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Study of Soil Under Earthquake Condition using PLAXIS Software.

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Abstract: The abstract presents a concise overview of the study involving the analysis of a multi-story building with a ground-plus-four (G+4) configuration in Seismic Zone 4. This research explores the structural behaviour, stability, and seismic performance of such buildings commonly found in urban environments prone to higher seismic activity. The investigation considers factors like material composition, load distribution, and vertical circulation systems. The study employs numerical simulations, potentially using software like PLAXIS, to assess the building's response to various loads and seismic forces. The goal is to enhance understanding of the building's behaviour and provide insights into its design and construction aspects, including foundation design, which are critical for ensuring both safety and functionality in Seismic Zone 4. By analysing a G+4 structure within this specific seismic context, this research contributes to the broader knowledge base concerning multi-story building dynamics and their implications for urban infrastructure development in areas of heightened seismic risk.

I. INTRODUCTION

Earthquakes are natural geological phenomena that pose significant threats to human life and infrastructure. When seismic waves promulgate through the Earth, the response of soil can significantly influence the ground motion experienced at the surface. Understanding the behaviour of soil under earthquake conditions is of paramount standing for earthquake engineering and geotechnical design. Accurate analysis of soil response during seismic events enables engineers to design resilient structures and implement appropriate mitigation measures to safeguard communities living in earthquake-prone regions.

The learning of soil behaviour under earthquake conditions has been a topic of extensive research in geotechnical engineering. Over the years, advancements in numerical simulation tools provided valuable visions into the complex behaviour of soils during earthquakes. PLAXIS Software, a widely-used geotechnical finite element software, has become a prevailing tool for conducting such analyses. By employing the Finite Element Method (FEM), PLAXIS allows engineers and researchers to model and analyse the response of soil to seismic loading with high precision and efficiency. This study aims to investigate the behaviour of different soil types under earthquake conditions using PLAXIS Software. The purposes of the study include analysing soil deformation, settlement, and shear stresses induced by seismic forces. Furthermore, the study aims to assess the liquefaction potential of soils, which can lead to catastrophic ground failures during earthquakes. Over the use of PLAXIS Software, this study pursues to gain valuable insights interested in the dynamic behaviour of soils also to contribute advancement of earthquake engineering practices.

The advantage of advanced geotechnical software, such as PLAXIS, has revolutionized the field of geotechnical engineering by providing powerful simulation capabilities for analysing and predicting the character of soil and rock structures. PLAXIS enables engineers to create accurate numerical models that simulate real-world scenarios, allowing for a better understanding of the multifaceted relations between soil, water, and construction processes.

This study aims to leverage the capabilities of PLAXIS software to simulate the character of slopes during road construction and investigate the impact of varying water table conditions on slope deformation. The water table, defined as the level at which the ground is saturated with water, plays a critical role in the stability and performance of slopes, as it affects soil properties, pore-water pressures and ultimately the resistance to deformation. By employing PLAXIS software, this research will contribute to improving the knowledge and understanding of slope behaviour during road construction on sloping terrains. The ability to simulate and analyse deformations under dissimilar water table circumstances provides valuable insights for potential hazards and failure mechanisms that can arise during the construction process and throughout the service life of the road. The conclusions of this work will have applied inferences for engineers and project stakeholders involved in road construction projects on slopes. By seeing the effects of water table fluctuations, engineers can make well-versed decisions regarding slope stability, construction techniques, and the implementation of appropriate mitigation measures. Ultimately, this will result in safer, more cost-effective, and sustainable road construction practices.



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II. SIMULATION WORK

A. Site Investigation and Data Collection:

The site investigation and data collection phase is a crucial process to understand the soil and geological conditions of a project site. It's vital for studying how the soil behaves during earthquakes using PLAXIS software. The process begins with reviewing existing data and literature, followed by a visual site inspection. Geotechnical boreholes are then drilled, and soil samples are collected. In-situ tests like SPT or CPT are done to determine soil properties. Laboratory tests analyse soil characteristics, and groundwater wells are installed to monitor water levels.

