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Free Vibrations of Simply Supported Rectangular Mindlin Plate at Higher Modes Using Coupled Displacement Field Method

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Abstract: An accurate closed form solution was developed to evaluate the free vibration behaviour of thin and moderately thick isotropic rectangular plates with simply supported boundary condition for all the edges. A novel methodology known as coupled displacement field (CDF) method, proposed by the author was used to study the free vibration of the rectangular plate. Here an admissible trial function which satisfies the boundary conditions was assumed for one of the variables (say total rotations) and another variable which is the lateral displacement field is derived in terms of the initial variable by using the coupling equations, where the two independent variables become dependent on one another. The proposed CDF method makes use of the energy formulation and results in only half the number of undetermined coefficients when compared with the conventional Rayleigh-Ritz method. The vibration problem gets simplified significantly due to the reduction in the number of undetermined coefficients. The plate problem was also solved in Rayleigh-Ritz method to show the efficacy and simplicity of the CDF method. The Primary focus was given to the effect of aspect ratio and slenderness ratio on the non-dimensional frequency parameter at higher modes. The numerical results obtained by the present methodology are validated with Rayleigh-Ritz method and results available in the existing literature where ever possible. The analysis of the plate problem is based on Mindlin plate theory and the effect of shear deformation, as well as rotary inertia, were included.

Keywords: coupled displacement field, coupling equation, large amplitude vibrations, moderately thick plates.

I. INTRODUCTION

The complex structures in various fields of engineering are made up of simple structural members like beams (slender or short), plates (thin or moderately thick) and shells (thin or moderately thick). If these structural members are subjected to severe dynamic environment they vibrate. One of the essential consideration in design of these structures is to evaluation of free vibration. The conventional energy methods provide an effective means in evaluating the fundamental frequency parameters of these structural members and the results obtained by these approaches will act as an upper limit for comparison.

Accuracy of assumed trial function define the efficiency of energy methods. For thin plates, shear deformation and rotary inertia effects can be neglected. Therefore only one admissible trail function is enough for the analysis. But for moderately thick plates, these effects have to be considered and they are introduced by choosing separate admissible trial functions for the total rotations and lateral displacement [1]. The vibration study on such plates have been reported in by Leissa [2, 3].

The coupled displacement field (CDF) concept was implemented in the finite element (FE) analysis and was reported in the open literature. Free vibration analysis of moderately thick beams of uniform thickness by CDF method was successfully demonstrated [4]. In this study, the present methodology was extended to moderately thick rectangular plates for evaluating free vibration behaviour of uniform shear deformable rectangular plates.

For thin plates the fundamental frequency parameter was evaluated in Refs. [5,6] with several configurations and boundary conditions. S. H. Hashemi *et al.* [7] used dimensionless equation of motion to solve the problem based on first order shear deformation theory of plates to analyze the transverse vibration behaviour of thick rectangular plates. Many researchers have successfully employed different shear deformation plate theories to solve problems related to free vibration analysis of plates [8-14]. Non linear analysis of plates was evaluated by using finite element method [19-27].

For the analysis of thin and moderately thick rectangular plates, shear deformation and rotary inertia effects are considered by choosing separate admissible functions for both total rotations and lateral displacement fields. Thus, if n term admissible trial function is chosen for the total rotation, another n term admissible trial function for the lateral displacement result in, and the vibration problem becomes 2n unknown coefficients. The vibration problem analyzed by classical Rayleigh Ritz method contain 2n homogeneous simultaneous equations which are to be solved which was explained in the present paper.

In the present methodology, for the analysis of moderately thick rectangular plates, if an n term admissible trial function is chosen for the total rotation, the admissible trail function for lateral displacement is derived in terms of total rotation by using coupling equation. Both the functions contain same undetermined coefficients.

The coupling equation used to couple the independent fields was derived from static equilibrium equations. The complexity of vibration problem will be reduced as the number of undetermined coefficients reduced to half when compared to classical Rayleigh Ritz method. The practicality of this method is verified by solving the vibration problem considering uniform and isotropic moderately thick rectangular plates with all edges simply supported condition for different slenderness and aspect ratios.

II. FIRST ORDER SHEAR DEFORMATION THEORY OF PLATES

In first order shear deformation plate theory (or FSDT), also known as the Mindlin plate theory, the displacements u,v and w are given by

$$u(x,y,z) = z\theta_x(x,y)\,. \qquad (1)$$

$$v(x,y,z) = z\theta_y(x,y)\,. \qquad (2)$$

$$w(x,y,z) = w(x,y)\,. \qquad (3)$$

where u is inplane displacement along x direction, v is inplane displacement along y direction and, w is transverse displacement along z direction, θ_x and θ_y denote rotations about the y and x axes respectively.

In this theory, shear correction factor k was introduced to correct the discrepancy between the actual transverse shear stress distribution and that computed using the kinematic relations of FSDT. A value of k=5/6, the widely used value of the shear correction factor, is used in the present study.

III. RAYLEIGH-RITZ METHOD FOR MODERATELY THICK RECTANGULAR PLATES

In the conventional Rayleigh Ritz method separate functions for total rotations (θ_x and θ_y) and transverse displacement (w) are to be assumed in the analysis of moderately thick rectangular plates. The functions for total rotations and transverse displacement that satisfy the kinematic boundary conditions can be assumed as

$$w = \sum_{i=1}^{n} \alpha_{1_i} f_{w_i}(x,y)\,. \qquad (4)$$

$$\theta_x = \sum_{i=1}^{n} \alpha_{2_i} f_{wx_i}(x,y)\,. \qquad (5)$$

$$\theta_y = \sum_{i=1}^{n} \alpha_{3_i} f_{wy_i}(x,y)\,. \qquad (6)$$

For rectangular plate the undetermined coefficients of total rotations are the same. Hence there are only two undetermined coefficients, i.e, α_{1i}, α_{2i}. To know the efficacy of the CDF method initially free vibration problems are solved using Rayleigh Ritz method. In the present Rayleigh Ritz method, for better understanding, a single term admissible function is taken for θ_x, θ_y and w.

Expressions for strain energy (U) and kinetic energy (T) for moderately thick rectangular plates are given as

$$U=\frac{D}{2}\int_0^b\int_0^a\left\{\left(\frac{\partial\theta_x}{\partial x}\right)^2+\left(\frac{\partial\theta_y}{\partial y}\right)^2+2\nu\frac{\partial\theta_x}{\partial x}\frac{\partial\theta_y}{\partial y}+2(1-\nu)\frac{\partial\theta_x}{\partial y}\frac{\partial\theta_y}{\partial x}\right\}dxdy$$

$$+\frac{kGh}{2}\int_0^b\int_0^a\left\{\left(\frac{dw}{dx}+\theta_x\right)^2+\left(\frac{dw}{dy}+\theta_y\right)^2\right\}dxdy \quad . \qquad (7)$$

$$T=\frac{\rho h w_L^2}{2}\int_0^b\int_0^a\left[w^2+\frac{h^2}{12}\left(\theta_x^2+\theta_y^2\right)\right]dxdy \, . (8)$$

where D is the flexural rigidity, G is rigidity modulus, h is thickness of the plate and ν is the poisson's ratio. By substituting the assumed functions, we derive the expressions for strain energy and kinetic energy.

The minimization of Lagranzian, with respect to undetermined coefficient α_1 and α_2 in case of single term admissible functions are given as:

$$\frac{\partial(U-T)}{\partial\alpha_1}=0 \, . \quad (9)$$

and

$$\frac{\partial(U-T)}{\partial\alpha_2}=0 \, . \quad (10)$$

By solving the above two homogeneous equations, the non-dimensional fundamental frequency parameter (λ) is obtained.

IV. EXPRESSION FOR NON-DIMENSIONAL FUNDAMENTAL FREQUENCY PARAMETER OF MODERATELY THICK RECTANGULAR PLATE USING RAYLEIGH RITZ METHOD

Single term admissible functions for total rotations (θ_x , θ_y) and transverse displacement (w) for the assumed fundamental mode are taken as

$$\theta_x=\alpha\frac{m\pi}{a}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \, . \qquad (11)$$

$$\theta_y=\alpha\frac{n\pi}{b}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \, . \qquad (12)$$

$$w=\beta\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \, . \quad (13)$$

substituting Eqs. (11-13) in Eq. (7) and Eq. (8) and after integration, the expression for strain energy and kinetic energy are obtained as

$$U=\frac{D}{2}\frac{ab}{4}\left\{\beta^2\left(p^2+q^2\right)^2+\left(\frac{k6(1-\nu)}{h^2}(\alpha+\beta)^2\left(p^2+q^2\right)\right)\right\} . \quad (14)$$

$$T=\frac{\rho h\omega_L^2}{2}\frac{ab}{4}\left(\alpha^2+\frac{h^2}{12}\beta^2\left(p^2+q^2\right)\right) . \quad (15)$$

where $p=\dfrac{m\pi}{a}$, $q=\dfrac{n\pi}{b}$ and $b=s\,a$

By minimizing the Lagranzian with respect to undetermined coefficient α and β, the following homogeneous quadratic equations are obtained.

$$\frac{D}{8}ab\left[\frac{12k(1-\nu)}{h^2}(\alpha+\beta)\left(p^2+q^2\right)\right]-\left[\frac{\rho h\omega_l^2 ab}{4}\alpha\right]=0. \qquad (16)$$

$$\frac{D}{8}ab\left[2\left(p^2+q^2\right)^2+\frac{12k(1-\nu)}{h^2}(\alpha+\beta)\left(p^2+q^2\right)\right] -\left[\frac{\rho h^3\omega_l^2 ab}{48}\left(p^2+q^2\right)\beta\right]=0 . \qquad (17)$$

$$\lambda^2\left(\frac{h}{a}\right)^4-\left(\begin{array}{l}6\left(2\pi^2\left(\frac{h}{a}\right)^2\left(\frac{m^2s^2+n^2}{s^2}\right)\right)\\ +\left(k(1-\nu)\left(12+\left(\frac{h}{a}\right)^2\left(\left(\frac{m^2s^2+n^2}{s^2}\right)\right)\pi^2\right)\right)\end{array}\right)\lambda +4k(1-\nu)\left(\left(\frac{m^2s^2+n^2}{s^2}\right)\right)\pi^4=0 . \qquad (18)$$

$$\lambda=\frac{\rho h\omega_L^2 a^4}{D}=\frac{G-\sqrt{H}}{2\left(\frac{h}{a}\right)^4} . \qquad (19)$$

where $G=\left(6\left(2\pi^2\left(\frac{h}{a}\right)^2\left(\left(\frac{m^2s^2+n^2}{s^2}\right)\right)\right)+\left(k(1-\nu)\left(12+\left(\frac{h}{a}\right)^2\left(\left(\frac{m^2s^2+n^2}{s^2}\right)\right)\pi^2\right)\right)\right)$ and

$$H=36\left(2\pi^2\left(\frac{h}{a}\right)^2\left(\left(\frac{m^2s^2+n^2}{s^2}\right)\right)\right)+\left(k(1-\nu)\left(12+\left(\frac{h}{a}\right)^2\left(\left(\frac{m^2s^2+n^2}{s^2}\right)\right)\pi^2\right)\right)^2 -4k(1-\nu)\left(\frac{h}{a}\right)^4\left(\left(\frac{m^2s^2+n^2}{s^2}\right)\right)\pi^4$$

