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Dynamical behaviour of Measles

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Abstract: We proposed a mathematical model of measles disease dynamics with vaccination by considering the total number of recovered individuals either from natural recovery or recovery due to vaccination. We tested for the existence and uniqueness of solution for the model using the Lipchitz condition to ascertain the efficacy of the model and proceeded to determine both the disease free equilibrium (DFE) and the endemic equilibrium (EE) for the system of the equations and vaccination reproduction number are given. In this research article we propose a four dimensional mathematical model with vaccinated class and analysed the model analytically. Our analytical result shows that vaccination is capable of reducing the number of exposed and infectious population.

Keywords: Meseales, Disease free equilibrium, Stability Analysis, Control.

I. INTRODUCTION

Measles remains a vital universal public health issue, particularly in developing nations. Measles (also known as rubeola or morbilli) represents one of the very highly transmissible diseases triggered by the genus Morbillivirus within the family Paramyxoviridae [1,2]. The virus is spread by coughing and sneezing, or direct contact with the infected [3]. The virus first comes in contact with host lung tissue, where it infects immune cells and it spreads throughout the body. As the virus travels in the blood, it infects capillaries in the skin that causes a rash on the skin [4]. Before the rash appears, measles has an incubation period for about 8-12 days followed by fever (390-40,50C), cough, coryza, conjunctivitis, and Koplik's spots [5] (small white spots in the mouth). These symptoms are followed by the appearance of a rash that starts on the face and neck and later it will spread to the body [3, 6, 7]. Although effective vaccines against measles infection are readily available, yet measles affects the mortality of children below five years of age [8], infecting ailing children in tens of millions yearly and resulting in deaths of about a million in number due primarily to intricate conditions that are coexistent with the disease examples of which are poor nourishment, diarrhea plus pneumonia [9].

It continues to remain highly infectious in the air or on the surface for up to two hours. Early symptoms include high-grade sore throat, cough, runny nose, blurry vision, and tiny white spots in the mouth; generally, 10–12 days after the infection appears. A later rash emerges, spreading downwards from the nose. The cycle of greatest infectiousness (meaning virus shedding) appears four days before the onset of rash and 4 days after the onset of rash. The average incubation period is 14 days, varying from 7 to 18 days [5]. In the real sense, some individuals who are vaccinated could still be vulnerable when the vaccination failed, or their immunity caused by the vaccine waned. Vaccination has reduced significantly global measles deaths by a 73% decrease between 2000 and 2018.Worldwide, measles is still prevalent in many developing countries, especially parts of Africa and Asia. Over 140,000 people died of measles in 2018. Between 2000 and 2018, global measles vaccination resulted in an 85 percent reduction in measles mortality, [10,11]. According to the World Health Organization (WHO), about 110,000 people died from measles especially children below the age of 6, despite the availability of a safe and efficient vaccine in 2017 [12]. To prevent measles, all health institutions recommend children to get the measles vaccine. Measles vaccine can be accepted by children and adults through MMR (Mumps, Measles, and Rubella) vaccines, MR (Measles and Rubella) vaccines and MMRV (Mumps, Measles, Rubella, and Varicella) vaccines. All of those vaccines consist of two doses. According to Centers for Disease Control and Prevention (CDC), one dose of MMR vaccine is 93% effective against measles and two doses of MMR vaccine are 97% effective against measles [7].

Some mathematical models have been introduced by many authors to describe the spread of measles, such as [13, 14, 15]. Different with previous model, we construct a mathematical model of measles that accommodates two-step of vaccination and quarantine in this article. The model differentiates individual who already get one time vaccination and two-time vaccination into two different compartments. In the other hand, quarantine strategy is also involved into the model to understand how important and crucial the quarantine is to control the spread of measles, if it is compared with the vaccination strategy. There has been an increasing interest in the use of deterministic compartmental models in recent decades to study the dynamics of measles and find measures to control and prevent the outbreak [16]. Bauch examined the implication of vaccination with regards to the effect of reaching herd immunity [17]. Mossong and Muller carried out a study on the modelling of measles re-emergence attributable to weakened immunity of vaccinated populations [18]. Zhang et al. examined the degree of the epidemic against the policy of discretional vaccination on Erdos-Renyi random graphs and Barabasi-Albert scale-free networks [19]. Momoh et al. designed a mathematical model to limit the spread of measles [20]. Fred et al. carried out a study on mathematical modelling on how vaccination curbs measles. In [21], the

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authors found that a wider gap between measles-infected and noninfected individuals is effective to control the spread of the disease. The impacts of the role of vaccination in controlling the spread of measles dynamics were investigated in [22, 23].

