed by Den Hartog. Following the inclusion of dry friction as a nonlinear parameter, it was concluded that the retuning of the linear TID is only necessary for low-amplitude loading scenarios, as the behaviour of the device converges to the ideal one for higher loads. However, retuning the TID to obtain optimal performance under low loading is detrimental if the host structure, then experiences higher amplitude forcing, while the reciprocal is not as critical. The paper suggests that future work could include the study of the effect of nonlinearities on the TID performance in more detail. Additionally, the paper proposes the development of a more comprehensive sub-structuring algorithm that can deal with the delays arising at the interface between the experimental and the numerical substructures. Finally, the paper suggests that further research could be conducted to investigate the potential of TID devices for civil engineering applications in more detail.

Zhang et al., ^[20] (2017) present a novel approach for designing passive vibration control systems using a structure-immittance approach. This paper proposes a new approach called the structure-immittance approach for designing linear passive vibration absorbers. This approach allows for investigating a class of absorber possibilities together while controlling the complexity, topology, and element values in resulting absorber configurations. The proposed structure-immittance approach provides a systematic way to design linear passive vibration absorbers, allowing for the investigation of a full set of possible networks with predetermined numbers of each element type. This approach provides the ability to control the complexity, topology, and element values in resulting absorber configurations, which can be useful in various applications such as building vibration suppression and automotive suspension design. The approach can help in optimizing the design of passive vibration control devices, leading to improved performance and reduced vibration levels. The paper proposes a new approach called the structure-immittance approach for designing linear passive vibration absorbers. This approach involves a general formulation process to obtain structural immittances, which can represent a full set of possible series-parallel networks with predetermined numbers of each element type. The proposed approach combines the advantages of both the structure-based and immittance-based approaches, providing the ability to investigate a class of absorber possibilities together while controlling the complexity, topology, and element values in resulting absorber configurations. The effectiveness of the proposed approach is demonstrated through two case studies on building vibration suppression and automotive suspension design. The paper proposes a new approach called the structure-immittance approach for designing linear passive vibration absorbers. The proposed approach provides a systematic way to investigate a full set of possible networks with predetermined numbers of each element type, while controlling the complexity, topology, and element values in resulting absorber configurations. The effectiveness of the proposed approach is demonstrated through two case studies on building vibration suppression and automotive suspension design. The results show that the proposed approach can help in optimizing the design of passive vibration

control devices, leading to improved performance and reduced vibration levels. The paper proposes a new approach called the structure-immittance approach for designing linear passive vibration absorbers. The proposed approach provides a systematic way to investigate a full set of possible networks with predetermined numbers of each element type, while controlling the complexity, topology, and element values in resulting absorber configurations. The effectiveness of the proposed approach is demonstrated through two case studies on building vibration suppression and automotive suspension design. The results show that the proposed approach can help in optimizing the design of passive vibration control devices, leading to improved performance and reduced vibration levels.

Lazar et al.,^[7] (2014) presents a design and performance analysis of inerter-based vibration control systems. This paper proposes a new type of passive vibration control system to reduce vibrations in civil engineering structures based on the inerter. The inerter is a device that was initially developed for high-performance suspensions in Formula 1 racing cars. The paper discusses the properties of the inerter and how it can be used to achieve a high level of vibration isolation with low amounts of added mass. The paper compares the inerter-based system to traditional tuned mass dampers (TMDs) and demonstrates how the performance could potentially be improved by using an inerter instead of a TMD. The practical implications of this paper are that the inerter-based system proposed in this paper could potentially offer an alternative to traditional tuned mass dampers (TMDs) for reducing vibrations in civil engineering structures subject to base excitation. The inerter system is shown to achieve an excellent level of vibration reduction with low amounts of added mass. The paper provides a comparison between the inerter-based system and TMDs, demonstrating how the performance could potentially be improved by using an inerter instead of a TMD. The findings of this paper could be useful for engineers and designers who are looking for new ways to reduce vibrations in structures. The paper proposes a new type of passive vibration control system based on the inerter. The inerter system is modelled inside a multi-storey building and is located on braces between adjacent storeys. Numerical simulations are used to compare the inerter-based system to traditional tuned mass dampers (TMDs) under a range of base excitation inputs, including an earthquake signal. The paper presents the results of these simulations and discusses the potential advantages of using an inerter-based system over a TMD system. The paper presents numerical results that show the inerter-based system achieves an excellent level of vibration reduction in a multi-storey building. The results also demonstrate that the inerter-based system could potentially offer improvement over traditional tuned mass dampers (TMDs) under a range of base excitation inputs, including an earthquake signal. The paper concludes that the inerter-based system could be a promising alternative to TMDs for reducing vibrations in civil engineering structures. The paper concludes that the inerter-based system proposed in this paper could potentially offer an alternative to traditional tuned mass dampers (TMDs) for reducing vibrations in civil engineering structures subject to base excitation. The inerter system achieves an excellent level of vibration reduction with low amounts of added mass. The paper provides a comparison between the inerter-based system and TMDs, demonstrating how the performance could potentially be improved by using an inerter instead of a TMD. The findings of this paper could be useful for engineers and designers who are looking for new ways to reduce vibrations in structures. The paper presents a theoretical study of the inerter-based system for reducing vibrations in civil engineering structures. However, the paper does not provide experimental validation of the proposed system. Therefore, the limitations of this paper are that the proposed system needs to be tested experimentally to validate its effectiveness in reducing vibrations in real-world structures. Additionally, the paper only considers a multi-storey building as a case study, and the proposed system's effectiveness needs to be tested for other types of structures.