V. COUPLED DISPLACEMENT FIELD (CDF) METHOD FOR MODERATELY THICK RECTANGULAR PLATES

In this section the detailed procedure for evaluating fundamental linear frequency parameter of uniform simply supported moderately thick rectangular plate based on present methodology is discussed in detail. The admissible trial functions for θ_x and θ_y are assumed in the functional form. They satisfy the boundary conditions and symmetric conditions for the fundamental mode.

$$\theta_x=\alpha\frac{m\pi}{a}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} . \qquad (20)$$

$$\theta_y=\alpha\frac{n\pi}{b}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} . \qquad (21)$$

VI. COUPLING EQUATIONS FOR MODERATELY THICK RECTANGULAR PLATES

The coupling equations for evaluating the transverse displacement distribution w in x and y directions are given as

$$\frac{dw}{dx}=-\theta_x+\frac{h^2}{3.5}\left[\frac{\partial^2\theta_x}{\partial x^2}+v\frac{\partial^2\theta_y}{\partial y\partial x}\right]+\frac{h^2}{10}\left[\frac{\partial^2\theta_x}{\partial y^2}+\frac{\partial^2\theta_y}{\partial y\partial x}\right]. \qquad (22)$$

$$\frac{dw}{dy}=-\theta_y+\frac{h^2}{3.5}\left[\frac{\partial^2\theta_y}{\partial y^2}+v\frac{\partial^2\theta_x}{\partial y\partial x}\right]+\frac{h^2}{10}\left[\frac{\partial^2\theta_y}{\partial x^2}+\frac{\partial^2\theta_x}{\partial y\partial x}\right]. \qquad (23)$$

substituting Eq. (20) and Eq. (21) in Eq. (22) and Eq. (23) and after integration and evaluation of the constant of integration we get

$$w=-\alpha\sin px\sin qy\left[1+\frac{h^2}{3.5}\left(p^2+vq^2\right)+\frac{h^2}{5}\left(q^2\right)\right]. \qquad (24)$$

$$U=\frac{D}{2}\alpha^2ab\left\{\left(p^2+q^2\right)^2+\left(\frac{k6(1-v)}{h^2}\left(p^2(A-1)^2+q^2(B-1)^2\right)\right)\right\}. \qquad (25)$$

where $A=\left[1+\frac{h^2}{3.5}\left(p^2+vq^2\right)+\frac{h^2}{5}\left(q^2\right)\right]$ and $B=\left[1+\frac{h^2}{3.5}\left(q^2+vp^2\right)+\frac{h^2}{5}\left(p^2\right)\right]$

Substituting Eq. (20), Eq. (21) and Eq. (24) in Eq. (8) and after simplification we find kinetic energy as

$$T=\frac{\rho h\omega_L^2}{2}\alpha^2\frac{ab}{4}\left(A^2+\frac{h^2}{12}\left(p^2+q^2\right)\right). \qquad (26)$$

By minimizing the Lagranzian with respect to undetermined coefficient α we get the non-dimensional fundamental frequency parameter.

$$\frac{\partial\left(U-T\right)}{\partial\alpha}=0. \qquad (27)$$

$$\lambda=\frac{\rho h\omega_L^2a^4}{D}=\frac{\pi^4\left[\left(m^2+\left(\frac{n}{s}\right)^2\right)^2+k6(1-v)\pi^2\left(m^2A_1^2+\left(\frac{nB_1}{s}\right)^2\right)\left(\frac{h}{a}\right)^2\right]}{\left[\left(1+\pi^2A_1\left(\frac{h}{a}\right)^2\right)^2+\frac{\pi^2}{12}\left(m^2+\left(\frac{n}{s}\right)^2\right)\left(\frac{h}{a}\right)^2\right]}. \qquad (28)$$

where $A_1=\frac{1}{3.5}\left(m^2+v\frac{n^2}{s^2}\right)+\frac{n^2}{5s^2}$ and $B_1=\frac{1}{3.5}\left(\frac{n^2}{s^2}+v\right)+\frac{m^2}{5}$.

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VII. LARGE AMPLITUDE FREE VIBRATIONS

By applying law of conservation of energy the total energy is constant at any instant of time neglecting damping. In order to evaluate the large amplitude free vibrations of plates, the work done due to the stretching of the central plane of the plate has to be derived. The stretching of the central plane in *x* direction causes tension and its expression can be given as

$$T_x = \frac{Eh}{2a}\int_0^a \left\{\frac{dw(x)}{dx}\right\}^2 dx \,. \quad (29)$$

$$T_x = \frac{Eh}{4}\alpha^2 \frac{\pi^2}{a^2} \,. \quad (30)$$

Shear flexible terms are not involved in the expression of tension because the tension in the plate is only caused by stretching of the mid plane due to bending and not due to shear. Similarly the tension in *y* direction can be given as

$$T_x = \frac{Eh}{2a}\int_0^a \left\{\frac{dw(x)}{dx}\right\}^2 dx \,. \quad (31)$$

$$T_x = \frac{Eh}{4}\alpha^2 \frac{\pi^2}{a^2} \,. \quad (32)$$

The expression for work done is given as

$$W = \frac{1}{4}\int_0^b\int_0^a \left\{T_x\left(\frac{dw}{dx}\right)^2 + T_y\left(\frac{dw}{dy}\right)^2\right\}dxdy \,. \quad (33)$$

Substituting Eq. (22), Eq. (23), Eq. (30) and Eq. (32) in Eq. (33) and after simplification we get

$$W = \alpha^4 \frac{Eh}{64} a^2 s\left(p^4 A^2 + q^4 B^2\right) . \quad (34)$$

By applying total energy principle

$$U + T + W = constant \,. \quad (35)$$

Substituting Eq. (25), Eq. (26), Eq. (34) and $D=Eh^3/12(1\text{-}v^2)$ in Eq. (35) and after simplification

$$\dot{z}^2 + \eta_1 z^2 + \eta_2 z^4 = \text{constant} \,. \quad (36)$$

where

$$\alpha_1 = \frac{\rho h \omega_L^2 a^4}{D} = \frac{\pi^4\left[\left(m^2 + \left(\frac{n}{s}\right)^2\right)^2 + k6(1-v)\pi^2\left(m^2 A_1^2 + \left(\frac{nB_1}{s}\right)^2\right)\left(\frac{h}{a}\right)^2\right]}{\left[\left(1 + \pi^2 A_1\left(\frac{h}{a}\right)^2\right)^2 + \frac{\pi^2}{12}\left(m^2 + \left(\frac{n}{s}\right)^2\right)\left(\frac{h}{a}\right)^2\right]} \,. \quad (37)$$

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$$\alpha_2=\frac{\rho h\omega_{NL}^2a^4}{D}=\frac{\pi^4\frac{3}{2}\left(1-v^2\right)\left[m^4\left(1+A_1\pi^2\left(\frac{h}{a}\right)^2\right)^2+\frac{n^4}{s^4}\left(1+B_1\frac{\pi^2}{s^2}\left(\frac{h}{a}\right)^2\right)^2\right]}{\left[\left(1+\pi^2A_1\left(\frac{h}{a}\right)^2\right)^2+\frac{\pi^2}{12}\left(m^2+\left(\frac{n}{s}\right)^2\right)\left(\frac{h}{a}\right)^2\right]}. \quad (38)$$

By following the harmonic balance method the expression for frequency ratio can be obtained as

$$\left[\frac{\omega_{NL}}{\omega_L}\right]^2=1+1.5\left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{\alpha_m}{h}\right)^2. \quad (39)$$

$$\frac{\alpha_2}{\alpha_1}=\frac{\frac{3}{2}\left(1-v^2\right)\left[m^4\left(1+A_1\pi^2\left(\frac{h}{a}\right)^2\right)^2+\frac{n^4}{z^4}\left(1+B_1\frac{\pi^2}{z^2}\left(\frac{h}{a}\right)^2\right)^2\right]}{\left[\left(m^2+\left(\frac{n}{z}\right)^2\right)^2+k6(1-v)\pi^2\left(m^2A_1^2+\left(\frac{nB_1}{z}\right)^2\right)\left(\frac{h}{a}\right)^2\right]}. \quad (40)$$

Eq. (39) gives the expression for frequency ratio in terms of thickness ratio (h/a) and maximum amplitude ratio (α_m/h).

VIII. RESULTS AND DISCUSSION

To show the simplicity, efficacy and ease of application of the proposed CDF method, initially linear free vibration problem of moderately thick rectangular plate with all edges simply supported boundary condition (Fig. 1) at higher modes is solved by using Rayleigh Ritz and CDF methods. Single term trigonometric admissible functions are used in these two methods to study the influence of plate thickness ratio and aspect ratio on the frequency parameter at fundamental mode and higher modes. Numerical results in the form of frequency parameter at different modes for a plate thickness ratio of 0.001 as a function of different aspect ratios is given in Table 1. For the purpose of validation the results obtained by the present CDF, RR methods along with other researchers are also included in Table 1 and they are in good agreement. It is observed from Table 1 that frequency parameter increases with mode number. It is also observed that frequency parameter decreases with increase of aspect ratio for a particular given mode sequence. Similar trend for the frequency parameter has been observed for other plate thickness ratios such as 0.05, 0.1, 0.15 and 0.2 which are given respectively in Table 2-5.