Valuable information on transmission and effective control of the measles epidemics as well as appropriate policies are very important. In this article, we study and analyze the behavior of solutions of transmission and control of measles infection by deterministic mathematical model; analyze the condition that determine the level of the effective prevention of the spread of measles using S-I-R-type compartmental model in an open population; analyze the qualitative properties of solutions of our model and give sufficient conditions for the model to be stable. For the S-I-R model to be fitting, once a person has recovered from the disease, they would acquire permanent immunity.

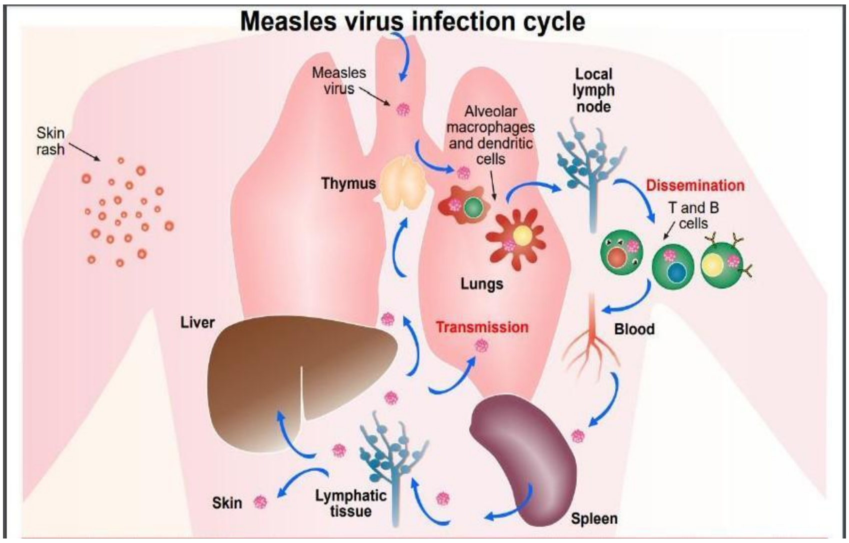

Fig. 1 Measles Virus Infection Cycle [24]

II. DEVELOPMENT OF MATHEMATICAL MODEL

We consider that prey population is facing an infectious disease, where the predator feeds on both healthy and infected preys .
Let
S = Susceptible Individuals.
E = Exposed Individuals.
I = Infected Individuals.
V = Vaccinated Individuals.
R = Recovered from disease.
Here, N=S+I+E.

$$\frac{ds}{dt} = (1-q)\,\pi - \frac{\lambda SI}{N} - d_1 S.$$
$$\frac{dE}{dt} = \frac{\beta SI}{N} - \gamma E - d_1 E.$$
$$\frac{dI}{dt} = \gamma E - \delta I - d_1 I + \frac{\lambda SI}{N} + \epsilon\lambda V.$$
$$\frac{dV}{dt} = \pi q - \omega V - d_1 V.$$
$$\frac{dR}{dt} = \sigma I + \omega V - d_1 R.$$

Where,

β = Transmission rate of Infection from Infected Individuals to Susceptible Individuals.
δ = Disease induced Death Rate.
d_1 = Natural Death Rate.
σ = Recovery rate from Disease.
λ = Infected class is increased by the contact rate to the Susceptible Class at a rate λ.
γ = Rate of Infection from Exposed to Infected Class.

