IARJSET
ISSN (Online) 2393-8021
ISSN (Print) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
6th National Conference on Science, Technology and Communication Skills – NCSTCS 2K23
Narula Institute of Technology, Agarpara, Kolkata, India
Vol. 10, Special Issue 3, September 2023

Dynamical behaviour of Giardiasis

Nikhilesh Sil[1], Baishali Chatterjee[2], Ranabir Paul[3] and Dibyendu Biswas[4]

Faculty, Department of Basic Science and Humanities, Narula Institute of Technology, Kolkata, India[1]

Student, Department of Electronics and Communication Engg., Narula Institute of Technology, Kolkata India[2,3]

Faculty, Department of Mathematics, City College of Commerce and Business Administration, Kolkata, India[4]

Abstract: In this paper, we presented a modelling approach to investigate the dynamics of Giardiasis in humans and domestic animals coupled with a contaminated environment. We computed the basic reproduction number R0 and employed it in analysing the effect of initial transmission and the stability of disease when an outbreak occurs. Results show that even when $\eta = 0$, R0 = R0h is greater than 4, showing that person-to-person transmission is the most significant in the dynamics of Giardiasis. An increase in η increases the value of R0 to some extent. Numerical simulations show that whenever there is an outbreak of Giardiasis in humans and domestic animals, the disease is likely to persist in the first two months and thereafter it will start to slow down to disease-free-equilibrium.

Keywords: Giardiasis, Disease free equilibrium, Stability Analysis, Control.

I. INTRODUCTION

Giardiasis, caused by the protozoa Giardia duodenalis (also known as Giardia lamblia and Giardia intestinalis), is an enteric infection prevalent in low-resource settings. It typically manifests with symptoms like flatulence and watery diarrhea[1–4]. In the United States, this disease is most commonly observed among international travelers, wilderness enthusiasts, and daycare workers. While many cases are asymptomatic, some patients experience severe symptoms leading to dehydration and weight loss. Fortunately, treatment with nitroimidazole or anthelminthic medication is often highly effective. Giardia duodenalis is responsible for this infection, and it is transmitted through the excretion of cysts by infected animals into freshwater sources. These cysts remain infectious for weeks to months. Seven genetic assemblages of Giardia have been identified, with genotypes A and B known to infect humans. Human infection occurs through the ingestion of cysts via contaminated water or direct person-to-person contact. Remarkably, even a small number of cysts, as few as 10, can lead to an infection [5–7].

After being excreted in feces, Giardia cysts become immediately infectious to new hosts, requiring no maturation or latent period [8]. These cysts can survive in various environments, particularly in water and at lower temperatures [9–11]. Sexual transmission of giardiasis, particularly among men who have sex with men, is a well-documented form of oral-anal and fecal-oral transmission [12]. Inadequate hygiene and sanitation practices significantly contribute to transmission. In modern times, daycares have become hotspots for giardiasis infections, primarily due to inadequate handwashing practices when handling and changing diapers. Within the intestinal system, the cysts undergo excystation and release trophozoites. These trophozoites are pear-shaped flagellated protozoa with two nuclei. Giardiasis is the most prevalent enteric protozoal infection globally, affecting approximately 2% of adults and 8% of children in developed countries. Estimates suggest that nearly 33% of the population in developing countries has been infected with giardiasis. In the United States, it is estimated that roughly 1.2 million individuals are affected, with most cases going undetected due to carriers being asymptomatic.

Giardia lamblia, initially observed by Antonie van Leeuwenhoek in 1681 within his own stool sample [13], was once viewed with uncertainty regarding its pathogenicity. However, it is now recognized as the predominant cause of protozoan-induced diarrhea in both humans and a wide range of domestic and wild animals globally [14]. Infections can lead to a spectrum of clinical manifestations, varying from asymptomatic colonization to acute or chronic diarrhea [15]. Notably, symptomatic infections have been documented in millions across Asia, Africa, and Latin America [16,17].

In 2012, the Centers for Disease Control and Prevention (CDC) documented 15,223 cases [18]. The most impacted demographic was children aged 0 to 4, and the majority of cases were reported in the northwest United States. Peak incidence typically coincides with late summer and early fall, primarily driven by outdoor water activities.

