

Concrete Pictorial Abstract (CPA) Approach in Mathematics Problem Solving

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Abstract: Problem solving requires analytical reasoning skills and comprehension. It is sometimes difficult to solve a problem due to misrepresentation of words problem. Thus, visual representation is an important factor in problem solving. This research determined the effects of the Concrete-Pictorial-Abstract-Approach on the Mathematics Problem Solving Performance of the First Year Bachelor of Industrial Technology students at Capiz State University, Main Campus. This study employs quasi-experimental research design, the matching only of pre-test and post-test of the experimental and control group. There were fifty-two (52) participants coming from two (2) different sections, twenty-six (26) of whom were the experimental group and the other twenty-six (26) were the control group. The research instrument used is a researcher made test in Mathematics problem solving validated by three experts in Mathematics teaching, and pilot tested at Capiz State University, Main Campus. The instrument underwent reliability test and the result show a reliability index of 0.76 Cronbach alpha. Descriptive statistics was aided by frequency, mean, and percentage standard deviation and mean gain while the inferential tools were the t-test for independent samples and Wilcoxon Signed Rank Test for related samples. All levels of significance were set at 0.05 alpha. Descriptively the pre-test of the mathematics problem solving performance of the two instructional groups was "low". The post-test of the mathematics problem solving performance of the experimental was "high" while the control group instructions was marked average. Mathematics problem solving performance mean gain of the experimental group showed development from "low" to "high". The inferential analysis revealed that there were no significant differences in the pre-test scores of the two groups of mathematics problem solving performance.

Keywords: quasi-experimental, block model approach, visualization

I. INTRODUCTION

Problem solving is mathematical skill students are exposed to throughout their entire schooling. This skill is taught and assessed in every grade level, from elementary until college. [1] Problem solving is taught in various forms, the most common being through word problems (Flores et al., 2016). [2] Zhu (2015) defines word problems as linguistically presented problems requiring arithmetic solutions. A student's mathematic ability is often assessed through their performance on word problems, whether it is on their daily or weekly quiz, summative assessments, or final examinations, and even in licensure examinations or career eligibility examination. A student will struggle to be successful in math if they are unable to successfully and accurately solve word problems.

Although there are many techniques and strategies existed to help students with problem-solving, there are teachers who are still using the old teaching method and teacher-centered. Lack of conceptual understanding happened due to inadequate exposure and use of thinking skills throughout the lesson. [3] A teacher must adapt to the teaching that responds to the strengths and needs of all learners (Briggs et.al, 2018). The teacher has to consider the needs of the children with different learning styles and abilities. Therefore, it is really important for teachers to change their ways of teaching based on students' needs and vary the strategies they taught to the students. It is vital for students to choose the best strategy when they attempt the word problem-solving questions.

[4] The use of visual representations has been found to help individuals solve different types of math related problems as they establish meaning within a problem and aid in determining problem-solving methods (Van Garderen, 2006).[5] Visual representation help students interpret and solve word problems using spatial relationships (Chu, Rittle-Johnson, & Fyfe, 2017).

A more specific approach of visual representation that deserves attention from teachers and education policymakers is the Concrete-Pictorial-Abstract (CPA) approach. The CPA approach is chosen as it has compatibility with the stages of cognitive development of primary school students. The CPA approach is a learning approach based on the concept of Bruner's learning theory regarding the stages of the cognitive development theory of "enactive-iconic-symbolic" cognitive development

As an educator, it is apparent that problem solving is not only a huge focus across grade levels, but also a huge deficit amongst students across grade levels. The level of importance and difficulty tied to problem solving is what drew interest to this topic. It is important to teach students strategies, such as using visual representations, so they can utilize them when solving word problems. Hence, the researcher is interested in utilizing visual representation approaches, specifically the CPA approach, to see how it affects the performance of students taking up mathematics in first year college.

II. OBJECTIVES OF THE STUDY

The general objective of this study is to determine the effects of the Concrete-Pictorial-Abstract approach in the Math problem solving performance of First Year students for the Academic Year 2021-2022.

Specifically, it aimed to attain the following objectives:

1. determine the level of Mathematics problem solving performance of the experimental and control groups during the pretest and post-test.
2. determine the mean gain in the mathematics problem solving performance of the experimental and control groups.
3. determine if there is a significant difference in the mathematics problem solving performance of the experimental and control groups during the pretest.
4. determine if there is a significant difference in the mathematics problem solving performance of the experimental and control groups during the post-test.
5. determine if there is a significant difference in the mean gains of the mathematics problem solving performance of the experimental and control groups.
6. determine if there is a significant difference in the pretest and post-test mean results of the mathematics problem solving performance of the experimental and control groups.

III. RESEARCH METHODOLOGY

This study attempted to ascertain the effects of visual representation (Concrete-Pictorial-Abstract) approach on the Math problem solving performance of First Year students. The goal afterwards was to develop an instructional material in problem solving.