ω = Rate of Recovered Individuals due to Vaccination.
π = Birth Rate.
q = Proportion of those Successfully Vaccinated at Birth.
ϵ = Rate of infection due to Vaccination

III. EQUILIBRIUM POINTS

There are fourteen equilibrium points of the system, trival equilibrium point is $E_0'(0,0,0,0,0)$,$E_1'(\frac{(1-q)\pi}{d_1},0,0,0,0)$, axial equilibrium point is $E_2'(0,0,0,\frac{\pi q}{\varepsilon\lambda+\omega+d_1},0)$, planar equilibrium points are $E_3'(\frac{(1-q)\pi}{d_1},0,0,\frac{\pi q}{\varepsilon\lambda+\omega+d_1},0)$, $E_4'(0,0,\frac{\varepsilon\lambda\pi q}{(\delta+\sigma+d_1)+(\varepsilon\lambda+\omega+d_1)},\frac{\pi q}{\varepsilon\lambda+\omega+d_1},0)$, $E_5'(0,0,0,\frac{\pi q}{\varepsilon\lambda+\omega+d_1},\frac{\omega\pi q}{d_1(\varepsilon\lambda+\omega+d_1)})$,$E_6'(S_6,E_6,I_6,0,0)$ which satisfies the equation : $(1-q)\pi - d_1S_6 - d_1E_6 - (\delta+\sigma+d_1)I_6 = 0$, $E_7'(S_7,0,I_7,V_7,0)$ which satisfies the equation : $\pi - d_1S_7 - d_1V_7 - (\delta+\sigma+d_1)I_7 - \omega V_7 = 0$, $E_8'(\frac{(1-q)\pi}{d_1},0,0,\frac{\pi q}{\varepsilon\lambda+\omega+d_1},\frac{\pi q}{\varepsilon\lambda+\omega+d_1},\frac{\omega\pi q}{d_1(\varepsilon\lambda+\omega+d_1)})$, [Disease Free Equilibrium Point] , $E_9'(0,0,\frac{\varepsilon\lambda\pi q}{(\delta+\sigma+d_1)+(\varepsilon\lambda+\omega+d_1)},\frac{\pi q}{\varepsilon\lambda+\omega+d_1},\frac{\pi q}{\varepsilon\lambda+\omega+d_1},\frac{\sigma\varepsilon\lambda+\omega(\delta+\sigma+d_1)}{(\delta+\sigma+d_1)})$, $E_{10}'(S_{10},E_{10},I_{10},V_{10},0)$ which satisfies the equation : $\pi - d_1S_{10} - d_1E_{10} - (\delta+\sigma+d_1)I_{10} - (\omega+d_1)V_{10} = 0$, $E_{11}'(S_{11},0,I_{11},V_{11},R_{11})$ which satisfies the equation : $\pi - \frac{\beta\beta S_{11}I_{11}}{S_{11}+I_{11}} - d_1S_{11} - (\delta+d_1)I_{11} - d_1V_{11} - d_1R_{11} = 0$, $E_{12}'(S_{12},E_{12},I_{12},0,R_{12})$ which satisfies the Equation : $(1-q)\pi - d_1S_{12} - d_1E_{12} - d_1R_{12} - (\delta+d_1)I_{12} = 0$, and the interior equilibrium point is **E**$^*(S^*,E^*,I^*,V^*,R^*)$ where S^*,E^*,I^*,V^*,R^* are positive and satisfy the given system of equations.

IV. EXISTENCE AND LOCAL STABILITY ANALYSIS OF THE EQUILIBRIUM POINTS

Lemma 1: The System (1) around $E_1'(\frac{(1-q)\pi}{d_1},0,0,0,0)$ is locally asymptotically stable .

Lemma 2: The System (1) around $E_3'(\frac{(1-q)\pi}{d_1},0,0,\frac{\pi q}{(\varepsilon\lambda+\omega+d_1)},0)$ is locally asymptotically stable .

Lemma 3: The System (1) around $E_4'(0,0,\frac{\varepsilon\lambda\pi q}{(\varepsilon\lambda+\omega+d_1)(\delta+\sigma+d_1)},\frac{\pi q}{(\varepsilon\lambda+\omega+d_1)},0)$ is locally asymptotically stable .

Lemma 4: The System (1) around $E_6'(S_6,E_6,I_6,0,0)$ is locally asymptotically stable .