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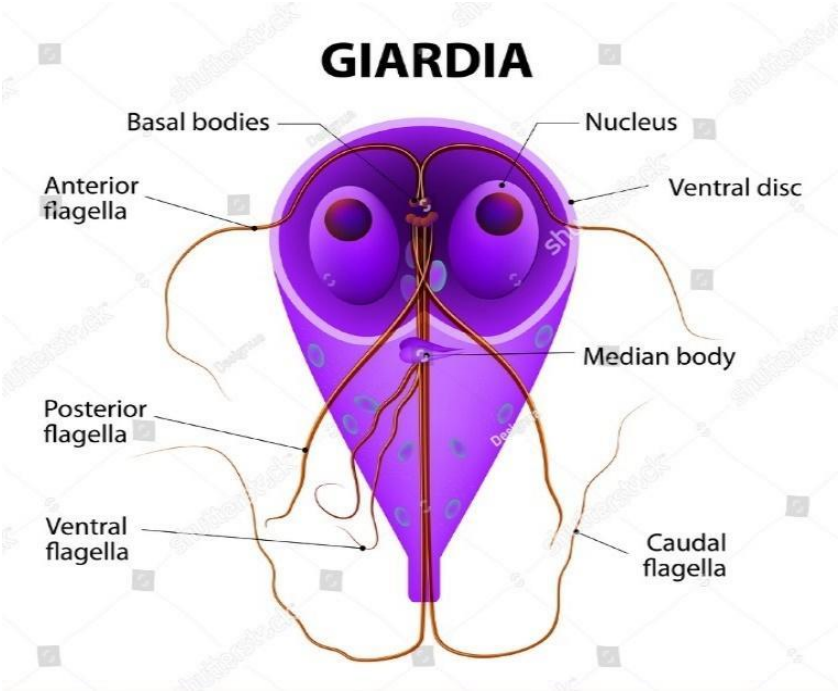

Fig. 1 schematic illustration of the virus.[19]

Globally, giardiasis incidence has been estimated at 2.8 × 10^8 cases annually [20]. However, numerous epidemiological studies suggest that these rates may be significantly underestimated. Prevalence rates of giardiasis range from 10 to 50% in developing countries [21, 22] and from 2 to 5% in developed nations [23, 24]. This discrepancy may be attributed to the substantial number of asymptomatic carriers who, despite lacking symptoms, play a pivotal role in disease transmission. Notably, Giardia infection in both animals and humans has been linked to growth retardation [25–26].

II. CAUSE AND SYMPTOMS

Giardia lamblia, commonly found in the feces of both animals and humans, thrives in various environments, including contaminated food, water, and soil. These hardy parasites can endure outside a host for extended periods, and accidental ingestion of these organisms can lead to infection. The primary mode of giardiasis transmission is through the consumption of water containing G. lamblia. Contaminated water sources include swimming pools, spas, and natural bodies of water like lakes. Such contamination often results from sources such as animal feces, diapers, and agricultural runoff. While acquiring giardiasis through food is less common due to heat effectively killing the parasites, poor hygiene practices during food handling or consumption of produce rinsed with contaminated water can facilitate the spread of the parasite. Additionally, giardiasis can spread through personal contact, such as unprotected sexual intercourse, facilitating transmission from one person to another. Activities like changing a child's diaper or exposure to the parasite while working in a daycare center are common routes of infection, particularly since children are at a heightened risk due to their exposure to feces during diaper changes or potty training. Giardia assumes a protective spore-like form known as cysts, which can endure harsh environmental conditions. These cysts are released through the bowel movements of infected individuals or animals. Giardiasis is transmitted when people accidentally ingest the parasite or its cysts, and astonishingly, just one to ten cysts are sufficient to cause infection. To provide perspective, approximately one million cysts could fit beneath a fingernail [27].

A person can become infected by several ways [28]:
• Ingesting contaminated drinking or recreational water.
• Touching their mouth with contaminated hands.
• Putting something in their mouth that has come into contact with the droppings of infected animals or the stool of infected humans.
• Eating raw or undercooked food that is contaminated.
• Inadequately washing their hands before preparing food, before eating, and after toileting or diapering.
•Exposure to the feces of an infected individual through sexual contact.