In order to attain these objectives, the researcher used the quasi-experimental specifically, the matching-only pretest-posttest design. Matching-only pretest-posttest design is an experimental design that researcher matches the participants in experimental and control groups. The experimental and control groups was randomly determined among the two sections FT-1A and FT-1B of the Bachelor of Industrial Technology Department. Group A was identified as the Experimental –Visual Representation group, while Group B was identified as the Control –Traditional group. The participants in Group A were matched with Group B in terms of their grade in Math 101 in the First Semester of AY 2021-2022.

The groups were match-paired, represented by letter M. In this study, the match-pairing was based on the participant's academic performance in Math 101 during the First Semester of the Academic Year 2021-2022. After the match-pairing, they were pretested (O1, O2) after which been exposed to different instructions. The experimental group (X) were taught using the visual representation (Concrete-Pictorial-Abstract) Approach in mathematics problem solving, and the control group (C) were taught mathematics problem solving using the traditional problem-solving approaches.

The data needed for this study were obtained by administering a researcher-made test. The initial draft of 50 multiple choice items researcher made test which includes the following topics; fraction problem, ratio problem, percent problem, number problem, and age problem was prepared based form the Table of Specifications. The researcher made test with the Table of Specification were then presented to the three math experts for face and content validation. The suggestions and recommendation of the math experts were considered for the revision and improvement of the researcher made test. After the revision and improvement of the researcher made test, the 50-item test then underwent pilot testing to 30 students from First Year Architectural Drafting in the BIT Department. The result of the pilot testing was used in item analysis and reliability test using the Cronbach alpha. The result of Cronbach alpha showed a reliability index of 0.76, which indicate that the instrument was reliable and acceptable. The item analysis was done to determine which of the items were rejected, revised and or retained and there were 34 items that were retained

In order to have an equal distribution of topics for the test items, six (6) items from each topic were included in the final number of test items which is 30 multiple choice items. The final version composed of 30 multiple choice items were used for the pre-test and post-test to the participants from the experimental group and the control group.

IV. RESULTS AND DISCUSSIONS

In the pre-test scores of participants in mathematics problem solving performance for both the experimental and control groups, results revealed that the pre-test mean scores in mathematics problem solving performance of the participants were in 11.46 and 11.26 respectively. The result showed similarity in the achievement test scores of the two groups of participants with slight differences in the mean score before the intervention. The mean scores in the performance test of both groups were verbally interpreted as “low achievement”.

It could likewise be noted that the standard deviation for the experimental group was 4.89 and 5.17 for the control group which tended to be nearly identical. This simply indicates that prior to intervention participants from each group did not possess sufficient concepts on problem solving. Therefore, the two groups were comparable at the onset. [6]According to Carden and Cline (2015), all students, even those who do not find word problem challenging, would benefit from visual imagery/visualization being promoted as a problem-solving strategy. Visualization strategies will increase the likelihood of successful students to be successful as they come across more complex problems.

A mean gain of 10.46 in mathematics problem solving performance was observed among experimental group participants which resulted from a post-test score of 21.92 higher than that of the pre-test score of 11.46 prior to the intervention. This goes to show that the participants in experimental group highly remarkably improved their low mathematical level in problem solving at the beginning to high level of performance after intervention was made in the instruction.

A mean gain of 4.55 in the problem solving performance was shown among the control group participant which resulted from a post-test score of 15.81 with a pre-test score of 11.26 priors to intervention. This suggests that problem solving performance in mathematics of the control group participants improved from a “low” performance at the beginning of the study to “average” performance after the post-test. Both groups improved their performance from a pre-test to post-test but the experimental groups participants was shown to perform high in problem solving than the control group counterpart.

[7]Findings of the study coincides with Johnson (2021) that when pupils used the CPA approach as part of their mathematics education, they were able to build on each stage towards a greater mathematical understanding of the concepts being learned, which in turn led to information and knowledge being internalized to a greater degree. Johnson (2021) reiterated that using the CPA approach, learners achieve a much deeper understanding of they do not have to resort to rote learning and are able to solve problems without having to memorizes.

The result of the test difference in problem solving performance of the two groups of participants revealed a t-value of -.523 and the probability value of .60 which was not significantly different at 0.05 alpha. This means that both respondents had similarly low mathematics problem solving performance prior to the conduct of the study. This implies that the two groups are comparable at the beginning of the intervention. This result can be attributed to the fact that the students from each group did not have sufficient strategy to solve non-routine problems prior to the study. More so, the result might be alluded to factors such as poor comprehension and mathematical analysis which are relative to each individual in the group.

Learning facts and contents in mathematics are important and students should learn how to use these facts to develop their thinking skills in solving problems. Mathematical problem-solving may help students to improve and develop the standard ability to solve-real life problem, to develop critical thinking skills and reasoning, to gain deeper understanding of concepts and to work in groups, cooperate with and interact with each other. Specifically, [8] it may also improve eagerness of an individual to try to analyze mathematical problems and to improve their determination and self-concepts, make the individual aware of the problem-solving strategies, and value of approaching problems in an orderly manner and that many problems can be solve in more than one way (Hoon, Kee, and Singh, 2013).