Proof: The System (1) around $E_6'(S_6,E_6,I_6,0,0)$, is locally asymptotically stable (LAS) if the roots of the characteristic equation $A_0\lambda^2 + A_1\lambda + A_2 = 0$ of the Jacobian Matrix satisfy the Routh-Hurwitz criteria i.e. $A_i>0$ (where, i=0,1,2)

where,

$A_0 = a_{33} + a_{13}a_{31}$.

$A_1 = -(a_{11}a_{33} + a_{11}a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31})$.

$A_2 = (a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{11}a_{12}a_{21}a_{33} + a_{11}a_{12}a_{23}a_{31} + a_{11}a_{13}a_{21}a_{32} - a_{11}a_{13}a_{22}a_{31})$

where,

$$a_{11} = -\frac{I_6(\beta+\lambda)(E_6+I_6)}{(S_6+E_6+I_6)^2}, a_{12} = \frac{\beta I_6(E_6+I_6)}{(S_6+E_6+I_6)^2}, a_{13} = \frac{\lambda I_6(E_6+I_6)}{(S_6+E_6+I_6)^2}, a_{21} = \frac{S_6I_6(\beta+\lambda)}{(S_6+E_6+I_6)^2}$$

$$a_{22} = -(\frac{\beta S_6I_6}{(S_6+E_6+I_6)^2} + \gamma + d_1), a_{23} = \gamma - \frac{\lambda S_6I_6}{(S_6+E_6+I_6)^2}, a_{31} = -\frac{S_6(\beta+\lambda)(E_6+S_6)}{(S_6+E_6+I_6)^2}$$

$$a_{32} = \frac{\beta S_6(E_6+S_6)}{(S_6+E_6+I_6)^2}, a_{33} = -(\delta+\sigma+d_1 - \frac{\lambda S_6(S_6+E_6)}{(S_6+E_6+I_6)^2})$$

Where,

S_6, E_6, I_6 satisfy the Equation

$(1-q)\pi - d_1S_6 - d_1E_6 - I_6(\delta+\sigma+d_1) = 0.$

The other two Eigen Values are $-(\varepsilon\lambda+\omega+d_1) and - d_1$

Therefore, E_6' is LAS.

Lemma 5: The System (1) around $E_7'(S_7,0,I_7,V_7,0)$ is locally asymptotically stable .

Proof: The System (1) around $E_7'(S_7,0,I_7,V_7,0)$ equation $A_0\lambda^2 + A_1\lambda + A_2 = 0$ of the Jacobian Matrix satisfy the Routh-Hurwitz criteria i.e. $A_i>0$ (where, i=0,1,2)

where,

$A_0 = a_{33} + a_{13}a_{31}$.

$A_1 = -(a_{11}a_{33} + a_{11}a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31})$.

$A_2 = (a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{11}a_{12}a_{21}a_{33} + a_{11}a_{12}a_{23}a_{31} + a_{11}a_{13}a_{21}a_{32} - a_{11}a_{13}a_{22}a_{31})$

where,
$a_{11} = -\frac{(I_7)^2(\beta+\lambda)}{(S_7+I_7)^2}, a_{12} = \frac{\beta(I_7)^2}{(S_7+I_7)^2}, a_{13} = \frac{\lambda(I_7)^2}{(S_7+I_7)^2}, a_{21} = \frac{S_7I_7(\beta+\lambda)}{(S_7+I_7)^2}, a_{22} = -(\frac{\beta S_7I_7}{(S_7+I_7)^2} + \gamma + d_1)$
$a_{23} = \gamma - \frac{\lambda S_7 I_7}{(S_7+I_7)^2}, a_{31} = -\frac{(S_7)^2(\beta+\lambda)}{(S_7+I_7)^2}, a_{32} = \frac{\beta(S_7)^2}{(S_7+I_7)^2}, a_{33} = -(\delta+\sigma+d_1 - \frac{\lambda(S_7)^2}{(S_7+I_7)^2})$
Where, S_7, I_7 satisfy the Equation
$\pi - d_1S_7 - d_1V_7 - I_7(\delta+\sigma+d_1) - \omega V_7 = 0.$
The other two Eigen Values are $-(\varepsilon\lambda+\omega+d_1) and - d_1$

Therefore, E_7^* is LAS.