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III. SYMPTOMS OF GIARDIASIS

Symptoms of giardiasis typically manifest within seven to 10 days after exposure, although onset can occur as early as three days or as late as 25 days afterward. These symptoms usually endure for two to six weeks but may persist for an extended period in some cases. Prolonged Giardia infection can lead to complications, such as arthritis or damage to the intestinal lining [29]. It's important to note that some individuals infected with Giardia may remain asymptomatic but still carry and transmit the disease. If you exhibit signs and symptoms of illness and have potential exposure to sources of Giardia parasites, it is advisable to seek medical attention [30]. Certain individuals can harbor Giardia parasites without displaying any symptoms. Symptoms of giardiasis typically become apparent one to two weeks after exposure and commonly include [31]: • Fatigue • Nausea • Diarrhea or greasy stools • Vomiting • Bloating and abdominal cramps • Weight loss • Excessive gas • Headaches • Abdominal pain

IV.DEVELOPMENT OF MATHEMATICAL MODEL

The model considers two populations, namely: humans and domestic animals coupled with contaminated water and food in the environment. There is a natural death rate in each stage because the infection may take a long time, and therefore individual may die naturally. The mode of transmission of giardiasis is the environment to host, host to host, and host to the environment.

$$\frac{dS_h}{dt} = \lambda h - \beta S_h I_h - \rho S_h W - \mu_h S_h$$

$$\frac{dI_h}{dt} = \beta S_h I_h + \rho S_h w - \gamma I_h - \mu_h I_h$$

$$\frac{dW}{dt} = \alpha I_h - \mu W$$

$$\frac{dR_h}{dt} = \gamma I_h - \mu_h R_h$$

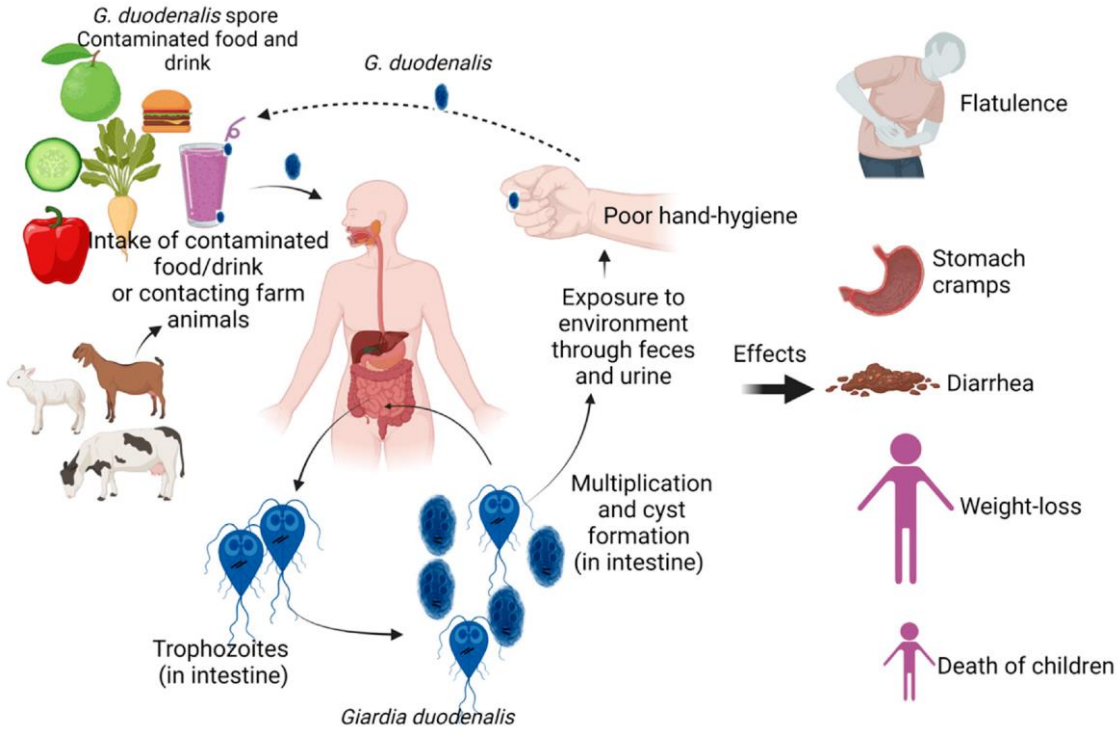

Fig. 2 Schematic illustration of the interaction between the virus and target cells. [32]