It is found from Table 1-5 that around 11.5 % reduction in frequency parameter has been observed in moderately thick plate (Plate thickness ratio=0.2) when compared to thin plate (Plate thickness ratio=0.001) for an aspect ratio 1 for fundamental mode and a variation of 55 % for mode sequence 44. Around 6.5 % reduction in frequency parameter has been observed in moderately thick plate (Plate thickness ratio=0.2) when compared to thin plate (Plate thickness ratio=0.001) for an aspect ratio of 10 for fundamental mode and a variation of 43 % for mode sequence of 44.

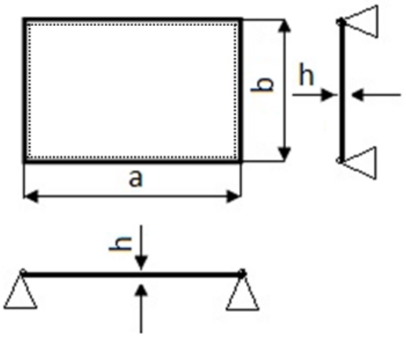

Fig.1. Uniform simply supported moderately thick rectangular plates

where u is inplane displacement along *x* direction, *v* is inplane displacement along *y* direction and, w is transverse displacement along *z* direction, θ_x and θ_y denote rotations about the *y* and *x* axes respectively. In this theory, shear correction factor k was introduced to correct the discrepancy between the actual transverse shear stress distribution and that computed using the kinematic relations of FSDT. A value of k=5/6, the widely used value of the shear correction factor, is used in the present study.

IX. FIRST ORDER SHEAR DEFORMATION THEORY OF PLATES

In first order shear deformation plate theory (or FSDT), also known as the Mindlin plate theory, the displacements *u*,*v* and *w* are given by

$$u(x,y,z)=z\theta_x(x,y)\,. \qquad (1)$$

$$v(x,y,z)=z\theta_y(x,y)\,. \qquad (2)$$

$$w(x,y,z)=w(x,y)\,. \qquad (3)$$

For the sake of comparison and validation of proposed method, the same results obtained by the finite element method (FEM) [20] and of other researchers [25, 26, 27] are included in table 6, table 7 for aspect ratio 1 and 2 respectively for fundamental mode and the agreement found is very good. The present results in terms time period ratio (T_{NL}/T_L) as a function of plate thickness ratio, aspect ratio and mode sequence are given respectively in Table 8-12 for different amplitude ratios such as 0.2, 0.4, 0.6, 0.8 and 1. The time period ratio is evaluated by taking reciprocal of frequency ratio given in Eq. (39). Table 8 shows the variation of time period ratio with different plate thickness ratios, aspect ratios and mode sequence for an amplitude ratio of 0.2.

Results show that there is around 2 % to 3 % (variation) decrease in time period for thin plate when nonlinearity is considered and this variation increases to around 3 % to 8 % for moderately thick plate for a thickness ratio of 0.2 and aspect ratio 1. With the increase in aspect ratio the time period ratio decreases. The decrease in time period considering nonlinearity is around 2 % to 4 % when the aspect ratio is increased from 1 to 10 for thin plate. As the plate thickness ratio is increased to 0.2 this variation increases to 3 % to 5% in fundamental mode and reaches 8% to 10% for 44 mode. Table 9-12 shows the variation of time period ratio for maximum amplitude ratios 0.4, 0.6, 0.8 1 respectively.

The observations of results from Table 9-12 are similar to table 8 but the variation is predominant with the increase in amplitude ratio. Results in Table 12 for maximum amplitude ratio 1 shows that there is around 30 % to 40 % (variation) decrease in time period for thin plate when nonlinearity is considered and this variation increases to around 40 % to 60 % for moderately thick plate for a thickness ratio of 0.2 and aspect ratio 1. This gives a clear indication from Table 8-12 that time period ratio decreases with increase of aspect ratio, amplitude ratio and plate thickness ratio. The effect of amplitude ratio is predominant on time period ratio. Shear deformation and rotary inertia contribution has been observed in all above tables for the given all edges simply supported plate.

X. CONCLUSION

The concept of coupled displacement field method (CDF) is successfully applied to study the free vibrations of moderately thick rectangular plates with all edges simply supported boundary condition. Elegant and closed form expressions for non-dimensional frequency parameter was derived in terms of plate thickness ratio (*h/a)* aspect ratio and mode number.

The closed form expression was obtained for the time period ratio as a function of maximum amplitude ratio α_m/h and plate thickness ratio h/a. The non-dimensional frequency parameter and time period results obtained by the present method are very accurate when compared to the published results.

It indicates that the CDF method can be applied for thin and moderately thick problems. From the present study it is evident that CDF method predicts the vibration behavior of rectangular plates accurately and even minimizes the computational effort.

Table 1. Linear frequency parameter ($\sqrt{\lambda}$) for thickness ratio 0.001

b/a	Ref	Mode Number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
1	CDF	19.7391	49.3476	98.6942	167.7781	49.3476	78.9557	128.3018	197.3849	98.6942	128.3018	177.6471	246.7289	167.7781	197.3849	246.7289	315.8089
	RR	19.7391	49.3476	98.6942	167.7780	49.3476	78.9557	128.3020	197.3850	98.6942	128.3200	177.6470	246.7290	167.7780	197.3850	246.7290	315.8090
	15	19.7482	49.3433	98.5957	167.4018	49.3433	78.9058	128.1059	196.8359	98.5957	128.1059	177.2171	245.8236	167.4018	196.8359	245.8236	314.2592
	16	19.7551	49.3878	98.7745	167.9134	49.3878	79.0204	128.4062	--	98.7745	128.4062	--	--	167.9134	--	--	--
	3	19.7392	49.3480	98.6960	--	49.3480	78.9568	128.3049	--	98.6960	128.3049	--	--	167.7833	--	--	--
	17	19.7391	49.3475	98.6943	--	49.3475	78.9556	128.3019	--	98.6943	128.3019	--	--	--	--	--	--
1.5	CDF	14.2561	27.4154	49.3476	80.0523	43.8646	57.0238	78.9557	109.6601	93.2113	106.3703	128.3018	159.0056	162.2953	175.4540	197.3849	228.0879
	RR	14.2561	27.4154	49.3476	80.0523	43.8646	57.0238	78.9557	109.6600	93.2113	106.3700	128.3020	159.0060	162.2950	175.4540	197.3850	228.0880
	15	14.2642	27.4240	49.3433	80.0003	43.8652	57.0103	78.9058	109.5292	93.1285	106.2499	128.1059	158.6740	161.9474	175.0361	196.8359	227.3279
	16	14.2681	27.4398	49.3868	80.1168	43.9008	57.0695	79.0194	109.7485	93.2866	106.4563	--	--	--	--	--	--
	3	14.2561	27.4156	49.3480	80.0535	43.8649	57.0244	78.9568	--	93.2129	106.3724	--	--	--	--	--	--
2	CDF	12.3370	19.7391	32.0760	49.3476	41.9455	49.3476	61.6843	78.9557	91.2923	98.6942	111.0308	128.3018	160.3763	167.7781	180.1143	197.3849
	RR	12.3370	19.7391	32.0760	49.3476	41.9455	49.3476	61.6843	78.9557	91.2923	98.6942	111.0310	128.3020	160.3760	167.7780	180.1140	197.3850
	15	12.3440	19.7482	32.0833	49.3433	41.9470	49.3433	61.6646	78.9058	91.2142	98.5957	110.8953	128.1059	160.0381	167.4018	179.6697	196.8359
	16	12.3469	19.7541	32.1020	49.3868	41.9796	49.3868	61.7337	79.0194	91.3673	--	--	--	--	--	--	--
2.5	CDF	11.4487	16.1861	24.0817	35.1356	41.0572	45.7946	53.6901	64.7438	90.4041	95.1413	103.0367	114.0902	159.4881	164.2252	172.1204	183.1737
	RR	11.4487	16.1861	24.0817	35.1356	41.0572	45.7946	53.6901	64.7438	90.4041	95.1413	103.0370	114.0900	159.4880	164.2250	172.1200	183.1740
	15	11.4560	16.1942	24.0904	35.1414	41.0600	45.7933	53.6805	64.7197	90.3282	95.0527	102.9261	113.9455	159.1540	163.8676	171.7202	182.7120
	16	11.4599	16.2012	24.1032	35.1641	41.0906	45.8318	53.7339	64.7967	--	--	--	--	--	--	--	--
	3	11.4487	16.1862	24.0818	35.1358	41.0576	45.7950	53.6906	--	--	--	--	--	--	--	--	--
3	CDF	10.9662	14.2561	19.7391	27.4154	40.5747	43.8646	49.3476	57.0238	89.9216	93.2113	98.6942	106.3703	159.0056	162.2953	167.7781	175.4540
	RR	10.9662	14.2561	19.7391	27.4154	40.5747	43.8646	49.3476	57.0238	89.9216	93.2113	98.6942	106.3700	159.0060	162.2950	167.7780	175.4540
	15	10.9730	14.2642	19.7482	27.4240	40.5770	43.8652	49.3433	57.0103	89.8462	93.1285	98.5957	106.2499	158.6740	161.9474	167.4018	175.0361
5	CDF	10.2644	11.4487	13.4226	16.1861	39.8729	41.0572	43.0311	45.7946	89.2198	90.4041	92.3779	95.1413	158.3038	159.4881	161.4619	164.2252
	RR	10.2644	11.4487	13.4226	16.1861	39.8729	41.0573	43.0312	45.7945	89.2197	90.4041	92.3779	95.1413	158.3038	159.4882	161.4621	164.2251
	15	10.2707	11.4560	13.4305	16.1942	39.8757	41.0600	43.0316	45.7933	89.1469	90.3282	92.2968	95.0527	157.9756	159.1540	161.1177	163.8676
10	CDF	9.9683	10.2644	10.7578	11.4487	39.5768	39.8729	40.3664	41.0572	88.9237	89.2198	89.7132	90.4041	158.0078	158.3038	158.7973	159.4881
	RR	9.9683	10.2644	10.7578	11.4487	39.5769	39.8729	40.3663	41.0573	88.9237	89.2197	89.7132	90.4041	158.0079	158.3038	158.7974	159.4882
	15	9.9744	10.2707	10.7646	11.4560	39.5803	39.8757	40.3696	41.0600	88.8515	89.1469	89.6388	90.3282	157.6813	157.9756	158.4665	159.1