Lemma 6: The System (1) around $E_8(\frac{(1-q)\pi}{d_1}, 0, 0, \frac{\pi q}{(\epsilon\lambda+\omega+d_1)}, \frac{\pi q\omega}{(\epsilon\lambda+\omega+d_1)d_1})$ is locally asymptotically stable .
Lemma 7: The System (1) around $E_9^*(0, 0, \frac{\varepsilon\lambda\pi q}{(\varepsilon\lambda+\omega+d_1)(\delta+\sigma+d_1)}, \frac{\pi q}{(\varepsilon\lambda+\omega+d_1)}, \frac{\pi q(\sigma\epsilon\lambda+\omega(\delta+\sigma+d_1))}{(\delta+\sigma+d_1)})$ is locally asymptotically stable .
Lemma 8: : The System (1) around $E_{10}^*(S_{10}, E_{10}, I_{10}, V_{10}, 0)$ is locally asymptotically stable .
Proof: The System (1) around $E_{10}^*(S_{10}, E_{10}, I_{10}, V_{10}, 0)$, is locally asymptotically stable (LAS) if the roots of the characteristic equation $A_0\lambda^2 + A_1\lambda + A_2 = 0$ of the Jacobian Matrix satisfy the Routh-Hurwitz criteria i.e. $A_i>0$ (where, i=0,1,2),
where,
$A_0 = a_{33} + a_{13}a_{31}.$
$A_1 = -(a_{11}a_{33} + a_{11}a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}).$
$A_2 = (a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{11}a_{12}a_{21}a_{33} + a_{11}a_{12}a_{23}a_{31} + a_{11}a_{13}a_{21}a_{32} - a_{11}a_{13}a_{22}a_{31})$
where,
$a_{11} = -\frac{I_{10}(\beta+\lambda)(E_{10}+I_{10})}{(S_{10}+E_{10}+I_{10})^2}, a_{12} = \frac{\beta I_{10}(E_{10}+I_{10})}{(S_{10}+E_{10}+I_{10})^2}, a_{13} = \frac{\lambda I_{10}(E_{10}+I_{10})}{(S_{10}+E_{10}+I_{10})^2}, a_{21} = \frac{S_{10}I_{10}(\beta+\lambda)}{(S_{10}+E_{10}+I_{10})^2}$
$a_{22} = -(\frac{\beta S_{10}I_{10}}{(S_{10}+E_{10}+I_{10})^2} + \gamma + d_1), a_{23} = \gamma - \frac{\lambda S_{10}I_{10}}{(S_{10}+E_{10}+I_{10})^2}, a_{31} = -\frac{S_{10}(\beta+\lambda)(E_{10}+S_{10})}{(S_{10}+E_{10}+I_{10})^2}$
$a_{32} = \frac{\beta S_{10}(E_{10}+S_{10})}{(S_{10}+E_{10}+I_{10})^2}, a_{33} = -(\delta+\sigma+d_1 - \frac{\lambda S_{10}(S_{10}+E_{10})}{(S_{10}+E_{10}+I_{10})^2})$

Where, S_{10}, E_{10}, I_{10} satisfy the Equation

$$\pi - d_1S_{10} - d_1V_{10} - I_{10}(\delta+\sigma+d_1) - (\omega+d_1)V_{10} = 0$$

The other two Eigen Values are $-(\varepsilon\lambda+\omega+d_1) and - d_1$
Therefore, E_{10}^* is LAS.

Lemma 9: The System (1) around $E_{11}^*(S_{11}, 0, I_{11}, V_{11}, R_{11})$ is locally asymptotically stable .
Proof: The System (1) around $E_{11}^*(S_{11}, 0, I_{11}, V_{11}, R_{11})$
equation $A_0\lambda^2 + A_1\lambda + A_2 = 0$ of the Jacobian Matrix satisfy the Routh-Hurwitz criteria i.e. $A_i>0$ (where, i=0,1,2)
where,
$A_0 = a_{33} + a_{13}a_{31}.$
$A_1 = -(a_{11}a_{33} + a_{11}a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}).$
$A_2 = (a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{11}a_{12}a_{21}a_{33} + a_{11}a_{12}a_{23}a_{31} + a_{11}a_{13}a_{21}a_{32} - a_{11}a_{13}a_{22}a_{31})$