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TABLE 1 PARAMETERS AND THEIR DESCRIPTION

Parameter	Description of the Parameters
S_h	Susceptible Human population
I_h	Infected Human population
β	Disease Transmitted from Infected Human
ρ	Susceptible Individual become infected with Contaminated Water
W	Infected water(Parasite)
μ_h	Natural Death Rate of Human
α	Infected Individuals Spread Pathogens into Water at a rate
μ	Natural decay rate of parasite caused Giardia from water

V. EXISTENCE AND LOCAL STABILITY ANALYSIS OF THE EQUILIBRIUM POINTS

There are five equilibrium points of the given system. The equilibrium points are given by

$$E_0(0,0,0,0),\ E_1(\frac{\lambda_h}{\mu_h},0,0,0),\ E_2(S_h^2, I_h^2, W^2, 0),$$

where,

$$S^2{}_h = \frac{\mu(\gamma+\mu_h)}{(\rho\alpha+\mu\beta)}, \qquad I^2{}_h = \frac{\rho S_h W}{(\gamma+\mu_h-\beta S_h)},\ W = \frac{\alpha I_h}{\mu}$$

$E_3(S_h^3, I_h^3, 0, R_h^3)$,
where,

$$S^3{}_h = \frac{(\gamma+\mu_h)}{\beta}, \qquad I^3{}_h = \frac{\beta\lambda_h-\mu(\gamma+\mu_h)}{\beta(\gamma+\mu_h)},\ R^3{}_h = \frac{\gamma I_h}{\mu_h}$$

and the interior point equilibrium is $E^*(S^*{}_h, I^*{}_h, W^*, R^*{}_h)$

Lemma 1: $E_1 = \left(\frac{\lambda_h}{u_h}, 0,0,0\right)$; This equilibrium point is LAS if

$$\mu(\gamma+\mu_h+\mu)-\beta\lambda_h > 0, and\ \mu_h\gamma\mu+\mu^2{}_h\mu-\mu\beta\lambda_h-\rho\lambda_h\alpha > 0$$

.

Proof. The eigenvalues of the matrix corresponding to the equilibrium point $E_1 = \left(\frac{\lambda_h}{u_h}, 0,0,0\right)$ are $-\mu_h, -\mu_h$, and other two eigenvalues will have negative real part if

$$\mu(\gamma+\mu_h+\mu)-\beta\lambda_h > 0, and\ \mu_h\gamma\mu+\mu^2{}_h\mu-\mu\beta\lambda_h-\rho\lambda_h\alpha > 0$$

Therefore the equilibrium $E_1 = \left(\frac{\lambda_h}{u_h}, 0,0,0\right)$ is always stable under the above condition.

Lemma 2: $E_2 = (S_h^2, I_h^2, W^2, 0)$ This equilibrium point is stable.

Proof. The eigen values of the matrix corresponding to the equilibrium point *is* $(-\mu_h)$, and two other eigen values are given by the characteristic equation, $\lambda^3 + a1\lambda^2 + a2\lambda + a3 = 0$

$$a1 = \beta I_h^2 + \rho W^2 + 2\mu_h - \beta S_h^2 + \gamma + \mu$$

$$a2 = -2\beta^2 S^2{}_h I^2{}_h - S^2{}_h(\rho W^2\beta + \beta\mu_h + \beta\mu - \rho\alpha + \beta\rho W^2) + I^2{}_h(\beta\gamma + \beta\mu_h + \beta\mu)$$
$$+ (\rho W^2\gamma + \mu_h\gamma + \rho W^2\mu_h + \mu^2{}_h + \gamma\mu + 2\mu_h\mu + \rho W^2\mu)$$

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$a3 = (\mu\gamma\beta + \mu u_h\beta)I_h^2\text{-}S_h^2(\text{-}\mu_h\rho\alpha+\mu\beta\mu_h)+(\gamma\mu\rho W^2+\mu\rho\mu_h W^2+\mu\mu_h{}^2+ \mu\gamma\ \mu_h)$