Seismic hazard assessment gauges earthquake potential. All data is compiled into a database for PLAXIS models. This helps simulate soil behaviour during earthquakes. For more details, refer to the provided PDF link.

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Parameter	Symbol	Sub Soil -1	Sub Soil-2	Units
General				
Material model	Model	Linear Elastic	Linear Elastic	
Type of behaviour	Туре	Drained	Drained	
Dry Unit weight	γ_{unsat}	17	18	kN/m ³
Wet Unit weight	$\gamma_{\rm sat}$	20	22	kN/m ³
Parameters				
Youngs's modulus	E'	30000	35000	kN/m ²
Poisson's ratio	υ'	0.2	0.3	
Rayleigh alpha		0.01	0.02	
Rayleigh Beta		0.01	0.02	

Table 1: Soil profile data.

B. Conceptualization of Model:

Numerical model setup is a crucial step in studying soil behaviour under earthquake conditions using PLAXIS software. This process involves creating a virtual representation of the site in a 2D environment, defining the soil layers, material properties, and boundary conditions built on the data found from site investigation and seismic hazard assessment.

Table 2: Building material data.

Parameter	Symbol	Beam	Units
Material type		Elastic	
Isotropic		Yes	
Axial stiffness	EA	5000000	kN/m
Flexural stiffness	EI	9000	kNm ² /m
Depth	d		m
Weight	W	5	kN/m/m
Poisson's ratio	ບ'	0.15	
Rayleigh alpha		0.01	
Rayleigh beta		0.01	

C. Building Description

Width of Building: 6m Height of Basement : 2m Height of each Floor : 3m Zone of the Location : Zone 4 (IS 1893:Part 1) Soil Type : Sandy Soil

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Fig. 1: Building Representation

By accurately defining the numerical model with appropriate soil layers, material properties, boundary conditions, and seismic input motions, PLAXIS enables engineers to simulate and study the dynamic reply of the soil and structures under realistic earthquake conditions. The numerical model serves as valued tool for measuring seismic vulnerability of site, identifying potential risks, and proposing appropriate mitigation measures to enhance seismic safety.

D. Ground Motion Selection:

a) Select representative earthquake ground motion records suitable for site's seismic hazard: The first step ground motion selection is to identify and gather earthquake ground motion records from seismic databases, historical records, or recorded data from nearby seismic events. The selection should be based on the site's location and proximity to active faults or seismic sources, considering the regional tectonics and seismic hazard assessment. The ground motion records should be representative of the potential seismic events that the site may experience.

b) Scale the ground motion records to various levels of peak ground acceleration (PGA):Once the appropriate ground motion records are selected, may need to be scaled to represent different levels of seismic intensity or peak ground acceleration (PGA). Scaling ground motion archives allows for simulating different earthquake scenarios with varying magnitudes, providing a range of seismic intensities to study the site's response under different seismic events. The scaling can be performed using various methods, like magnitude-scaling relationship, distance-scaling relationship, or empirical relationships specific to seismic region. The objective to simulate ground motions with different PGA values, representing a spectrum of potential earthquake intensities that the site might encounter.



Fig. 2: Accelerogram Graph.

E. Geometric Modelling and Mesh Generation:

Material modelling is a critical aspect of studying soil behaviour under earthquake conditions using PLAXIS software. In this process, suitable soil constitutive models are defined built on results of laboratory testing and in-situ measurements. These models are mathematical representations the soil's mechanical properties and behaviour. The choice of constitutive models depends on the specific soil types present at the site and their response to seismic loading. Different soil models, such as Mohr-Coulomb, Hardening Soil, Modified Cam-Clay, Soft Soil, and Duncan-Chang models, among others, are available in PLAXIS to accurately capture the behaviour of various soil types. By accurately defining soil constitutive models and considering nonlinearity and dynamic properties, PLAXIS provides a robust platform for simulating realistic soil behaviour under seismic loading. This comprehensive material modelling enables engineers to conduct reliable seismic analyses, predict soil responses during earthquakes, measure the seismic presentation of structures, and design effective mitigation measures to enhance seismic safety at the site. Geometric modelling and mesh generation are fundamental processes in studying soil behaviour below earthquake circumstances using PLAXIS