where,

$$a_{11} = -\frac{(I_{11})^2(\beta+\lambda)}{(S_{11}+I_{11})^2}, a_{12} = \frac{\beta(I_{11})^2}{(S_{11}+I_{11})^2}, a_{13} = \frac{\lambda(I_{11})^2}{(S_{11}+I_{11})^2}, a_{21} = \frac{S_{11}I_{11}(\beta+\lambda)}{(S_{11}+I_{11})^2}$$

$a_{22} = -(\frac{\beta S_{11}I_{11}}{(S_{11}+I_{11})^2} + \gamma + d_1), a_{23} = \gamma - \frac{\lambda S_{11}I_{11}}{(S_{11}+I_{11})^2}, a_{31} = -\frac{(S_{11})^2(\beta+\lambda)}{(S_{11}+I_{11})^2}, a_{32} = \frac{\beta(S_{11})^2}{(S_{11}+I_{11})^2}$
$a_{33} = -(\delta+\sigma+d_1 - \frac{\lambda(S_{11})^2}{(S_{11}+I_{11})^2})$

Where,
S_{11}, I_{11} satisfy the Equation
$\pi - \frac{\beta S_{11}I_{11}}{S_{11}+I_{11}} - d_1S_{11} - I_{11}(\delta+d_1) - d_1V_{11} - d_1R_{11} = 0.$
The other two Eigen Values are $-(\varepsilon\lambda+\omega+d_1) and - d_1$
Therefore, E_{11}^* is LAS.

Lemma 10: The System (1) around $E_{12}^*(S_{12}, E_{12}, I_{12}, 0, R_{12})$ is locally asymptotically stable.

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Proof: The System (1) around $E_{12}'(S_{12}, E_{12}, I_{12}, 0, R_{12})$, is locally asymptotically stable (LAS) if the roots of the characteristic equation $A_0\lambda^2 + A_1\lambda + A_2 = 0$ of the Jacobian Matrix satisfy the Routh-Hurwitz criteria i.e. $A_i>0$ (where, i=0,1,2)
where,
$A_0 = a_{33} + a_{13}a_{31}$.
$A_1 = -(a_{11}a_{33} + a_{11}a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31})$.
$A_2 = (a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{11}a_{12}a_{21}a_{33} + a_{11}a_{12}a_{23}a_{31} + a_{11}a_{13}a_{21}a_{32} - a_{11}a_{13}a_{22}a_{31})$
where,

$$a_{11} = -\frac{I_{12}(\beta+\lambda)(E_{12}+I_{12})}{(S_{12}+E_{12}+I_{12})^2}, a_{12} = \frac{\beta I_{12}(E_{12}+I_{12})}{(S_{12}+E_{12}+I_{12})^2}, a_{13} = \frac{\lambda I_{12}(E_{12}+I_{12})}{(S_{12}+E_{12}+I_{12})^2}, a_{21} = \frac{S_{12}I_{12}(\beta+\lambda)}{(S_{12}+E_{12}+I_{12})^2}$$

$$a_{22} = -(\frac{\beta S_{12}I_{12}}{(S_{12}+E_{12}+I_{12})^2} + \gamma + d_1), a_{23} = \gamma - \frac{\lambda S_{12}I_{12}}{(S_{12}+E_{12}+I_{12})^2}, a_{31} = -\frac{S_{12}(\beta+\lambda)(E_{12}+S_{12})}{(S_{12}+E_{12}+I_{12})^2}$$

$$a_{32} = \frac{\beta S_{12}(E_{12}+S_{12})}{(S_{12}+E_{12}+I_{12})^2}, a_{33} = -(\delta + \sigma + d_1 - \frac{\lambda S_{12}(S_{12}+E_{12})}{(S_{12}+E_{12}+I_{12})^2})$$

Where,
S_{12}, E_{12}, I_{12} satisfy the Equation
$(1-q)\pi - d_1S_{12} - d_1E_{12} - d_1R_{12} - I_{12}(\delta + d_1) = 0$.
The other two Eigen Values are $-(\varepsilon\lambda + \omega + d_1) and - d_1$
Therefore, E_{12}' is LAS.