The equilibrium point E_2 is LAS if $a1 > 0, a3 > 0$ *and* $a1a2 - a3 > 0$

Lemma 3: For $E_3 = (S_h^3, I_h^3, 0, R_h^3)$ This equilibrium point is LAS stable if
A>0, B>0 and AB-C>0.
Where $A = \beta I_h^3 + 2\mu_h - \beta S_h^3 + \gamma + \mu$

$$B = -2\beta^2 S^3{}_h\, I^3{}_h - S^3{}_h(\beta\mu_h + \beta\mu - \rho\alpha) + I^3{}_h(\beta\gamma + \beta\mu_h + \beta\mu) + (\mu_h\gamma + \mu^2{}_h + \gamma\mu + 2\mu_h\mu)$$

$C = (\mu\gamma\beta + \mu\mu_h\beta)I_h^3\text{-}S_h^3(\text{-}\mu_h\alpha\rho+\mu\beta\mu_h)+(\mu\mu_h{}^2+ \mu\gamma\mu_h)$

Proof. The eigenvalues of the matrix corresponding to the equilibrium point $is\ (-\mu_h)$ and other three eigenvalues are given by the characteristic equation,$\lambda^3 + A\lambda^2 + B\lambda + C = 0$,
Where,

$$A = \beta I_h^3 + 2\mu_h - \beta S_h^3 + \gamma + \mu$$

$$B = -2\beta^2 S^3{}_h\, I^3{}_h - S^3{}_h(\beta\mu_h + \beta\mu - \rho\alpha) + I^3{}_h(\beta\gamma + \beta\mu_h + \beta\mu) + (\mu_h\gamma + \mu^2{}_h + \gamma\mu + 2\mu_h\mu)$$

$C = (\mu\gamma\beta + \mu\mu_h\beta)I_h^3\text{-}S_h^3(\text{-}\mu_h\ \rho\alpha+\mu\beta\mu_h)+(\mu\mu_h{}^2+ \mu\mu_h\ \gamma)$

Therefore the equilibrium point E_3 is LAS if A>0, B>0 and AB-C>0.

Lemma 4: The interior point equilibrium is, $E^*(S^*{}_h, I^*{}_h, W^*, R^*{}_h)$, which is LAS if

Proof. The eigenvalues of the matrix corresponding to the equilibrium point $is(-\mu_h)$
and two other eigenvalues are given by the characteristic equation,$\lambda^3 + a1\lambda^2 + a2\lambda + a3=0$

$$a1 = \beta I_h^* + \rho W^* + 2\mu_h - \beta S_h^* + \gamma + \mu$$

$$a2 = -2\beta^2 S^*{}_h\, I^*{}_h - S^*{}_h(\rho W^*\beta + \beta\mu_h + \beta\mu - \rho\alpha + \beta\rho W^*) + I^*{}_h(\beta\gamma + \beta\mu_h + \beta\mu) + (\rho W^*\gamma + \mu_h\gamma + \rho W^*\mu_h + \mu^2{}_h + \gamma\mu + 2\mu_h\mu + \rho W^*\mu)$$

$a3 = (\mu\gamma\beta + \mu\mu_h\beta)I_h^*\text{-}S_h^*(\text{-}\mu_h\alpha\rho+\mu\beta\mu_h)+(\gamma\mu\rho W^*+\mu\rho\mu_h W^*+\mu\mu_h{}^2+\mu\mu_h\gamma)$

VI. CONCLUSION

The last two years have been really challenging the COVID-19 pandemic changed so much about the way we lived and worked. Now, just as we're starting to ease back into regular life, giardiasis started its infection .The majority of presenting patients will be non-toxic and may only require oral rehydration for initial fluid resuscitation. In more severe cases, intravenous (IV) fluids may be needed [33].

In this research article, we used a modelling approach to investigate the dynamics of giardiasis coupled with a contaminated environment. To study the effect of initial transmission of the disease we find the equilibrium point of the given system of equations and find the existence condition of the equilibrium and then we analysed the stability of the several equilibrium points [30-34]. The analysis of the stability of equilibrium points indicates that both the disease-free equilibrium and endemic equilibrium of the model system are locally and globally asymptotically stable.

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