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software. Geometric modelling involves creating a digital representation of the site, encompassing soil layers, structural elements, and other pertinent features. The 2D or 3D model is built to accurately replicate the site's subsurface conditions based on geotechnical investigations. Additionally, it incorporates structural elements like buildings, retaining walls, and foundation systems to simulate their interaction with the soil during seismic events. This geometric model serves as the foundation for essential material properties, boundary conditions and seismic input motions in subsequent analyses. In conclusion, geometric modelling and mesh generation form the groundwork for precise numerical simulations of soil behaviour below earthquake conditions. The geometric model captures the site's actual configuration and incorporates relevant structural elements, while the mesh provides a discrete representation of the domain for numerical analysis. These essential steps enable reliable seismic analyses, allowing for a comprehensive evaluation of the soil's response to earthquake loading and the seismic performance of structures at the site.



Fig 3: Mesh Generated.

III. INTERPRETATION AND EVALUATION OF RESULTS

In the interpretation and evaluation of results phase, the output data obtained from the mathematical analysis using PLAXIS software is thoroughly analysed to measure the seismic performance of the site and structures. This step involves carefully examining the calculated displacements, stresses, strains, and pore water pressures for different seismic scenarios to understand how the soil and structures behave under earthquake loading.

Liquefaction Potential: Liquefaction potential assessment is crucial in earthquake analysis. It's about the soil losing strength due to increased pore-water pressures during seismic events. Engineers study pore-water pressures to locate regions prone to liquefaction. High pressures signify areas at risk, where soil may weaken, causing instability and reduced bearing capacity. This threatens structures' integrity, as the soil's ability to support loads diminishes, potentially leading to settlements, tilting, or even collapse.

Settlement: Settlement analysis is crucial to determine the extent to which the soil and structures may experience vertical displacement or sinking during seismic events. Excessive settlement can cause structural damage, affect the performance of foundations, and compromise the stability of structures.

Structural Stability: The analysis results are used to assess the stability of the structures under different earthquake intensities. Engineers examine factors such as stresses and strains in the structural elements to ensure that the buildings and foundations can withstand the seismic forces without experiencing failure or collapse.

Performance Evaluation: By comparing the results from different seismic scenarios, engineers evaluate the performance of the site and structures under varying levels of seismic loading. They can identify the serious areas that might experience higher levels of deformation, stress concentration, or liquefaction, enabling them to prioritize mitigation measures for vulnerable zones.

Risk Assessment: The evaluation of results allows for a comprehensive risk assessment, which helps in understanding the potential hazards posed by earthquakes to the site and structures. Engineers can determine the likelihood of different damage states and the severity of consequences, helping in developing effective risk mitigation strategies. Stipulated Deformation Behaviour :

Deformed Mesh: A deformed mesh is a grid used in arithmetic simulations similar finite element analysis (FEA) to represent how materials deform under different loads. The mesh adjusts its shape based on calculated displacements, showing how the structure changes during analysis. This helps engineers visualize effects like settlement, stress distribution, and deformation, aiding in design and safety evaluations.



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Fig 4: Deformed mesh of Sub Soil-1



Fig 5: Deformed mesh of Sub Soil-2

Total Displacement in Arrows: This visual representation helps engineers understand the overall deformation pattern of structure or soil. It provides valuable insights into areas of potential concern, such as excessive settlement, tilting, or stress concentrations, which stay critical for assessing stability and safety of the system under analysis.