Lemma 11: The System (1) around $E'^*(S^*, E^*, I^*, V^*, R^*)$ is locally asymptotically stable.

Proof: The System (1) around $E'^*(S^*, E^*, I^*, V^*, R^*)$ is locally asymptotically stable (LAS) if the roots of the characteristic equation $A_0\lambda^2 + A_1\lambda + A_2 = 0$ of the Jacobian Matrix satisfy the Routh-Hurwitz criteria i.e. $A_i>0$ (where, i=0,1,2)
where,
$A_0 = a_{33} + a_{13}a_{31}$.
$A_1 = -(a_{11}a_{33} + a_{11}a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31})$.
$A_2 = (a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{11}a_{12}a_{21}a_{33} + a_{11}a_{12}a_{23}a_{31} + a_{11}a_{13}a_{21}a_{32} - a_{11}a_{13}a_{22}a_{31})$
where,

$$a_{11} = -\frac{I^*(\beta+\lambda)(E^*+I^*)}{(S^*+E^*+I^*)^2}, a_{12} = \frac{\beta I^*(E^*+I^*)}{(S^*+E^*+I^*)^2}, a_{13} = \frac{\lambda I_{10}(E^*+I^*)}{(S^*+E^*+I^*)^2}, a_{21} = \frac{S^*I^*(\beta+\lambda)}{(S^*+E^*+I^*)^2}$$

$$a_{22} = -(\frac{\beta S^*I^*}{(S^*+E^*+I^*)^2} + \gamma + d_1), a_{23} = \gamma - \frac{\lambda S^*I^*}{(S^*+E^*+I^*)^2}, a_{31} = -\frac{S^*(\beta+\lambda)(E^*+S^*)}{(S^*+E^*+I^*)^2}$$

$$a_{32} = \frac{\beta S^*(E^*+S^*)}{(S^*+E^*+I^*)^2}, a_{33} = -(\delta + \sigma + d_1 - \frac{\lambda S^*(E^*+S^*)}{(S^*+E^*+I^*)^2})$$

Where,
S^*, E^*, I^* satisfy the Equation
$\pi - d_1S^* - d_1E^* - d_1V^* - d_1R^* - I^*(\delta + d_1) = 0$.
The other two Eigen Values are $-(\varepsilon\lambda + \omega + d_1) and - d_1$
Therefore, E'^* is LAS.

V. CONCLUSION AND DISCUSSION

Measles, an acute viral respiratory illness, presents with a prodrome featuring high fever (up to 105°F), malaise, cough, coryza, and conjunctivitis. Typically, the rash emerges approximately 14 days following exposure, starting from the head and extending to the trunk and lower extremities. This childhood infection, caused by a virus, was previously widespread but can now be effectively prevented through vaccination. Also known as rubeola, measles is highly contagious, particularly dangerous for young children, and can lead to severe complications, even fatalities. In this research article we examine the dynamics of measles by using a mathematical model with five compartments: susceptible, exposed, infected, vaccinated, and recovered. The boundary of solutions is proved, the basic reproduction number is calculated. Here we ten biologically feasible equilibrium point. We find the existence condition and check stability condition of each equilibrium point. This research article suggest that an increase in vaccination of population can reduce the total infected population in measles. Vaccination policy can reduce the fatality of the disease. As a result we can suggest that vaccination policy can be adopted to control the disease measles.

