



Awareness Program to Control Giardiasis: A Mathematical Approach

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Abstract: We propose and analyse a mathematical model of Giardiasis with awareness program to study the impact of awareness programs on an infectious disease outbreak. These programs induce behavioural changes in the population, which divide the susceptible class into two subclasses, aware susceptible and unaware susceptible. The system can have a disease-free equilibrium and an endemic equilibrium. The expression of the basic reproduction number and the conditions for the stability of the equilibria are derived. We further improve and study the model by introducing two time-delay factors, one for the time lag in memory fading of aware people and one for the delay between cases of disease occurring and mounting awareness programs. The delayed system has positive bounded solutions. We show that under certain conditions on the parameters, the system is permanent.

Keywords: Epidemic model, Giardiasis, Awareness programs, Stability analysis.

I. INTRODUCTION

Diarrheal disease is a prominent global health concern, ranking among the top ten causes of mortality and disability-adjusted life-years across all age groups. This makes diarrhea a critical threat on the global health landscape. In 2016, approximately 1.6 million cases of diarrheal deaths were reported, with a staggering 90% of these fatalities occurring in South Asia and sub-Saharan Africa [1]. Alarmingly, one in every nine child deaths can be attributed to diarrhea, resulting in 2,195 deaths daily and a total of 801,000 child deaths annually. This figure surpasses the combined cases of malaria, measles, and AIDS [2]. Of the reported cases, around 250 million individuals suffer from giardiasis, a parasitic disease caused by *Giardia duodenalis* (also known as *G. lamblia* and *G. intestinalis*) [3]. Diarrhea is the second leading cause of morbidity and mortality due to infectious diseases among children, with pneumonia as the primary cause. Therefore, it is imperative to implement measures to control the spread of giardiasis to mitigate the risk of increased diarrhea-related deaths [4]. Addressing giardiasis primarily relies on two key strategies: ensuring access to clean water and promoting good sanitation practices. Unfortunately, these measures are often lacking in disadvantaged community settings, profoundly impacting the well-being of children and their socioeconomic conditions. Consequently, giardiasis was included in the WHO Neglected Diseases Initiative in 2004 [5]. Notably, giardiasis also holds significant clinical and economic importance in the context of livestock and pet animals, necessitating a One Health approach to comprehensively control giardiasis.

In recent decades, *Giardia* has experienced a significant shift from its former status as a commensal organism to being recognized as a substantial pathogen. The complex nature of *G. duodenalis*, characterized by its high diversity, has created a situation where our understanding of immunological mechanisms, disease presentations, and pathophysiology remains limited and underexplored. Additionally, the virulence characteristics of this parasite and the factors governing host responses are still not well-defined, leading to critical gaps in knowledge that warrant thorough investigation of giardiasis.

In light of these considerations, this review endeavors to reexamine and update the state of giardiasis in Malaysia. Particular emphasis will be placed on the prevalence of the disease in humans, animals, and the environment, along with an exploration of associated risk factors. This comprehensive analysis will draw from studies conducted in the country over the past two decades. The information gathered in this review will serve as a valuable contribution to the body of literature on giardiasis in Malaysia. It will be instrumental in providing insights to healthcare professionals, researchers, policymakers, and other stakeholders, enabling them to formulate strategies for the prevention and control of giardiasis within the country.

II. SEARCH STRATEGY

To compile our research, we conducted searches on major academic databases, namely PubMed, Science Direct, and Scopus. The search terms we employed included MeSH keywords such as "*Giardia duodenalis*," "*Giardia intestinalis*," "*Giardia lamblia*," "intestinal protozoa," and "Malaysia." We limited our search to articles published within the last two decades, spanning from 2000 to 2020.



Our screening process encompassed the evaluation of all pertinent abstracts, systematic reviews, and articles that provided data on various aspects of giardiasis in Malaysia, including its epidemiology, prevalence, clinical manifestations, diagnostic methods, interventions, and genotyping. Articles that did not align with the specified keywords as mentioned in their titles or abstracts were excluded from our study. Additionally, any studies with ambiguous or contradictory information were also removed from our selection.

