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An Approach to Relate Time, Extent and Energy of a System

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Abstract: This paper uses some fundamental concepts of physics, takes into account the uncertainties at quantum scale, applies the laws of thermodynamics and kinetic theory of gases and arrives at an inequality which provides a relation between time, extent and energy of a system.

Keywords: Heisenberg's Uncertainty Principle, Average Kinetic Energy, 1st Law of Thermodynamics, Total Internal Energy, Specific Heat, Universal Gas Constant, Avogadro's Number, Uncertainty in Position, Absolute Zero, Planck's Temperature, Dimensional Analysis

INTRODUCTION

This paper starts with the universal formulas and we equate them throughout making assumptions and logic to finally arrive at certain inequality.

The Heisenberg's Uncertainty Principle [1] is expressed as,

$$\Delta x \Delta p \geq \frac{h}{4\pi} \quad \dots (1)$$

The average kinetic energy of a particle [2] of mass m and velocity v_{avg} is given by,

$$K_{avg} = \frac{1}{2} m v_{avg}^2 \quad \dots (2)$$

From 1st law of thermodynamics [3] where U is the total internal energy, Q is the heat in the system and w is the work done on the system,

$$U = Q + w \quad \dots (3)$$

Equation relating heat and temperature where m is mass, C_p is specific heat of a system and$(T_1 - T_2)$ is the chance in temperature,

$$Q = mC_p(T_1 - T_2) \quad \dots (4)$$

Work is defined as w and F is force and x is the distance moved,

$$w = Fx \quad \dots (5)$$

Relating specific heat [4] to temperature (T) and energy (E),

$$C_p = \frac{dE}{dT} \quad \dots (6)$$

Relating Universal gas constant(R) to temperature (T), average kinetic energy (K_{avg}) and N_a in the equation is Avogadro's number.

$$K_{avg} = \frac{3RT}{2N_a} \quad \dots (7)$$

From eqn. (7) and (2)

$$\frac{1}{2} m v_{avg}^2 = \frac{3RT}{2N_a} \quad \dots (8)$$

or,

$$\Delta v_{avg}^2 = \frac{3R\Delta T}{mN_a} \quad \dots (9)$$

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$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

Where x is the distance bounded in one dimension by the isolated box with length d.
We know that particle is somewhere on the axis connecting the edges of the box.

In eqn. (9) uncertainty in average velocity is related to the uncertainty in the temperature only as other three are constants. For uncertainty alone in velocity Δv_{avg}^2 nearing absolute zero where v is otherwise tending to zero (as motion ceases to absolute zero).

Substituting,
$\Delta p = m\Delta v_{avg}$

Eqn. (1) and squaring both sides,
$$(\Delta x)^2 \Delta v_{avg}^2 \geq \frac{h^2}{16\pi^2 m^2} \quad \dots (10)$$
Substituting Δv_{avg} from eqn. (9)

$$(\Delta x)^2 \frac{3R\Delta T}{mN_a} \geq \frac{h^2}{16\pi^2 m^2} \quad \dots (11)$$

Taking the mass from the momentum to RHS and bringing the uncertainties to the LHS, we get

$$\Delta T (\Delta x)^2 \geq \frac{N_a h^2}{48\pi^2 m R} \quad \dots (12)$$

To express in terms of uncertainty in position we first note that.
$$\Delta x = d \quad \dots (13)$$

Since particle only along the axis and within‘d’, anywhere.
$(\Delta x)^2 = d^2$
Therefore,
$$\Delta T \geq \frac{N_a h^2}{48\pi^2 m R d^2} \quad \dots (14)$$

There would be uncertainty in in heat as there is uncertainty in temperatureΔT, in the equation for heat equation (4)

$$\Delta Q = mC_p(T_1 - T_2 + \Delta T) \quad \dots (15)$$

At temperature approaching absolute zero,
$T_1 - T_2 = 0$
Hence,
$$\Delta Q = mC_p \Delta T \quad \dots (16)$$

Rearranging.
$\Delta T = \frac{\Delta Q}{mC_p}$ … (17)

Substituting ΔT in eqn. (14):

$\frac{\Delta Q}{mC_p} \geq \frac{N_a h^2}{48\pi^2 mRd^2}$ … (18)

Solving forΔQ, we get
$\Delta Q \geq \frac{N_a h^2 C_p}{48\pi^2 Rd^2}$ … (19)

Since,
$U = Q + w$,
Mathematically in total internal energy, total heat and work done are related by:
$\Delta U = \Delta Q + \Delta w$ … (20)

ΔU of an isolated system is 0,
Hence,
$\Delta Q = -\Delta w$ … (21)

And from eqn. (5),
$-\Delta w = -\Delta(Fx)$ … (22)

Expressing,
-$\Delta(Fx) = -(\Delta F \Delta x + \delta)$ … (23)