REFERENCES

[1]. Garba, S. M., Safi, M. A., & Usaini, S. (2017). Mathematical model for assessing the impact of vaccination and treatment on measles transmission dynamics. *Mathematical Methods in the Applied Sciences*, *40*(18), 6371-6388.
[2]. Griffin, D. E. (2016). The immune response in measles: virus control, clearance and protective immunity. *Viruses*, *8*(10), 282.
[3]. World Health Organization (2018), Measles. Retrieved from http://www.who.int/news-room/fact-sheets/ detail/measles.
[4]. David S. (2015). What does measles actually do? Retrieved from http://www.sciencemag.org/news/2015/ 01/what-does-measles-actually-do

[5]. Perry, R. T., & Halsey, N. A. (2004). The clinical significance of measles: a review. *The Journal of infectious diseases*, *189*(Supplement_1), S4-S16.
[6]. Fiebelkorn, A. P., Seward, J. F., & Orenstein, W. A. (2014). A global perspective of vaccination of healthcare personnel against measles: systematic review. *Vaccine*, *32*(38), 4823-4839.
[7]. Centers for Disease Control and Prevention, Measles, Mumps, and Rubella (MMR) Vaccination: What Everyone Should Know. Retrieved from, (2018), https://www.cdc.gov/vaccines/vpd/mmr/public/index.html
[8]. Roberts, M. G., & Tobias, M. I. (2000). Predicting and preventing measles epidemics in New Zealand: application of a mathematical model. *Epidemiology & Infection*, *124*(2), 279-287.
[9]. WHO. World Health Organization. Measles, preprint (2018). Available at https://www.who.int/ news-room/fact-sheets/detail/measles.
[10]. Ejima, K., Omori, R., Aihara, K., & Nishiura, H. (2012). Real-time investigation of measles epidemics with estimate of vaccine efficacy. *International journal of biological sciences*, *8*(5), 620.
[11]. Mossong, J., & Muller, C. P. (2003). Modelling measles re-emergence as a result of waning of immunity in vaccinated populations. *Vaccine*, *21*(31), 4597-4603.
[12]. Bolarin, G. (2014). On the dynamical analysis of a new model for measles infection.
[13]. Prawoto B. P. (2017). Stability and simulation of measles transmission model with and without vaccination. Far East Journal of Mathematical Sciences, 102(2), pp. 271-281.
[14]. Momoh, A.A., Ibrahim, M.O., Uwanta, I.J., Manga, S.B. (2013). Modelling the e_ect of vaccination on the transmission dynamics of measles, International Journal of Pure and Applied Mathematics, 88(3), pp. 381-390.
[15]. Oghre, E. O., & Ako, I. I. (2011). A mathematical model for measles disease. *Far East Journal of Mathematical Sciences*, *54*(1), 47-63.
[16]. Bauch, C. T., & Earn, D. J. (2004). Vaccination and the theory of games. *Proceedings of the National Academy of Sciences*, *101*(36), 13391-13394.
[17]. Mossong, J., & Muller, C. P. (2003). Modelling measles re-emergence as a result of waning of immunity in vaccinated populations. *Vaccine*, *21*(31), 4597-4603.
[18]. Zhang, H., Zhang, J., Zhou, C., Small, M., & Wang, B. (2010). Hub nodes inhibit the outbreak of epidemic under voluntary vaccination. *New Journal of Physics*, *12*(2), 023015.
[19]. Momoh, A. A., Ibrahim, M. O., Uwanta, I. J., & Manga, S. B. (2013). Mathematical model for control of measles epidemiology. *International Journal of Pure and Applied Mathematics*, *87*(5), 707-718.
[20]. Mose, O. F., Sigey, J. K., Okello, J. A., Okwoyo, J. M., & Giterere, J. (2014). Mathematical modeling on the control of measles by vaccination: Case study of KISII County, Kenya.
[21]. Adewale, S. O., Mohammed, I. T., & Olopade, I. A. (2014). Mathematical analysis of effect of area on the dynamical spread of measles.
[22]. Peter, O. J., Afolabi, O. A., Victor, A. A., Akpan, C. E., & Oguntolu, F. A. (2018). Mathematical model for the control of measles. *Journal of Applied Sciences and Environmental Management*, *22*(4), 571-576.
[23]. Smith, R. K., Archibald, A., MacCarthy, E., Liu, L., & Luke, N. S. (2016). A mathematical investigation of vaccination strategies to prevent a measles epidemic. *The North Carolina Journal of Mathematics and Statistics*, *2*, 29-44.
[24]. https://www.immunopaedia.org.za/clinical-cases/infectious-diseases/a-case-of-sudden-rapid-neurological-deterioration-in-an-hiv-positive-27-year-old-female