III. GLOBAL SCENARIO OF GIARDIASIS

Giardiasis exhibits a marked variation in prevalence across different regions, with rates reported as high as 20–40% in low-income countries, particularly affecting young children under the age of 5. In contrast, the prevalence is lower at 2–7% in high-income countries, where nearly 40% of cases are linked to individuals returning from areas with high endemicity [6]. The transmission of giardiasis is significantly influenced by socio-economic factors and hygiene conditions. In areas lacking access to proper hygiene and sanitation infrastructure, poverty and subpar living conditions contribute to its spread. In more developed regions with stringent sanitation standards, giardiasis is considered a reemerging infection, often associated with travel and potentially contaminated water sources. Moreover, Giardiasis is also responsible for various cases of foodborne illnesses and outbreaks in daycare centers, primarily transmitted through direct fecal-oral routes, zoonotic sources, or person-to-person contact. High-risk groups include immunocompromised individuals, young children, and patients with cystic fibrosis [7].

Between 2004 and 2010, a total of 199 waterborne protozoan outbreaks were reported worldwide, with Giardia identified as the causative agent in 35.2% (70) of these outbreaks [8]. Subsequently, from 2011 to 2016, there were at least 381 documented outbreaks attributed to waterborne transmission of parasitic protozoa, with Giardia species implicated in 37% (142) of these cases [9]. The significant public health impact of these outbreaks has led many countries, including the United States, the United Kingdom, Canada, and Australia, to classify giardiasis as a notifiable infection for the purpose of public health surveillance. Over the period from 1995 to 2016, the National Notifiable Diseases Surveillance System (NNDSS) received reports of 435,186 cases of giardiasis, of which 5.1% were associated with outbreaks. In 2016, the incidence rate was 6.4 cases per 100,000 populations [10]. Canada reported a rate of 19.6 giardiasis cases per 100,000 individuals in 2015, while in 2017, the South West of England had the highest rate of Giardia with 20.1 laboratory reports per 100,000 populations [11]. It's worth noting that while Denmark does not include giardiasis in its national health surveillance registers, the disease is systematically reported in Norway and Sweden, and Finland has a voluntary-based registration system [12].

IV. DEVELOPMENT OF MATHEMATICAL MODEL

$$\frac{dS_h}{dt} = \lambda_h - \beta S_h I_h - \rho S_h W - \mu_h S_h + \gamma_1 I_h - \mu_1 S_h + \alpha_1 A$$

$$\frac{dA}{dt} = \mu_1 S_h - \alpha_1 A + \delta \rho A W - \mu_h A$$

$$\frac{dI_h}{dt} = \beta S_h I_h + \rho S_h W - \mu_h I_h - \gamma_1 I_h - \delta \rho A W$$

$$\frac{dW}{dt} = \gamma_2 I_h - \mu W$$

μ_1 = Rate of median campaign to increase awareness population.

α_1 = Rate of loss of awareness due to neglect and concentration.

δ = Reduction of infection due to awareness.

ρ = Aware individuals A become infected with contaminated water at a rate.

V. EXISTENCE AND LOCAL STABILITY ANALYSIS OF THE EQUILIBRIUM POINTS

There are four equilibrium points of the aggregated system, trivial equilibrium point $E_0(0,0,0,0)$, planer equilibrium points

$$E_1\left(\frac{\lambda_h}{\mu_h + \mu_1}, 0, 0, 0\right), E_2\left(\frac{\lambda_h + \alpha_1 A}{\mu_h + \mu_1}, \frac{\mu_1 S_h}{\mu_h + \alpha_1}, 0, 0\right), E_3(S_h^3, 0, I_h^3, W)$$



Where S_h^3, I_h^3 and W satisfy the equations

$$\frac{dS_h}{dt} = \lambda_h - \beta S_h I_h - \rho S_h W - \mu_h S_h + \gamma_1 I_h - \mu_1 S_h$$

$$\frac{dI_h}{dt} = \beta S_h I_h + \rho S_h W - \mu_h I_h - \gamma_1 I_h - \delta \rho A W$$

$$\frac{dW}{dt} = \gamma_2 I_h - \mu W$$

and the interior point equilibrium is given by $E^*(S_h^*, A^*, I_h^*, W^*)$,

where S_h^*, A^*, I_h^* and W^* satisfy the equations

$$\frac{dS_h^*}{dt} = \lambda_h - \beta S_h^* I_h^* - \rho S_h^* W^* - \mu_h S_h^* + \gamma_1 I_h^* - \mu_1 S_h^* + \alpha_1 A^*$$

$$\frac{dA}{dt} = \mu_1 S_h^* - \alpha_1 A + \delta \rho A W^* - \mu_h A^*$$

$$\frac{dI_h^*}{dt} = \beta S_h^* I_h^* + \rho S_h^* W^* - \mu_h I_h^* - \gamma_1 I_h^* - \delta \rho A^* W^*$$

$$\frac{dW^*}{dt} = \gamma_2 I_h^* - \mu W^*$$

Lemma 1: The equilibrium point $E_0(0,0,0,0)$ is always unstable.