The significant difference in the post-test mathematics performance score of the experimental group participants was due to the fact that students’ exposure to the strategy used (block model) resulted to remarkably higher scores compared to the control group who were not exposed to the same treatment. Perhaps, these remarkable differences can be attributed to the intervention done in varied forms of problems that caused students to pull together their cognitive, affective, and psychomotor skills during on line classes learning which resulted to a maximum performance in the subject. Zimmerman and [9] Zimmerman and Cunningham (2021) validate the findings of this research that the use of visualization has often been cited as a powerful problem representation process for solving problems and that the use of visual imagery can be an important help for all sorts of problems, including problems in which nothing geometric is evident.

The wide disparity in the students' mean gain score in mathematics performance could be alluded to the effectiveness by the block model approach in solving varied non-routine problem. Likewise, the idea of lesson fitting tend to make students' actively engaged and pushed them to their fullest potentials, hence the result. [10] Flevaras and Penny (2021) claimed that visual representations enable pupils to make connections between their own experience and mathematical concepts and therefore gain insight in to these abstract mathematical ideas. Drawings help students think through the approximation of reality and abstractions of scientific concepts and engage in meaning making and problem solving around such concepts. [11] The findings of Arcavi (2013) agreed that the use of visual representations has been found to help individuals solve different types of math related problems. He further stated that the two benefits of visual representations being they establish meaning within a problem and aid in determining problem solving methods. It is imperative to a student's success that they are able to determine the meaning of each part of the problem and determine an appropriate method to solve the problem.

Researcher have further explained that visual representations as a semiotic and epistemological tool often help students process information more efficiently than verbal in written representations by organizing and high lighting key concepts, thus making information accessible for further thinking and action. There was a significant difference in the pre-test and post-test mean score result in mathematics problem solving performance of both the experimental and control group participants as shown by the $t(25) = -7.810$; $p = .000$ and $t(25) = -3.517$; $p = .002$ respectively. The big difference observed in the mean score of the experimental groups was maybe due to the genuinely challenging tasks brought about by the block model approach that help students stay focus, meet the expectation for quality of works, find and use the strategy to solve the non-routine problems during the conduct of the study.

However, a slight increase was observed in the pre-test and post-test mean score result in the control group with a t-value of -3.517 and having p-value of .002 which is also significant at 0.05 alpha. This significant result was in favor of the experimental group participants. Visualization is a mediating system that is experienced, learned, and developed through classroom interactions. Visualization intersects and creates meaning through classroom interactions. Visualization intersects and creates meaning through classroom interactions. It formulates one's thoughts and one's thoughts are formulated and communicated through visualization. Accordingly, in this study, [12] it is understand visual representations as cognitive, dialogical, and collective tools in science classrooms. (Tibell & Harms, 2017).

V. CONCLUSION

The participants of both groups (experimental and control group) did not possess adequate conceptual understanding, comprehension and mathematical analysis before the start of the study.

The experimental group performed higher score in Mathematics problem solving. This is evident in the increase in the pre-test to the post-test score of the participants thus, CPA approach is an effective strategy in performing the mathematics problem solving performance of students.

The increase in the mean gain of the experimental group participants in mathematics problem solving performance was not comparable with the control group. This indicates that intervention brought about positive benefits to the students.

The participants of the two instructional group were comparable in their mathematics problem solving performance prior to intervention that is the approach was appropriate and effective for the study since the two groups had the same level of performance prior to the onset of the study.

The participants in the experimental group were better off in understanding mathematics concepts, can think of new ideas, can interpret, analyze and solve problems than those in the control group instruction.

VI. RECOMMENDATIONS

Curriculum planner may consider Concrete-Pictorial-Approach as baseline information in designing the curriculum. Policy Makers of the university and other institutions may implement realistic policy in terms of the used of Concrete-Pictorial-Approach, this will help students find mathematics instruction not boring but more enjoyable, engaging, and active that will lead them to think critically and creatively.

With the advent of Concrete-Pictorial-Approach as an approach to mathematical problem solving in service trainings and workshops be designed by the school to update teacher in different fields of endeavor the use of this effective instructional strategy thus making learning engaging.

The researcher may develop instructional materials to meet its learner's ability, creativity and critical thinking skills thus engaging them in the activities to gain higher achievement in mathematics. Writers may develop their textbooks in supportive of Concrete-Pictorial-Approach strategy.

The researcher recommends that mathematics teachers give time to students' engagement along with making students more comfortable with the study of mathematics itself especially mathematics problem solving.

Future researchers can endeavor which instructional strategy apart from Concrete-Pictorial-Approach work best for different cultures, disabilities, and economic status.

To avoid confusion, the family name must be written as the last part of each author name (e.g. John A.K. Smith). Each affiliation must include, at the very least, the name of the company and the name of the country where the author is based (e.g. Causal Productions Pty Ltd, Australia).

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