Table 2. Linear frequency parameter ($\sqrt{\lambda}$) for thickness ratio 0.05

b/a	Ref	Mode Number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
1	CDF	19.5624	48.2697	94.5465	156.3867	48.2697	76.2603	121.4466	181.9428	94.5465	121.4466	164.9652	223.393	156.3867	181.9428	223.393	279.2334
	RR	19.5624	48.2695	94.5454	156.3797	48.2695	76.25988	121.4438	181.9308	94.54544	121.4438	164.9567	223.3676	156.3797	181.9308	223.3676	279.1775
1.5	CDF	14.1635	27.0767	48.2697	77.2839	43.0091	55.5933	76.2603	104.5818	89.496	101.5803	121.4466	148.7099	151.5945	163.0641	181.9428	207.8956
	RR	14.1635	27.0767	48.2695	77.2834	43.0090	55.59317	76.2599	104.5801	89.4952	101.579	121.4438	148.7041	151.5883	163.0561	181.9308	207.8759
2	CDF	12.2675	19.5624	31.6142	48.2697	41.162	48.2697	60.0167	76.2603	87.7231	94.5465	105.8302	121.4466	149.9127	156.3867	167.1003	181.9428
	RR	12.2675	19.5624	31.6141	48.2695	41.162	48.26956	60.0165	76.25988	87.72235	94.54544	105.8286	121.4438	149.9066	156.3797	167.0916	181.9308
2.5	CDF	11.3889	16.0669	23.8197	34.5828	40.3061	44.8636	52.4186	62.9111	86.9016	91.2763	98.5308	108.6117	149.1334	153.2835	160.1687	169.7428
	RR	11.3888	16.0669	23.8196	34.5828	40.3061	44.86357	52.4185	62.9109	86.90086	91.27541	98.52964	108.6099	149.1278	153.2772	160.1612	169.7333
3	CDF	10.9113	14.1635	19.5624	27.0767	39.8408	43.0091	48.2697	55.5933	86.4551	89.496	94.5465	101.5803	148.7099	151.5945	156.3867	163.0641
	RR	10.9113	14.1635	19.5624	27.0767	39.8408	43.00907	48.2696	55.5932	86.45438	89.4952	94.54544	101.579	148.7041	151.5883	156.3797	163.0561
5	CDF	10.2162	11.3889	13.3405	16.0669	39.1638	40.3061	42.2073	44.8636	85.8054	86.9016	88.7264	91.2763	148.0937	149.1334	150.8644	153.2835
	RR	10.2162	11.3888	13.3405	16.0669	39.1637	40.30608	42.2072	44.8636	85.80466	86.90086	88.72559	91.27541	148.0881	149.1278	150.8582	153.2772
10	CDF	9.9228	10.2162	10.705	11.3889	38.878	39.1638	39.6399	40.3061	85.5311	85.8054	86.2623	86.9016	147.8336	148.0937	148.527	149.1334
	RR	9.92285	10.2162	10.7049	11.3888	38.8780	39.16376	39.6399	40.3061	85.5304	85.80466	86.26152	86.90086	147.8279	148.0881	148.5214	149.1278

Table 3. Linear frequency parameter ( ) for thickness ratio 0.1

b/a	Ref	Mode Number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
1	CDF	19.0651	45.4857	85.0656	133.7399	45.4857	69.8084	106.7420	152.7853	85.0656	106.7420	140.1940	182.6167	133.7399	152.7853	182.6167	221.0848
	RR	19.0650	45.4826	85.0380	133.6211	45.4826	69.7943	106.6836	152.6086	85.0380	106.6836	140.0571	182.3239	133.6211	152.6086	182.3239	220.6003
	15	19.0805	45.5237	85.1326	133.8230	45.5237	69.8649	106.8188	152.8689	85.1326	106.8188	140.2780	182.6981	133.8230	152.8689	182.6981	221.1653
	16	19.0805	45.5197	85.1070	133.7292	45.5197	69.8501	106.7694	--	--	106.7694	--	--	133.7292			
	18	19.0736	45.4861	84.9963	--	45.4861	69.7750	106.5985	--	84.9963	106.5985	--	--	--	--	--	--
	7	19.0649	45.4826	85.0380	133.6212	45.4826	69.7943	106.6836	--	85.0380	106.6836	--	--	--	--	--	--
	17	19.0653	45.4869	85.0646	--	45.4869	69.8093	106.7350	--	85.0646	106.7350	--	--	--	--	--	--
1.5	CDF	13.8983	26.1455	45.4857	70.6747	40.7660	51.9739	69.8084	93.2527	80.8969	90.8170	106.7420	127.9089	130.1062	138.7692	152.7853	171.6043
	RR	13.8983	26.1451	45.4826	70.6601	40.7640	51.9690	69.7943	93.2151	80.8737	90.7827	106.6836	127.8053	129.9969	138.6366	152.6086	171.3578
	15	13.9096	26.1666	45.5237	70.7322	40.8002	52.0172	69.8649	93.3241	80.9613	90.8873	106.8188	127.9913	130.1891	138.8527	152.8689	171.6867
	16	13.9096	26.1666	45.5197	70.7173	40.7972	52.0112	69.8501	93.2905	80.9386	90.8557	--	--	--	--	--	--
	18	13.9076	26.1558	45.4861	70.6541	40.7647	51.9658	69.7750	--	80.8379	--	--	--	--	--	--	--
2	CDF	12.0674	19.0651	30.3616	45.4857	39.0967	45.4857	55.8483	69.8084	79.4256	85.0656	94.2625	106.7420	128.8260	133.7399	141.7905	152.7853
	RR	12.0674	19.0650	30.3610	45.4826	39.0950	45.4826	55.8419	69.7943	79.4038	85.0380	94.2236	106.6836	128.7202	133.6211	141.6489	152.6086
	15	12.0773	19.0805	30.3863	45.5237	39.1289	45.5237	55.8941	69.8649	79.4886	85.1326	94.3346	106.8188	128.9090	133.8230	141.8742	152.8689
	16	12.0773	19.0805	30.3853	45.5197	39.1260	45.5197	55.8872	69.8511	79.4679	--	--	--	--	--	--	--
2.5	CDF	11.2159	15.7278	23.0918	33.0970	38.3209	42.4357	49.1728	58.3660	78.7424	82.3702	88.3311	96.5058	128.2320	131.3893	136.5932	143.7611
	RR	11.2159	15.7277	23.0916	33.0961	38.3193	42.4334	49.1688	58.3585	78.7212	82.3455	88.2998	96.4637	128.1277	131.2768	136.4668	143.6137
	15	11.2248	15.7409	23.1105	33.1244	38.3526	42.4705	49.2139	58.4139	78.8051	82.4351	88.4001	96.5787	128.3143	131.4722	136.6767	143.8448
	16	11.2248	15.7409	23.1105	33.1224	38.3506	42.4675	49.2080	58.4060	--	--	--	--	--	--	--	--
3	CDF	10.7523	13.8983	19.0651	26.1455	37.8986	40.7660	45.4857	51.9739	78.3707	80.8969	85.0656	90.8170	127.9089	130.1062	133.7399	138.7692
	RR	10.7523	13.8983	19.0650	26.1451	37.8971	40.7640	45.4826	51.9690	78.3499	80.8737	85.0380	90.7827	127.8053	129.9969	133.6211	138.6366
	15	10.7606	13.9096	19.0805	26.1666	37.9298	40.8002	45.5237	52.0172	78.4337	80.9613	85.1326	90.8873	127.9913	130.1891	133.8230	138.8527
5	CDF	10.0765	11.2159	13.1047	15.7278	37.2834	38.3209	40.0422	42.4357	77.8293	78.7424	80.2588	82.3702	127.4384	128.2320	129.5507	131.3893
	RR	10.0765	11.2159	13.1046	15.7277	37.2819	38.3193	40.0402	42.4334	77.8089	78.7212	80.2362	82.3455	127.3362	128.1277	129.4430	131.2768
	15	10.0850	11.2248	13.1154	15.7409	37.3144	38.3526	40.0752	42.4705	77.8914	78.8051	80.3223	82.4351	127.5212	128.3143	129.6340	131.4722
10	CDF	9.7909	10.0765	10.5518	11.2159	37.0234	37.2834	37.7161	38.3209	77.6006	77.8293	78.2101	78.7424	127.2398	127.4384	127.7693	128.2320
	RR	9.7909	10.0765	10.5518	11.2159	37.0220	37.2819	37.7146	38.3193	77.5805	77.8089	78.1893	78.7212	127.1381	127.3362	127.6660	128.1277
	15	9.7985	10.0850	10.5601	11.2248	37.0537	37.3144	37.7471	38.3526	77.6632	77.8914	78.2727	78.8051	127.3226	127.5212	127.8521	128.3143