Lemma 2: The System (1) around $E_1(\frac{\lambda_h}{\mu_h + \mu_1}, 0, 0, 0)$ is locally asymptotically stable (LAS) if the the roots of the characteristic

equation $\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0$ of the Jacobian Matrix satisfy the following conditions, i.e $A_i > 0$ (where, $i=1,2,3,4$), together with $A_1A_2 - A_3 > 0$ and $A_1A_2A_3 - A_1^2A_4 - A_3^2 > 0$.

Where,

$$A_1 = a_{11} + a_{22} + a_{33} + a_{44}$$

$$A_2 = -(a_{11}a_{22} + a_{11}a_{33} + a_{11}a_{44} + a_{22}a_{33} + a_{22}a_{44} + a_{33}a_{44} - a_{34}a_{43} - a_{12}a_{21})$$

$$A_3 = a_{11}a_{22}a_{33} + a_{11}a_{22}a_{44} + a_{11}a_{44}a_{33} + a_{22}a_{44}a_{33} - a_{11}a_{34}a_{43} - a_{22}a_{34}a_{43} - a_{12}a_{21}a_{33} - a_{12}a_{21}a_{44}$$

$$A_4 = -a_{11}a_{22}a_{33}a_{44} + a_{11}a_{22}a_{34}a_{43} - a_{12}a_{21}a_{34}a_{43} + a_{12}a_{21}a_{33}a_{44}$$

Where, $a_{ij}, 1 \leq i \leq 4, 1 \leq j \leq 4$ are the elements of the variational matrix.

Lemma 3: The System around $E_2(\frac{\lambda_h + \alpha_1 A}{\mu_h + \mu_1}, \frac{\mu_1 S_h}{\mu_h + \alpha_1}, 0, 0)$ is locally asymptotically stable (LAS) if

the roots of the characteristic equation $\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0$ of the Jacobian Matrix satisfy the following conditions, i.e $A_i > 0$ (where, $i=1,2,3,4$), together with $A_1A_2 - A_3 > 0$ and $A_1A_2A_3 - A_1^2A_4 - A_3^2 > 0$.

Where,

$$A_1 = a_{11} + a_{22} + a_{33} + a_{44}$$

$$A_2 = -(a_{11}a_{22} + a_{11}a_{33} + a_{11}a_{44} + a_{22}a_{33} + a_{22}a_{44} + a_{33}a_{44} - a_{34}a_{43} - a_{12}a_{21})$$



$$A_3 = a_{11}a_{22}a_{33} + a_{11}a_{22}a_{44} + a_{11}a_{44}a_{33} + a_{22}a_{44}a_{33} - a_{11}a_{34}a_{43} - a_{22}a_{34}a_{43} - a_{12}a_{21}a_{33} - a_{12}a_{21}a_{44}$$

$$A_4 = -a_{11}a_{22}a_{33}a_{44} + a_{11}a_{22}a_{34}a_{43} - a_{12}a_{21}a_{34}a_{43} + a_{12}a_{21}a_{33}a_{44}$$

Where, $a_{ij}, 1 \leq i \leq 4, 1 \leq j \leq 4$ are the elements of the variational matrix.

Lemma 4: The System around $E_3 (S_h^3, 0, I_h^3, W)$ is locally asymptotically stable (LAS) if the the roots of the characteristic equation $\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0$ of the Jacobian Matrix satisfy the following conditions, i.e $A_i > 0$ (where, $i=1,2,3,4$), together with $A_1A_2 - A_3 > 0$ and $A_1A_2A_3 - A_1^2A_4 - A_3^2 > 0$.