Fig 6: Total Displacement in Arrows of sub soil-1



Fig 7: Total Displacement in Arrows of sub soil-2

Total Incremental Displacement: The accumulated displacement of a point or element in a structure or soil after a series of load increments or steps in a numerical simulation. It's the total sum of small displacement changes that occur at each step of the analysis. In finite element analysis (FEA) or other numerical simulations, the behaviour of a system is typically calculated step by step.

Each step involves applying a load incrementally and calculating the resulting displacement. The "Total Incremental Displacement" sums up these small displacements over all the steps to give the final overall displacement of the point or element.



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Fig 8: Total Incremental Displacement sub soil-1



Fig 9: Total Incremental Displacement sub soil-2

Vertical Displacement in Arrows: It is a visual representation used in numerical simulations to depict the vertical displacement of points or elements within a structure or soil. This graphical method uses arrows superimposed on the original geometry to show how much and in which direction each point has moved vertically due to applied loads or external factors. By displaying vertical displacements in arrows, engineers can quickly grasp the overall deformation pattern and identify areas of concern. This information is crucial for assessing the structural integrity, stability, and safety of the system under analysis, particularly in scenarios such as seismic events or other dynamic loads.



Fig 10: Vertical Displacement sub soil-1



Fig 11: Vertical Displacement sub soil-2

Total Velocities in Shading : It is a visual representation technique used in numerical simulations, such as finite element analysis (FEA), to depict the velocities of points or elements within a structure or soil. Instead of using arrows, this method employs shading or colour gradients to indicate the magnitude of velocities at different points. In this visualization, areas with higher velocities are represented with darker or more intense shading, while areas with lower velocities are depicted with lighter shades. This shading approach provides a quick overview of the velocity distribution across the analysed system.



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Fig 12: Total Velocities sub soil-1



Fig 13: Total Velocities sub soil-2

Total Accelerations in Shading: It is a visualization technique used in numerical simulations, like finite element analysis (FEA), to represent the accelerations experienced by points or elements within a structure or soil. Instead of using arrows or vectors, this method employs shading or color gradients to indicate the magnitude of accelerations at different points. In this visual representation, areas with higher accelerations are typically depicted with darker or more intense shading, while areas with lower accelerations are shown with lighter shades. This shading approach provides a visual overview of how accelerations are distributed throughout the analysed system.



Fig 14: Total Accelerations sub soil-1



Fig 15: Total Accelerations sub soil-2

IV. RESULTS AND DISCUSSIONS

The results obtained from the seismic analysis using PLAXIS software deliver valuable visions into the behaviour of the soil and structures under earthquake loading. The analysis outcomes are thoroughly evaluated and discussed to evaluate the seismic performance of the site and structures and identify potential vulnerabilities. The discussions revolve around the following key aspects:

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Liquefaction Potential: The analysis results are examined to determine areas with a high liquefaction potential. Locations where pore water pressures exceed critical thresholds during earthquake are identified as susceptible liquefaction. The discussions focus on understanding the factors contributing to liquefaction, such as soil properties, groundwater conditions, and seismic forces.

Settlement Analysis: The level of settlement experienced by the soil and structures during seismic events is assessed. The discussions aim to understand the implications of settlement on the stability and performance of the structures. High settlement zones are identified and evaluated to determine the need for soil improvement or structural design modifications.

Structural Stability: The analysis results are used to evaluate the stability of the structures under different earthquake intensities. The discussions focus on identifying critical structural elements and understanding their behaviour under seismic forces. Any potential structural weaknesses or failure modes are discussed, and recommendations for structural design modifications or retrofitting options are proposed.

Mitigation Measures: Based on analysis results and discussions, suitable mitigation dealings are proposed to enhance the seismic resilience of the site and structures. These observations might include soil improvement techniques, foundation design enhancements, retrofitting options, or structural design modifications. The discussions highlight the effectiveness of each mitigation measure and its potential impact on the site's overall seismic performance.