Table 4. Linear frequency parameter ($\sqrt{\lambda}$) for thickness ratio 0.15

b/a	Ref	Mode Number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
1	CDF	18.3295	41.8944	74.6564	112.3571	41.8944	62.3084	91.7247	126.5732	74.6564	91.7247	117.2031	148.3837	112.3571	126.5732	148.3837	175.8471
	RR	18.3285	41.8771	74.5533	112.0589	41.87708	62.2476	91.5442	126.1828	74.55327	91.5442	116.8743	147.8421	112.0589	126.1828	147.8421	175.112
1.5	CDF	13.4941	24.8214	41.8944	63.0180	37.8069	47.4409	62.3084	81.163	71.3127	79.2353	91.7247	107.9521	109.6151	116.1359	126.5732	140.391
	RR	13.49378	24.8186	41.8771	62.95506	37.79471	47.41498	62.24757	81.03277	71.22226	79.11346	91.5442	107.681	109.3339	115.8136	126.1828	139.9057
2	CDF	11.7589	18.3295	28.6205	41.8944	36.3498	41.8944	50.7157	62.3084	70.1274	74.6564	81.9602	91.7247	108.6466	112.3571	118.3972	126.5732
	RR	11.75874	18.3285	28.6158	41.87708	36.3391	41.87708	50.68353	62.24757	70.04113	74.55327	81.82652	91.5442	108.3711	112.0589	118.0606	126.1828
2.5	CDF	10.9479	15.2163	22.0396	31.0606	35.6706	39.2583	45.0563	52.8297	69.576	72.4969	77.261	83.7274	108.1968	110.5845	114.5032	119.8686
	RR	10.94774	15.2158	22.0377	31.05439	35.66048	39.24436	45.03432	52.79309	69.49173	72.402	77.14739	83.5857	107.924	110.2973	114.1915	119.5224
3	CDF	10.5052	13.4941	18.3295	24.8214	35.3003	37.8069	41.8944	47.4409	69.2758	71.3127	74.6564	79.2353	107.9521	109.6151	112.3571	116.1359
	RR	10.50505	13.4938	18.3285	24.81862	35.29051	37.79471	41.87708	47.41498	69.19256	71.22226	74.55327	79.11346	107.681	109.3339	112.0589	115.8136
5	CDF	9.8585	10.9479	12.7433	15.2163	34.76	35.6706	37.1759	39.2583	68.8382	69.576	70.7989	72.4969	107.5955	108.1968	109.1951	110.5845
	RR	9.8584	10.9477	12.7431	15.21581	34.75083	35.66048	37.16423	39.24436	68.75645	69.49173	70.71032	72.402	107.3266	107.924	108.9165	110.2973
10	CDF	9.5847	9.8585	10.3135	10.9479	34.5315	34.7600	35.1401	35.6706	68.6532	68.8382	69.146	69.576	107.4449	107.5955	107.8463	108.1968
	RR	9.5846	9.8584	10.3134	10.94774	34.52246	34.75083	35.13047	35.66048	68.57215	68.75645	69.06323	69.49173	107.1765	107.3266	107.5756	107.924

Table 5. Linear frequency parameter ($\sqrt{\lambda}$) for thickness ratio 0.2

b/a	Ref	Mode Number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
1	CDF	17.4521	38.1963	65.3264	95.1531	38.1963	55.2712	78.9699	106.1494	65.3264	78.9699	98.9140	122.8308	95.1531	106.1494	122.8308	143.5898
	RR	17.4486	38.1522	65.1453	94.7658	38.1522	55.1501	78.6970	105.6863	65.1453	78.6970	98.5004	122.2608	94.7658	105.6863	122.2608	142.9066
	15	17.4665	38.2172	65.3519	95.2719	38.2172	55.2916	79.0194	106.3476	65.3519	79.0194	99.0570	123.2096	95.2719	106.3476	123.2096	144.3031
	16	17.4625	38.1827	65.1978	94.8423	38.1827	55.1948	78.7606	--	65.1978	78.7606	--	--	94.8423	--	--	--
	7	17.4485	38.1521	65.1452	94.7658	38.1521	55.1500	78.6969	--	65.1452	78.6969	--	--	94.7658	--	--	--
	17	17.4523	38.1883	65.3135	--	38.1883	55.2543	78.9865	--	65.3135	78.9865	--	--	--	--	--	--
1.5	CDF	12.9935	23.3132	38.1963	55.8539	34.6923	42.9011	55.2712	70.5591	62.6206	69.0132	78.9699	91.7221	93.0188	98.0870	106.1494	116.7411
	RR	12.9923	23.3038	38.1522	55.7295	34.6591	42.8395	55.1501	70.3437	62.4565	68.8081	78.6970	91.3591	92.6466	97.6791	105.6863	116.2084
	15	13.0038	23.3308	38.2172	55.8743	34.7137	42.9219	55.2916	70.5909	62.6434	69.0421	79.0194	91.8217	93.1256	98.2243	106.3476	117.0450
	16	13.0028	23.3229	38.1827	55.7746	34.6870	42.8735	55.1948	70.4003	62.5071	68.8633	--	--	--	--	--	--
2	CDF	11.3714	17.4521	26.6855	38.1963	33.4350	38.1963	45.6542	55.2712	61.6585	65.3264	71.1974	78.9699	92.2639	95.1531	99.8387	106.1494
	RR	11.3707	17.4486	26.6709	38.1522	33.4054	38.1522	45.5809	55.1501	61.5002	65.1453	70.9778	78.6970	91.8970	94.7658	99.4187	105.6863
	15	11.3809	17.4665	26.7049	38.2172	33.4563	38.2172	45.6748	55.2916	61.6814	65.3519	71.2300	79.0194	92.3660	95.2719	99.9875	106.3476
	16	11.3799	17.4625	26.6921	38.1827	33.4325	38.1827	45.6175	55.1948	61.5500	--	--	--	--	--	--	--
2.5	CDF	10.6089	14.5915	20.8178	28.8317	32.8473	35.9403	40.8851	47.4224	61.2104	63.5802	67.4261	72.6103	91.9131	93.7739	96.8204	100.9770
	RR	10.6083	14.5897	20.8113	28.8129	32.8192	35.9033	40.8312	47.3412	61.0549	63.4101	67.2314	72.3812	91.5487	93.3963	96.4214	100.5490
	15	10.6174	14.6040	20.8337	28.8513	32.8685	35.9612	40.9059	47.4429	61.2329	63.6035	67.4538	72.6454	92.0133	93.8841	96.9492	101.1343
	16	10.6164	14.6010	20.8278	28.8365	32.8458	35.9326	40.8644	47.3796	--	--	--	--	--	--	--	--
3	CDF	10.1915	12.9935	17.4521	23.3132	32.5265	34.6923	38.1963	42.9011	60.9663	62.6206	65.3264	69.0132	91.7221	93.0188	95.1531	98.0870
	RR	10.1910	12.9923	17.4486	23.3038	32.4992	34.6591	38.1522	42.8395	60.8123	62.4565	65.1453	68.8081	91.3591	92.6466	94.7658	97.6791
	15	10.1996	13.0038	17.4665	23.3308	32.5475	34.7137	38.2172	42.9219	60.9880	62.6434	65.3519	69.0421	91.8217	93.1256	95.2719	98.2243
5	CDF	9.5802	10.6089	12.2932	14.5915	32.0580	32.8473	34.1483	35.9403	60.6103	61.2104	62.2038	63.5802	91.4439	91.9131	92.6915	93.7739
	RR	9.5798	10.6083	12.2922	14.5897	32.0319	32.8192	34.1167	35.9033	60.4583	61.0549	62.0422	63.4101	91.0828	91.5487	92.3216	93.3963
	15	9.5881	10.6174	12.3035	14.6040	32.0783	32.8685	34.1694	35.9612	60.6324	61.2329	62.2266	63.6035	91.5422	92.0133	92.7966	93.8841
10	CDF	9.3208	9.5802	10.0105	10.6089	31.8596	32.0580	32.3877	32.8473	60.4597	60.6103	60.8607	61.2104	91.3264	91.4439	91.6396	91.9131
	RR	9.3205	9.5798	10.0101	10.6083	31.8341	32.0319	32.3608	32.8192	60.3087	60.4583	60.7072	61.0549	90.9661	91.0828	91.2771	91.5487
	15	9.3284	9.5881	10.0188	10.6174	31.8798	32.0783	32.4082	32.8685	60.4812	60.6324	60.8823	61.2329	91.4236	91.5422	91.7387	92.0133

Table 6. T_{NL}/T_L values of fundamental mode for aspect ratio 1

h/a	α_m / h									
	0.2		0.4		0.6		0.8		1.0	
	CDF	FEM [19]	CDF	FEM [19]	CDF	FEM [19]	CDF	FEM [19]	CDF	FEM [19]
0.001	0.9801	0.9818 0.9821* 0.9809$ 0.9783#	0.9270	0.9327 0.9338* 0.9297$ 0.9210#	0.8548	0.8652 0.8673* 0.8602$ 0.8451#	0.7773	0.7916 0.7943* 0.7853$ 0.7653#	0.7029	0.7199 0.7233* 0.7131$ 0.6901#
0.05	0.9799	0.9807	0.9260	0.9290	0.8532	0.8586	0.7751	0.7827	0.7005	0.7098
0.1	0.9790	0.9788	0.9233	0.9226	0.8484	0.8475	0.7687	0.7681	0.6931	0.6931
0.15	0.9777	0.9766	0.9188	0.9153	0.8406	0.8350	0.7585	0.7520	0.6814	0.6752
0.2	0.9758	0.9740	0.9126	0.9069	0.8300	0.8212	0.7447	0.7347	0.6660	0.6562

Values are taken from * [27], $[26], #[25]

Table 7. T_{NL}/T_L values of fundamental mode for aspect ratio 2

h/a	α_m / h									
	0.2		0.4		0.6		0.8		1.0	
	CDF	FEM [19]	CDF	FEM [19]	CDF	FEM [19]	CDF	FEM [19]	CDF	FEM [19]
0.001	0.9733	0.9768	0.9043	0.9161	0.8162	0.8367	0.7272	0.7550	0.6465	0.6795
0.05	0.9731	0.9761	0.9037	0.9140	0.8151	0.8332	0.7258	0.7506	0.6450	0.6747
0.1	0.9724	0.9749	0.9017	0.9100	0.8118	0.8266	0.7217	0.7423	0.6406	0.6658
0.15	0.9714	0.9733	0.8984	0.9049	0.8065	0.8186	0.7151	0.7325	0.6333	0.6554
0.2	0.9700	0.9714	0.8939	0.8990	0.7991	0.8092	0.7060	0.7213	0.6235	0.6437