Where,

$$A_1 = a_{11} + a_{22} + a_{33} + a_{44}$$

$$A_2 = -(a_{11}a_{22} + a_{11}a_{33} + a_{11}a_{44} + a_{22}a_{33} + a_{22}a_{44} + a_{33}a_{44} - a_{34}a_{43} - a_{12}a_{21} - a_{13}a_{31})$$

$$A_3 = a_{11}a_{22}a_{33} + a_{11}a_{22}a_{44} + a_{11}a_{44}a_{33} + a_{22}a_{44}a_{33} - a_{11}a_{34}a_{43} - a_{22}a_{34}a_{43} + a_{14}a_{31}a_{43} - a_{12}a_{21}a_{33} - a_{12}a_{21}a_{44} + a_{13}a_{21}a_{32} - a_{22}a_{31}a_{13} - a_{31}a_{44}a_{13}$$

$$A_4 = -a_{11}a_{22}a_{33}a_{44} + a_{11}a_{22}a_{34}a_{43} - a_{12}a_{21}a_{34}a_{43} + a_{14}a_{21}a_{32}a_{43} - a_{14}a_{22}a_{31}a_{43} + a_{12}a_{21}a_{33}a_{44} - a_{13}a_{21}a_{32}a_{44} + a_{13}a_{22}a_{31}a_{44}$$

Where, $a_{ij}, 1 \leq i \leq 4, 1 \leq j \leq 4$ are the elements of the variational matrix.

Lemma 5: The System around the interior equilibrium point $E^* (S_h^*, A^*, I_h^*, W^*)$ is locally asymptotically stable(LAS) if the roots of the characteristic equation $\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0$ of the Jacobian Matrix satisfy the following conditions, i.e $A_i > 0$ (where, $i=1,2,3,4$), together with $A_1A_2 - A_3 > 0$ and $A_1A_2A_3 - A_1^2A_4 - A_3^2 > 0$.

Where,

$$A_1 = a_{11} + a_{22} + a_{33} + a_{44}$$

$$A_2 = -(a_{11}a_{22} + a_{11}a_{33} + a_{11}a_{44} + a_{22}a_{33} + a_{22}a_{44} + a_{33}a_{44} - a_{34}a_{43} - a_{12}a_{21} - a_{13}a_{31})$$

$$A_3 = a_{11}a_{22}a_{33} + a_{11}a_{22}a_{44} + a_{11}a_{44}a_{33} + a_{22}a_{44}a_{33} - a_{11}a_{34}a_{43} - a_{22}a_{34}a_{43} + a_{24}a_{32}a_{43} + a_{14}a_{31}a_{43} - a_{12}a_{21}a_{33} - a_{12}a_{21}a_{44} + a_{13}a_{21}a_{32} - a_{22}a_{31}a_{13} - a_{31}a_{44}a_{13}$$

$$A_4 = -a_{11}a_{22}a_{33}a_{44} + a_{11}a_{22}a_{34}a_{43} - a_{11}a_{24}a_{32}a_{43} - a_{12}a_{21}a_{34}a_{43} + a_{12}a_{24}a_{31}a_{43} + a_{14}a_{21}a_{32}a_{43} - a_{14}a_{22}a_{31}a_{43} + a_{12}a_{21}a_{33}a_{44} - a_{13}a_{21}a_{32}a_{44} + a_{13}a_{22}a_{31}a_{44}$$

Where, $a_{ij}, 1 \leq i \leq 4, 1 \leq j \leq 4$ are the elements of the variational matrix.

VI. CONCLUSION

Giardiasis is a diarrheal illness resulting from infection with the microscopic parasite Giardia duodenalis, commonly referred to as "Giardia." Following infection in humans or animals, this parasite takes up residence in the intestinal tract and is excreted in feces. When expelled from the host's body, Giardia can endure for extended periods, ranging from weeks to months. Notably, Giardia is distributed widely and can be encountered in diverse regions across the United States and globally. In this research article we have considered the effect of awareness programs on the disease giardiasis where the human population is divided into three different classes, susceptible, aware and infected classes. A deterministic mathematical model was developed for the transmission dynamics of giardiasis with awareness. It was shown that the model is mathematically and epidemiologically well posed. Environment and vaccination strategies are discussed especially in the case of the succession of two epidemics with two different viruses. The equilibrium of the model equation was obtained and analysed analytically.



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