Design Recommendations: The analysis results form the basis for design recommendations aimed at improving the seismic performance of future constructions. The discussions focus on suggesting changes to building codes, regulations, and land use planning to reduce seismic risks and ensure the construction of earthquake-resistant structures.

Sensitivity Analysis: Sensitivity analyses may be conducted to recognize the effect of varying parameters on analysis results. Discussions around the significance of each parameter's impact on the site's seismic behaviour and the importance of accurate data collection and material modelling.

Comparison with Standards: The analysis outcomes are associated with relevant seismic design standards and guidelines to evaluate the structures' compliance with safety requirements. Any deviations or areas where additional reinforcement may be necessary are discussed.

Limitations: The discussions also include a consideration of the limitations of the analysis and potential uncertainties in the results. Factors such as simplifications made in the numerical model, assumptions in material properties, and variations in ground motion records may be addressed.

Overall, the results and discussions provide a comprehensive understanding of the site's seismic behaviour and serve as a basis for informed decision-making. The proposed mitigation measures and design recommendations are aimed at enhancing the seismic resilience of the site and structures, reducing potential risks, and ensuring the safety and stability of occupants during seismic events. By addressing the identified vulnerabilities and implementing the recommended measures, engineers can create earthquake-resistant infrastructure and contribute to building resilient communities in earthquake-prone regions.

Point A: Reference point at top of building.

Point B: Reference point at bottom of building.

Point C: Reference point at bottom of considered soil profile.



Fig 16: Points Representation.

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Results of Subsoil-1



Fig 17: Displacement V/S Time graph.

Results of Subsoil-2





V. CONCLUSION

• In conclusion, the study of soil under earthquake conditions using PLAXIS software plays a crucial role in earthquake engineering and geotechnical analysis. It provides a comprehensive understanding of how the soil and structures interact during seismic events and enables engineers to assess the seismic performance of sites and structures, identify potential vulnerabilities, and propose effective mitigation measures.

• The methodology followed in this study begins with a detailed geotechnical investigation, which involves collecting data on soil properties, stratigraphy, and groundwater conditions. This data serves as the foundation for the subsequent numerical modelling using PLAXIS software. The creation of 2D numerical models allows engineers to simulate the complex behaviour of soil and structures under earthquake loading, providing a realistic representation of real-world scenarios.

• Ground motion selection is a critical step in the analysis, as it involves choosing appropriate earthquake records that are representative of the site's seismic hazard. Scaling these ground motion records to various levels of peak ground acceleration (PGA) helps evaluate the structures' response under different seismic intensities, providing insights into potential damage levels.

• Material modelling in PLAXIS is essential for accurately capturing the nonlinear and dynamic character of soil during seismic events. Defining appropriate soil constitutive models based on laboratory testing and in-situ measurements ensures that the numerical analysis closely represents the actual soil response.



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• Dynamic analysis using PLAXIS enables the evaluation of displacements, stresses, strains, and pore-water pressures during earthquake loading. The interpretation and evaluation of the results deliver valued insights into the site's seismic performance, including liquefaction potential, settlement characteristics, and structural stability.

• Based on analysis results, engineers can propose suitable mitigation measures and design recommendations to enhance seismic resilience of the site and structures. These recommendations may involve reinforcing critical structural elements, adopting soil improvement techniques, retrofitting existing structures, or updating building codes and regulations to ensure safer construction practices in seismic regions.

• The results of study have far-reaching inferences for seismic risk assessment, urban planning, and disaster mitigation. By implementing the proposed mitigation measures, communities can better withstand effect of earthquakes, reduce potential risks, and safeguard lives and assets during seismic events.

• Overall, study of soil under earthquake conditions using PLAXIS software empowers engineers to make wellversed decisions in designing earthquake-resistant structures and developing resilient infrastructure. It significantly contributes to advancing earthquake engineering practices and fostering safer and supportable urban growth in earthquake-prone areas. As seismic hazards continue to pose challenges to communities worldwide, the visions gained from to study are instrumental in building a safer and more resilient future.

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