Table 8. T_{NL}/T_L values for amplitude ratio 0.2

h/a	b/a	Mode number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
0.001	1	0.9801	0.9733	0.968	0.9655	0.9733	0.9801	0.9773	0.9733	0.968	0.9773	0.9801	0.9786	0.9655	0.9733	0.9786	0.9801
	1.5	0.9773	0.9786	0.9733	0.9694	0.968	0.9773	0.9801	0.9786	0.9647	0.9716	0.9773	0.9799	0.9633	0.968	0.9733	0.9773
	2	0.9733	0.9801	0.9773	0.9733	0.9655	0.9733	0.9786	0.9801	0.9633	0.968	0.9733	0.9773	0.9625	0.9655	0.9694	0.9733
	2.5	0.9702	0.9792	0.9795	0.9765	0.9641	0.9702	0.9759	0.9792	0.9627	0.9659	0.9702	0.9742	0.9621	0.9641	0.967	0.9702
	3	0.968	0.9773	0.9801	0.9786	0.9633	0.968	0.9733	0.9773	0.9623	0.9647	0.968	0.9716	0.9619	0.9633	0.9655	0.968
	5	0.9641	0.9702	0.9759	0.9792	0.9621	0.9641	0.967	0.9702	0.9617	0.9627	0.9641	0.9659	0.9616	0.9621	0.963	0.9641
	10	0.9621	0.9641	0.967	0.9702	0.9616	0.9621	0.963	0.9641	0.9615	0.9617	0.9621	0.9627	0.9614	0.9616	0.9618	0.9621
0.05	1	0.9799	0.9724	0.9659	0.9616	0.9724	0.979	0.9753	0.9697	0.9659	0.9753	0.9777	0.975	0.9616	0.9697	0.975	0.9758
	1.5	0.9771	0.9786	0.9733	0.9695	0.9671	0.9766	0.9796	0.9783	0.9625	0.9696	0.9756	0.9786	0.9593	0.9643	0.9699	0.9744
	2	0.9731	0.9801	0.9774	0.9736	0.9645	0.9724	0.978	0.9798	0.9611	0.9659	0.9714	0.9758	0.9585	0.9616	0.9657	0.97
	2.5	0.97	0.9791	0.9795	0.9767	0.9631	0.9693	0.9752	0.9787	0.9604	0.9638	0.9682	0.9724	0.9581	0.9602	0.9632	0.9666
	3	0.9678	0.9771	0.9801	0.9788	0.9623	0.9671	0.9725	0.9767	0.96	0.9625	0.9659	0.9697	0.9578	0.9593	0.9616	0.9643
	5	0.9639	0.97	0.9757	0.9791	0.9611	0.9631	0.966	0.9693	0.9594	0.9604	0.9619	0.9638	0.9575	0.9581	0.959	0.9602
	10	0.9619	0.9639	0.9667	0.97	0.9606	0.9611	0.962	0.9631	0.9592	0.9594	0.9598	0.9604	0.9574	0.9575	0.9577	0.9581
0.1	1	0.979	0.9697	0.9595	0.9501	0.9697	0.9758	0.9693	0.9591	0.9595	0.9693	0.9705	0.9644	0.9501	0.9591	0.9644	0.9632
	1.5	0.9766	0.9783	0.9733	0.9696	0.9643	0.9744	0.9781	0.9773	0.9559	0.9637	0.9707	0.9745	0.9476	0.9533	0.9599	0.9655
	2	0.9724	0.9798	0.9777	0.9745	0.9616	0.97	0.9762	0.9788	0.9544	0.9597	0.9659	0.9711	0.9467	0.9502	0.955	0.9602
	2.5	0.9693	0.9787	0.9796	0.9774	0.9602	0.9666	0.973	0.9772	0.9537	0.9573	0.9622	0.9671	0.9462	0.9486	0.952	0.9561
	3	0.9671	0.9767	0.98	0.9791	0.9593	0.9643	0.97	0.9747	0.9533	0.9559	0.9597	0.9639	0.946	0.9477	0.9502	0.9534
	5	0.9631	0.9693	0.9752	0.9788	0.9581	0.9602	0.9632	0.9666	0.9527	0.9537	0.9553	0.9574	0.9456	0.9462	0.9472	0.9486
	10	0.9611	0.9631	0.966	0.9693	0.9575	0.9581	0.959	0.9602	0.9524	0.9527	0.9531	0.9537	0.9454	0.9456	0.9459	0.9462
0.15	1	0.9777	0.9652	0.9493	0.9319	0.9652	0.9705	0.9597	0.9422	0.9493	0.9597	0.9588	0.9474	0.9319	0.9422	0.9474	0.9432
	1.5	0.9756	0.9779	0.9731	0.9692	0.9597	0.9707	0.9753	0.975	0.9453	0.9542	0.9624	0.9675	0.9291	0.9358	0.9439	0.9511
	2	0.9714	0.9794	0.9781	0.9756	0.9568	0.9659	0.9731	0.9768	0.9436	0.9495	0.9567	0.9632	0.928	0.9321	0.9379	0.9443
	2.5	0.9682	0.9781	0.9797	0.9782	0.9553	0.9622	0.9693	0.9744	0.9428	0.9469	0.9525	0.9583	0.9274	0.9302	0.9343	0.9392
	3	0.9659	0.9759	0.9798	0.9796	0.9544	0.9597	0.966	0.9714	0.9423	0.9453	0.9496	0.9545	0.9271	0.9291	0.9321	0.9359
	5	0.9619	0.9682	0.9743	0.9782	0.9531	0.9553	0.9585	0.9622	0.9417	0.9428	0.9446	0.9469	0.9267	0.9274	0.9286	0.9302
	10	0.9598	0.9619	0.9648	0.9682	0.9525	0.9531	0.954	0.9553	0.9414	0.9417	0.9421	0.9428	0.9265	0.9267	0.927	0.9274
0.2	1	0.9758	0.9591	0.9354	0.9082	0.9591	0.9632	0.9466	0.9201	0.9354	0.9466	0.9432	0.925	0.9082	0.9201	0.925	0.9172
	1.5	0.9744	0.9773	0.9724	0.9679	0.9533	0.9655	0.9713	0.9713	0.931	0.9412	0.9511	0.9576	0.9048	0.9127	0.9227	0.9319
	2	0.97	0.9788	0.9783	0.9765	0.9502	0.9602	0.9686	0.9735	0.9291	0.9358	0.9443	0.9523	0.9035	0.9084	0.9154	0.9233
	2.5	0.9666	0.9772	0.9797	0.979	0.9486	0.9561	0.9642	0.9704	0.9282	0.9328	0.9392	0.9462	0.9029	0.9061	0.911	0.917

	3	0.9643	0.9747	0.9793	0.98	0.9477	0.9534	0.9603	0.9666	0.9277	0.931	0.9359	0.9416	0.9026	0.9048	0.9084	0.913
	5	0.9602	0.9666	0.9731	0.9774	0.9462	0.9486	0.9521	0.9562	0.9269	0.9282	0.9302	0.9328	0.9021	0.9029	0.9043	0.9061
	10	0.9581	0.9602	0.9632	0.9666	0.9456	0.9462	0.9472	0.9486	0.9266	0.9269	0.9274	0.9282	0.9018	0.9021	0.9024	0.9029

Table 9. T_{NL}/T_L values for amplitude ratio 0.4

h/a	b/a	Mode number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
0.001	1	0.927	0.9043	0.8878	0.88	0.9043	0.927	0.9175	0.9043	0.8878	0.9175	0.927	0.9219	0.88	0.9043	0.9219	0.927
	1.5	0.9175	0.9219	0.9043	0.892	0.8878	0.9175	0.927	0.9219	0.8777	0.899	0.9175	0.9261	0.8736	0.8878	0.9043	0.9175
	2	0.9043	0.927	0.9175	0.9043	0.88	0.9043	0.9219	0.927	0.8736	0.8878	0.9043	0.9175	0.8712	0.88	0.892	0.9043
	2.5	0.8945	0.9238	0.9248	0.9147	0.8759	0.8945	0.9128	0.9238	0.8716	0.8815	0.8945	0.9074	0.87	0.8759	0.8846	0.8945
	3	0.8878	0.9175	0.927	0.9219	0.8736	0.8878	0.9043	0.9175	0.8705	0.8777	0.8878	0.899	0.8694	0.8736	0.88	0.8878
	5	0.8759	0.8945	0.9128	0.9238	0.87	0.8759	0.8846	0.8945	0.8688	0.8716	0.8759	0.8815	0.8684	0.87	0.8725	0.8759
	10	0.87	0.8759	0.8846	0.8945	0.8684	0.87	0.8725	0.8759	0.8681	0.8688	0.87	0.8716	0.868	0.8684	0.8691	0.87
0.05	1	0.926	0.9014	0.8813	0.8683	0.9014	0.9233	0.9109	0.8929	0.8813	0.9109	0.9188	0.9099	0.8683	0.8929	0.9099	0.9126
	1.5	0.9169	0.9217	0.9044	0.8924	0.8849	0.915	0.9253	0.921	0.871	0.8928	0.912	0.9217	0.8618	0.8764	0.8936	0.9079
	2	0.9037	0.9267	0.9179	0.9055	0.8771	0.9017	0.9199	0.926	0.8669	0.8814	0.8984	0.9124	0.8594	0.8684	0.8808	0.8939
	2.5	0.8938	0.9234	0.925	0.9155	0.873	0.8917	0.9105	0.9222	0.8649	0.8749	0.8883	0.9017	0.8582	0.8642	0.8731	0.8835
	3	0.8871	0.9169	0.9269	0.9223	0.8706	0.885	0.9017	0.9154	0.8638	0.8711	0.8814	0.8929	0.8576	0.8618	0.8684	0.8765
	5	0.8752	0.8938	0.9123	0.9235	0.867	0.873	0.8817	0.8918	0.8621	0.8649	0.8693	0.8749	0.8566	0.8582	0.8608	0.8642
	10	0.8692	0.8752	0.8839	0.8938	0.8654	0.867	0.8696	0.873	0.8614	0.8621	0.8633	0.8649	0.8562	0.8566	0.8573	0.8582
0.1	1	0.9233	0.8929	0.8625	0.836	0.8929	0.9126	0.8919	0.8612	0.8625	0.8919	0.8955	0.8767	0.836	0.8612	0.8767	0.8732
	1.5	0.915	0.921	0.9044	0.8927	0.8764	0.9079	0.9201	0.9173	0.852	0.8748	0.8961	0.9083	0.8293	0.8447	0.8635	0.8803
	2	0.9017	0.926	0.9189	0.9084	0.8684	0.8939	0.9139	0.9226	0.8478	0.8629	0.8812	0.8974	0.8267	0.8362	0.8495	0.8642
	2.5	0.8917	0.9222	0.9253	0.9177	0.8642	0.8835	0.9034	0.9171	0.8457	0.8561	0.8703	0.885	0.8255	0.8318	0.8412	0.8526
	3	0.885	0.9154	0.9265	0.9236	0.8618	0.8765	0.894	0.909	0.8446	0.8521	0.8629	0.8753	0.8248	0.8293	0.8362	0.8449
	5	0.873	0.8918	0.9106	0.9224	0.8582	0.8642	0.8732	0.8836	0.8429	0.8458	0.8503	0.8561	0.8238	0.8255	0.8282	0.8318
	10	0.867	0.873	0.8817	0.8918	0.8566	0.8582	0.8608	0.8642	0.8422	0.8429	0.8441	0.8458	0.8234	0.8238	0.8245	0.8255
0.15	1	0.9188	0.8793	0.8336	0.7892	0.8793	0.8955	0.8629	0.815	0.8336	0.8629	0.8604	0.8285	0.7892	0.815	0.8285	0.8175
	1.5	0.912	0.9196	0.9037	0.8914	0.8628	0.8961	0.9109	0.9098	0.823	0.8471	0.871	0.8861	0.7823	0.7986	0.8193	0.8387
	2	0.8984	0.9246	0.92	0.9119	0.8545	0.8812	0.9038	0.9158	0.8186	0.8344	0.8544	0.8733	0.7797	0.7896	0.8039	0.8204
	2.5	0.8883	0.9201	0.9256	0.9206	0.8503	0.8703	0.8918	0.9081	0.8165	0.8272	0.8424	0.8588	0.7784	0.785	0.795	0.8073
	3	0.8814	0.9127	0.9257	0.9251	0.8478	0.8629	0.8815	0.8983	0.8153	0.823	0.8345	0.848	0.7777	0.7823	0.7896	0.7989
	5	0.8693	0.8883	0.9077	0.9206	0.8441	0.8503	0.8595	0.8704	0.8135	0.8165	0.8211	0.8273	0.7767	0.7784	0.7812	0.785
	10	0.8633	0.8693	0.8781	0.8883	0.8425	0.8441	0.8467	0.8503	0.8128	0.8135	0.8148	0.8165	0.7763	0.7767	0.7774	0.7784
0.2	1	0.9126	0.8612	0.7977	0.7352	0.8612	0.8732	0.8265	0.7613	0.7977	0.8265	0.8175	0.7727	0.7352	0.7613	0.7727	0.7548
	1.5	0.9079	0.9173	0.9016	0.8875	0.8447	0.8803	0.8979	0.898	0.7868	0.8123	0.8387	0.8569	0.7282	0.7451	0.7672	0.789
	2	0.8939	0.9226	0.9208	0.9148	0.8362	0.8642	0.8897	0.9052	0.7823	0.7988	0.8204	0.8419	0.7255	0.7357	0.7508	0.7687
	2.5	0.8835	0.9171	0.9254	0.9233	0.8318	0.8526	0.8761	0.8952	0.7801	0.7913	0.8073	0.8254	0.7242	0.7309	0.7414	0.7545