δ Is some unknown uncertainty.
Since,
$F = \frac{dp}{dt}$
We may express RHS of eqn. (23) as,
$-(\Delta F \Delta x + \delta) = -(\Delta \frac{dp}{dt} \Delta x + \delta) = -(m\Delta \frac{dv}{dt} \Delta x + \delta)$ … (24)

Since time is absolute in quantum scale (that is there is no uncertainty in time),
$-(\Delta F \Delta x + \delta) = -(m \frac{d\Delta v}{dt} \Delta x + \delta)$ … (25)

From eqn. (22) and (21) and substituting the value of ΔQ from eqn. (19) we have,
$-\Delta(Fx) \geq \frac{N_a h^2 C_p}{48\pi^2 Rd^2}$ … (26)

Replacing $\Delta(Fx)$ for eqn. (23), we have,

$-(m\Delta \frac{dv}{dt} \Delta x + \delta) \geq \frac{N_a h^2 C_p}{48\pi^2 Rd^2}$

Putting,

$$m\Delta x \frac{d\Delta v}{dt} \Delta \leq -\frac{N_a h^2 C_p}{48\pi^2 Rd^2} - \delta$$

Rearranging,
$\Delta x d\Delta v \geq \frac{N_a h^2 C_p dt}{48\pi^2 mRd^2} - \frac{\delta}{m} dt$ … (27)

Integrating both sides of equation 27

$$\Delta x \int_0^{\Delta v} d\Delta v \leq \int_0^t -\frac{N_a h^2 C_p dt}{48\pi^2 mRd^2} - \int_0^t \frac{\delta}{m} dt$$

Where t is the time particle exists,
Expressing,

$$\int_0^t \frac{\delta}{m} dt = \delta^*$$

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$\Delta v \Delta x \leq -\frac{N_a h^2 C_p t}{48\pi^2 mRd^2} - \delta^*$ … (28)

Also $\Delta v \Delta x \geq \frac{h}{4m\pi}$

$\frac{h}{4m\pi} \leq \Delta v \Delta x \leq -\frac{N_a h^2 C_p t}{48\pi^2 mRd^2} - \delta^*$ … (29)

We thus conclude,
$\frac{h}{4m\pi} \leq -\frac{N_a h^2 C_p t}{48\pi^2 mRd^2} - \delta^*$

$0 \geq +\frac{N_a h^2 C_p t}{48\pi^2 mRd^2} + \delta^* + \frac{h}{4m\pi}$ … (30)

Putting,
$\delta^* + \frac{h}{4m\pi} = -\kappa$ … (31)

We now have,
$\kappa \geq \frac{N_a h^2 C_p t}{48\pi^2 mRd^2}$ … (32)

From eqn. (6),

$$dE = C_p dT$$

10^{32} is called the Planck's Temperature, temperature above it does not exist. Assuming that temperature upto Planck's temperature is achieved in the universe is given by the integral,

$\int_0^E dE = \int_0^{10^{32}} C_p dT$ … (33)

$E = 10^{32} C_p$ … (34)

Substituting C_p from eqn. (34) in eqn. (32), we obtain,

$\kappa \geq \frac{N_a h^2 E t}{48\pi^2 mRd^2} 10^{-32}$ … (34)

On rearranging,

$$\frac{\kappa 48\pi^2 mRd^2}{N_a h^2 E} 10^{32} \geq t$$

$t \leq \frac{\kappa 48\pi^2 mRd^2}{N_a h^2 E} 10^{32}$ … (35)

So t is the total time the universe could exists as it is the maximum time any particle could exist.
'd' is the extent of the box. Taking the isolated system, d would the extent of the universe.
E is the total energy of the universe including U + energy converted from mass = U + mc^2
κ is an unknown factor.

This inequality relates the total energy in the universe, to its extent and its age. Thus if we know total energy, we can determine the age of the universe. The physical interpretation of κ is not known but has the dimension determined by a dimensional analysis of kappa constant and kappa factor:

$$t \leq \frac{\kappa 48\pi^2 mRd^2}{N_a h^2 E} 10^{32}$$

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$$C_\kappa = \frac{48\pi^2 mR}{N_a h^2} 10^{32}$$

Where C_κ is a constant and κ is the kappa factor.

$$t \leq C_\kappa \frac{\kappa d^2}{E}$$

Dimension of C_κ is arrived at, $[L^{-2}K^{-1}]$

$$\kappa \geq \frac{tE}{C_\kappa d^2}$$

Dimension of κ is,
$[ML^2T^{-1}K]$

Where,
M – Mass
L – Length
T – Time
K – Temperature

CONCLUSION

Thus we conclude, that the energy, physical extent of an isolated system and period of its existence could be related by an inequality. Applying the approach to our universe and taking maximum value of energy and temperature possible, we can arrive at the upper limit of the age of the universe.

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