Smith and Wang,^[16] (2003) investigates the potential benefits of using inerters in passive vehicle suspensions. This paper introduces a new mechanical element called the inerter, which can potentially improve the vehicle dynamics of a passively suspended vehicle by using suspension struts employing inerters as well as springs and dampers. The paper presents a comparative study of several simple passive suspension struts, each containing at most one damper and inerter, and demonstrates improved performance in a quarter-car and full-car model. The introduction of this paper discusses the traditional suspension strut, which employs springs and dampers only and avoids the mass element. It is argued that such a strut has limited dynamic characteristics in comparison to suspension struts employing inerters as well as springs and dampers. The paper proposes the use of a new mechanical element called the inerter, which can potentially improve the vehicle dynamics of a passively suspended vehicle. In this paper, the authors conducted a comparative study of several simple passive suspension struts, each containing at most one damper and inerter, to investigate the potential performance advantages of the inerter. They used a quarter-car model and a full-car model to demonstrate the improved performance of the inerter in comparison to a conventional passive suspension strut. The authors also built and tested a prototype inerter and presented experimental results that demonstrate a characteristic phase advance property that cannot be achieved with conventional passive struts consisting of springs and dampers only. The results of the paper demonstrate that the use of the inerter in suspension struts can improve the vehicle dynamics of a passively suspended vehicle. The comparative study of several simple passive suspension struts, each containing at most one damper and inerter, showed improved performance in a quarter-car and full-car model. The experimental results of the prototype inerter also demonstrated a characteristic phase advance property that cannot be achieved with conventional passive struts consisting

of springs and dampers only. The paper concludes that the inerter can potentially improve the vehicle dynamics of a passively suspended vehicle. The comparative study of several simple passive suspension struts, each containing at most one damper and inerter, showed improvements in a quarter-car and full-car model. The improvements were demonstrated across a wide range of static suspension stiffnesses, and for certain combinations of measures, good simultaneous improvement was obtained. The paper suggests that the inerter has the potential to be a useful addition to passive suspension systems. In the paper, the authors suggest several future works, including the investigation of the inerter's potential in active suspension systems, the development of a more detailed model of the inerter, and the exploration of the inerter's potential in other engineering applications. The authors also suggest that further experimental work is needed to validate the inerter's performance in more complex suspension systems.

Pietrosanti et al., ^[14] (2017) present The Tuned Mass Damper Inerter (TMDI) which is a recently developed passive vibration control device that can effectively reduce the structural response to dynamic loads. The paper discusses the design and performance evaluation of a Tuned Mass Damper Inerter (TMDI) system to reduce dynamic vibrations. TMDI is a system that utilizes the properties of an inerter, a mechanical device that can produce a force proportional to the relative acceleration between its terminals. The system consists of a primary structure equipped with a classical linear Tuned Mass Damper (TMD) and a secondary structure whose mass is connected to the ground via an inerter. The paper compares the optimal design of TMDI using three different methodologies and evaluates its performance and robustness compared to conventional and non-conventional TMDs. The paper concludes that TMDI systems are more effective and robust in reducing dynamic response than classical TMDs. The paper uses three different design methodologies to obtain the optimal design of the Tuned Mass Damper Inerter (TMDI) system. These methodologies are displacement minimization, acceleration minimization, and maximization of the ratio between the energy dissipated in the secondary system and the total input energy. The paper compares the results obtained from these methodologies and evaluates the performance and robustness of the TMDI system using a sensitivity analysis. The paper concludes that the Tuned Mass Damper Inerter (TMDI) system is more effective and robust in reducing dynamic response than classical Tuned Mass Dampers (TMDs). The TMDI system outperforms conventional and non-conventional TMDs in terms of reducing the relative displacement variance of the primary structure, as well as the rms and peak values of the primary and secondary structures. The paper also demonstrates the effectiveness of the optimally designed TMDI system in reducing the dynamic response of the primary structure under earthquake base excitation. The paper concludes that the Tuned Mass Damper Inerter (TMDI) system is a more effective and robust solution for reducing dynamic vibrations compared to conventional and non-conventional Tuned Mass Dampers (TMDs). The paper proposes three different design methodologies for the optimal design of the TMDI system and evaluates its performance and robustness using a sensitivity analysis. The results show that the TMDI system outperforms classical TMDs in terms of reducing the relative displacement variance of the primary structure, as well as the rms and peak values of the primary and secondary structures. The paper also demonstrates the effectiveness of the optimally designed TMDI system in reducing the dynamic response of the primary structure under earthquake base excitation. The paper suggests some future works related to the Tuned Mass Damper Inerter (TMDI) system. These include: Investigating the effectiveness of the TMDI system for multi-degree-of-freedom structures. Studying the performance of the TMDI system under non-stationary excitations. Developing a control strategy for the TMDI system to improve its performance. Evaluating the effectiveness of the TMDI system for reducing vibrations in real structures.

III. CONCLUSION

The conclusion highlights the potential of fluid inerter dampers as a promising technology for vibration control in civil-structural engineering. The review provides an overview of the current state of research on fluid inerter dampers, covering their working principle, design considerations, advantages, and limitations. It is evident that fluid inerter dampers have several advantages over traditional vibration control devices, including low mass, low energy consumption, and high durability. These advantages make them suitable for various structural applications and improve the performance of structures during extreme loading events.

However, the review also highlights the need for further research to optimize the design of fluid inerter dampers for different structural applications and explore their potential for energy harvesting and other innovative uses. The review indicates that fluid inerter dampers are still a developing technology, and future research could provide a deeper understanding of their performance and broaden their potential applications.

In conclusion, fluid inerter dampers represent a valuable addition to the toolkit of civil-structural engineers. Their potential advantages over traditional vibration control devices make them a promising technology for improving the performance of structures during extreme loading events. As further research is conducted to optimize their design and explore their potential uses, fluid inerter dampers are expected to gain increasing recognition within the engineering community in the years to come.

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