	3	0.8765	0.909	0.9243	0.9264	0.8293	0.8449	0.8647	0.8836	0.7789	0.7869	0.7989	0.8134	0.7235	0.7283	0.7358	0.7455
	5	0.8642	0.8836	0.9037	0.9178	0.8255	0.8318	0.8413	0.8528	0.7771	0.7801	0.785	0.7914	0.7225	0.7242	0.7271	0.731
	10	0.8582	0.8642	0.8732	0.8836	0.8238	0.8255	0.8282	0.8318	0.7764	0.7771	0.7784	0.7801	0.7221	0.7225	0.7232	0.7242

Table 10. T_{NL}/T_L values for amplitude ratio 0.6

h/a	b/a	Mode number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
0.001	1	0.8548	0.8162	0.7895	0.7772	0.8162	0.8548	0.8383	0.8162	0.7895	0.8383	0.8548	0.8459	0.7772	0.8162	0.8459	0.8548
	1.5	0.8383	0.8459	0.8162	0.7961	0.7895	0.8383	0.8548	0.8459	0.7736	0.8074	0.8383	0.8532	0.7673	0.7895	0.8162	0.8383
	2	0.8162	0.8548	0.8383	0.8162	0.7772	0.8162	0.8459	0.8548	0.7673	0.7895	0.8162	0.8383	0.7636	0.7772	0.7961	0.8162
	2.5	0.8002	0.8493	0.8511	0.8335	0.7709	0.8002	0.8304	0.8493	0.7643	0.7795	0.8002	0.8212	0.7619	0.7709	0.7844	0.8002
	3	0.7895	0.8383	0.8548	0.8459	0.7673	0.7895	0.8162	0.8383	0.7626	0.7736	0.7895	0.8074	0.7609	0.7673	0.7772	0.7895
	5	0.7709	0.8002	0.8304	0.8493	0.7619	0.7709	0.7844	0.8002	0.7601	0.7643	0.7709	0.7795	0.7595	0.7619	0.7657	0.7709
	10	0.7619	0.7709	0.7844	0.8002	0.7595	0.7619	0.7658	0.7709	0.7591	0.7601	0.7619	0.7643	0.7589	0.7595	0.7605	0.7619
0.05	1	0.8532	0.8114	0.7792	0.7594	0.8114	0.8484	0.8271	0.7976	0.7792	0.8271	0.8406	0.8255	0.7594	0.7976	0.8255	0.83
	1.5	0.8372	0.8455	0.8163	0.7967	0.7849	0.8342	0.8519	0.8443	0.7635	0.7973	0.8291	0.8456	0.7497	0.7717	0.7987	0.822
	2	0.8151	0.8544	0.839	0.8181	0.7727	0.8118	0.8425	0.8531	0.7573	0.7794	0.8065	0.8297	0.7461	0.7595	0.7785	0.7991
	2.5	0.7991	0.8486	0.8513	0.8349	0.7664	0.7957	0.8264	0.8465	0.7543	0.7694	0.7902	0.8118	0.7443	0.7533	0.7667	0.7827
	3	0.7883	0.8374	0.8546	0.8467	0.7628	0.785	0.8119	0.8347	0.7526	0.7635	0.7794	0.7976	0.7434	0.7497	0.7595	0.7718
	5	0.7698	0.7991	0.8294	0.8487	0.7574	0.7664	0.7798	0.7957	0.7501	0.7543	0.7608	0.7694	0.742	0.7443	0.7481	0.7533
	10	0.7608	0.7698	0.7832	0.7991	0.755	0.7574	0.7612	0.7664	0.7491	0.7501	0.7519	0.7543	0.7414	0.742	0.743	0.7443
0.1	1	0.8484	0.7976	0.7506	0.7125	0.7976	0.83	0.796	0.7487	0.7506	0.796	0.8018	0.7721	0.7125	0.7487	0.7721	0.7667
	1.5	0.8342	0.8443	0.8164	0.7972	0.7717	0.822	0.8428	0.8381	0.7354	0.7692	0.8027	0.8228	0.7033	0.7248	0.7522	0.7776
	2	0.8118	0.8531	0.8407	0.8229	0.7595	0.7991	0.8323	0.8471	0.7293	0.7512	0.7791	0.8049	0.6998	0.7128	0.7318	0.7533
	2.5	0.7957	0.8465	0.8519	0.8387	0.7533	0.7827	0.8147	0.8377	0.7263	0.7413	0.7623	0.785	0.6981	0.7067	0.7199	0.7362
	3	0.785	0.8347	0.854	0.8488	0.7497	0.7718	0.7994	0.8239	0.7247	0.7354	0.7513	0.77	0.6972	0.7033	0.7129	0.7251
	5	0.7664	0.7957	0.8266	0.8468	0.7443	0.7533	0.7667	0.7828	0.7223	0.7263	0.7328	0.7413	0.6959	0.6981	0.7018	0.7067
	10	0.7574	0.7664	0.7798	0.7957	0.742	0.7443	0.7481	0.7533	0.7213	0.7223	0.724	0.7263	0.6953	0.6959	0.6968	0.6981
0.15	1	0.8406	0.7761	0.7093	0.6506	0.7761	0.8018	0.7512	0.684	0.7093	0.7512	0.7475	0.7022	0.6506	0.684	0.7022	0.6873
	1.5	0.8291	0.8419	0.8151	0.7951	0.7511	0.8027	0.8272	0.8253	0.6947	0.7282	0.7634	0.7868	0.642	0.6625	0.6897	0.7164
	2	0.8065	0.8507	0.8427	0.8287	0.739	0.7791	0.8153	0.8354	0.6888	0.7103	0.7387	0.7669	0.6387	0.6511	0.6695	0.6912
	2.5	0.7902	0.8429	0.8524	0.8436	0.7328	0.7623	0.7959	0.8225	0.6859	0.7005	0.7215	0.7453	0.6371	0.6453	0.6579	0.6739
	3	0.7794	0.8302	0.8525	0.8515	0.7293	0.7513	0.7796	0.8063	0.6844	0.6948	0.7105	0.7295	0.6363	0.642	0.6511	0.663
	5	0.7608	0.7903	0.8218	0.8436	0.724	0.7328	0.7462	0.7625	0.6821	0.6859	0.6922	0.7006	0.635	0.6371	0.6406	0.6453
	10	0.7519	0.7608	0.7743	0.7903	0.7217	0.724	0.7277	0.7328	0.6811	0.6821	0.6837	0.686	0.6345	0.635	0.6359	0.6371
0.2	1	0.83	0.7487	0.6614	0.586	0.7487	0.7667	0.6996	0.6164	0.6614	0.6996	0.6873	0.6302	0.586	0.6164	0.6302	0.6087
	1.5	0.822	0.8381	0.8117	0.7889	0.7248	0.7776	0.8056	0.8058	0.6476	0.6804	0.7164	0.7424	0.5781	0.5973	0.6235	0.6503
	2	0.7991	0.8471	0.8441	0.8337	0.7128	0.7533	0.7925	0.8176	0.642	0.6628	0.6912	0.7209	0.575	0.5865	0.604	0.6252
	2.5	0.7827	0.8377	0.8521	0.8483	0.7067	0.7362	0.7711	0.8013	0.6393	0.6533	0.6739	0.698	0.5736	0.5811	0.593	0.6083

	3	0.7718	0.8239	0.8501	0.8538	0.7033	0.7251	0.754	0.7828	0.6378	0.6477	0.663	0.6819	0.5728	0.5781	0.5866	0.5978
	5	0.7533	0.7828	0.8151	0.8389	0.6981	0.7067	0.72	0.7364	0.6356	0.6393	0.6453	0.6533	0.5716	0.5736	0.5768	0.5811
	10	0.7443	0.7533	0.7667	0.7828	0.6959	0.6981	0.7018	0.7068	0.6346	0.6356	0.6371	0.6393	0.5711	0.5716	0.5724	0.5736

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Table 11. T_{NL}/T_L values for amplitude ratio 0.8

h/a	b/a	Mode number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
0.001	1	0.7773	0.7272	0.6943	0.6796	0.7272	0.7773	0.7555	0.7272	0.6943	0.7555	0.7773	0.7655	0.6796	0.7272	0.7655	0.7773
	1.5	0.7555	0.7655	0.7272	0.7024	0.6943	0.7555	0.7773	0.7655	0.6753	0.7162	0.7555	0.7752	0.6679	0.6943	0.7272	0.7555
	2	0.7272	0.7773	0.7555	0.7272	0.6796	0.7272	0.7655	0.7773	0.6679	0.6943	0.7272	0.7555	0.6636	0.6796	0.7024	0.7272
	2.5	0.7073	0.77	0.7723	0.7493	0.6721	0.7073	0.7453	0.77	0.6644	0.6823	0.7073	0.7335	0.6616	0.6721	0.6881	0.7073
	3	0.6943	0.7555	0.7773	0.7655	0.6679	0.6943	0.7272	0.7555	0.6624	0.6753	0.6943	0.7162	0.6605	0.6679	0.6796	0.6943
	5	0.6721	0.7073	0.7453	0.77	0.6616	0.6721	0.6881	0.7073	0.6595	0.6644	0.6721	0.6823	0.6588	0.6616	0.6661	0.6721
	10	0.6616	0.6721	0.6881	0.7073	0.6588	0.6616	0.6661	0.6721	0.6583	0.6595	0.6616	0.6644	0.6581	0.6588	0.66	0.6616
0.05	1	0.7751	0.7212	0.682	0.6587	0.7212	0.7687	0.741	0.7042	0.682	0.741	0.7585	0.739	0.6587	0.7042	0.739	0.7447
	1.5	0.7541	0.765	0.7274	0.7031	0.6888	0.7501	0.7734	0.7633	0.6634	0.7038	0.7436	0.7651	0.6475	0.6731	0.7056	0.7346
	2	0.7258	0.7767	0.7564	0.7296	0.6742	0.7217	0.761	0.775	0.6562	0.6822	0.7151	0.7444	0.6433	0.6588	0.6811	0.706
	2.5	0.7059	0.769	0.7727	0.7512	0.6668	0.7018	0.7402	0.7662	0.6527	0.6704	0.6952	0.7217	0.6414	0.6516	0.6671	0.6861
	3	0.6929	0.7543	0.777	0.7665	0.6626	0.6888	0.7218	0.7508	0.6508	0.6634	0.6822	0.7041	0.6403	0.6475	0.6588	0.6732
	5	0.6708	0.706	0.7441	0.7692	0.6563	0.6668	0.6827	0.7019	0.648	0.6527	0.6603	0.6704	0.6387	0.6414	0.6457	0.6516
	10	0.6603	0.6708	0.6868	0.706	0.6536	0.6563	0.6608	0.6668	0.6468	0.648	0.65	0.6527	0.638	0.6387	0.6398	0.6414
0.1	1	0.7687	0.7042	0.6485	0.6059	0.7042	0.7447	0.7022	0.6464	0.6485	0.7022	0.7093	0.6735	0.6059	0.6464	0.6735	0.6672
	1.5	0.7501	0.7633	0.7274	0.7037	0.6731	0.7346	0.7614	0.7552	0.6312	0.6701	0.7105	0.7356	0.5958	0.6194	0.6504	0.6801
	2	0.7217	0.775	0.7587	0.7357	0.6588	0.706	0.7477	0.767	0.6244	0.6492	0.6818	0.7131	0.5921	0.6062	0.6272	0.6516
	2.5	0.7018	0.7662	0.7734	0.756	0.6516	0.6861	0.7253	0.7547	0.6211	0.6379	0.662	0.6889	0.5903	0.5996	0.614	0.6321
	3	0.6888	0.7508	0.7761	0.7693	0.6475	0.6732	0.7063	0.737	0.6193	0.6313	0.6493	0.6711	0.5893	0.5959	0.6063	0.6198
	5	0.6668	0.7019	0.7404	0.7667	0.6414	0.6516	0.6672	0.6862	0.6166	0.6211	0.6283	0.6379	0.5879	0.5903	0.5942	0.5996
	10	0.6563	0.6668	0.6827	0.7019	0.6387	0.6414	0.6457	0.6516	0.6155	0.6166	0.6185	0.6211	0.5873	0.5879	0.5889	0.5903
0.15	1	0.7585	0.6783	0.6023	0.5406	0.6783	0.7093	0.6492	0.5753	0.6023	0.6492	0.645	0.5947	0.5406	0.5753	0.5947	0.5788
	1.5	0.7436	0.7602	0.7259	0.7011	0.6491	0.7105	0.7412	0.7387	0.5866	0.6232	0.6633	0.6911	0.5318	0.5528	0.5813	0.6101
	2	0.7151	0.7718	0.7613	0.7431	0.6353	0.6818	0.7261	0.7517	0.5803	0.6035	0.635	0.6674	0.5285	0.541	0.56	0.5829
	2.5	0.6952	0.7614	0.774	0.7625	0.6283	0.662	0.702	0.7352	0.5773	0.5929	0.6158	0.6424	0.5269	0.5351	0.5481	0.5646
	3	0.6822	0.745	0.7743	0.7729	0.6244	0.6493	0.6824	0.7149	0.5756	0.5867	0.6037	0.6246	0.5261	0.5318	0.5411	0.5533
	5	0.6603	0.6952	0.7343	0.7625	0.6185	0.6283	0.6435	0.6622	0.5732	0.5773	0.584	0.5929	0.5248	0.5269	0.5304	0.5352
	10	0.65	0.6603	0.6761	0.6953	0.616	0.6185	0.6227	0.6283	0.5721	0.5732	0.5749	0.5773	0.5243	0.5248	0.5257	0.5269
0.2	1	0.7447	0.6464	0.5516	0.4767	0.6464	0.6672	0.5918	0.5063	0.5516	0.5918	0.5788	0.5199	0.4767	0.5063	0.5199	0.4987
	1.5	0.7346	0.7552	0.7216	0.6936	0.6194	0.6801	0.7141	0.7143	0.5376	0.5714	0.6101	0.6391	0.4692	0.4876	0.5133	0.5403
	2	0.706	0.767	0.7631	0.7495	0.6062	0.6516	0.6979	0.729	0.5318	0.5531	0.5829	0.6151	0.4663	0.4773	0.4941	0.515
	2.5	0.6861	0.7547	0.7736	0.7687	0.5996	0.6321	0.6724	0.7087	0.5291	0.5433	0.5646	0.5902	0.4649	0.4721	0.4835	0.4983

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	3	0.6732	0.737	0.771	0.776	0.5959	0.6198	0.6524	0.6862	0.5276	0.5377	0.5533	0.573	0.4642	0.4692	0.4773	0.4882
	5	0.6516	0.6862	0.7259	0.7563	0.5903	0.5996	0.6141	0.6324	0.5254	0.5291	0.5352	0.5434	0.4631	0.4649	0.468	0.4721
	10	0.6414	0.6516	0.6672	0.6863	0.5879	0.5903	0.5942	0.5996	0.5244	0.5254	0.5269	0.5291	0.4626	0.4631	0.4639	0.4649

Table 12. T_{NL}/T_L values for amplitude ratio 1.0

h/a	b/a	Mode number															
		11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
0.001	1	0.7029	0.6465	0.611	0.5954	0.6465	0.7029	0.678	0.6465	0.611	0.678	0.7029	0.6894	0.5954	0.6465	0.6894	0.7029
	1.5	0.678	0.6894	0.6465	0.6196	0.611	0.678	0.7029	0.6894	0.5909	0.6346	0.678	0.7005	0.5832	0.611	0.6465	0.678
	2	0.6465	0.7029	0.678	0.6465	0.5954	0.6465	0.6894	0.7029	0.5832	0.611	0.6465	0.678	0.5787	0.5954	0.6196	0.6465
	2.5	0.6249	0.6945	0.6972	0.6711	0.5876	0.6249	0.6666	0.6945	0.5795	0.5983	0.6249	0.6535	0.5766	0.5876	0.6044	0.6249
	3	0.611	0.678	0.7029	0.6894	0.5832	0.611	0.6465	0.678	0.5775	0.5909	0.611	0.6346	0.5755	0.5832	0.5954	0.611
	5	0.5876	0.6249	0.6666	0.6945	0.5766	0.5876	0.6044	0.6249	0.5745	0.5795	0.5876	0.5983	0.5738	0.5766	0.5813	0.5876
	10	0.5766	0.5876	0.6044	0.6249	0.5738	0.5766	0.5813	0.5876	0.5733	0.5745	0.5766	0.5795	0.5731	0.5738	0.575	0.5766
0.05	1	0.7005	0.64	0.5979	0.5736	0.64	0.6931	0.6618	0.6215	0.5979	0.6618	0.6814	0.6596	0.5736	0.6215	0.6596	0.666
	1.5	0.6765	0.6888	0.6468	0.6204	0.6051	0.672	0.6984	0.687	0.5785	0.6212	0.6647	0.689	0.5621	0.5886	0.623	0.6547
	2	0.645	0.7023	0.6791	0.6492	0.5898	0.6406	0.6843	0.7003	0.5711	0.5981	0.6333	0.6656	0.5579	0.5737	0.597	0.6235
	2.5	0.6234	0.6935	0.6976	0.6732	0.5821	0.619	0.6609	0.6902	0.5675	0.5857	0.6119	0.6405	0.5559	0.5663	0.5824	0.6023
	3	0.6095	0.6767	0.7027	0.6906	0.5777	0.6052	0.6407	0.6728	0.5656	0.5786	0.5982	0.6215	0.5548	0.5622	0.5738	0.5887
	5	0.5862	0.6235	0.6652	0.6936	0.5712	0.5821	0.5987	0.6191	0.5627	0.5675	0.5754	0.5858	0.5532	0.5559	0.5603	0.5663
	10	0.5753	0.5862	0.603	0.6235	0.5684	0.5712	0.5758	0.5821	0.5614	0.5627	0.5647	0.5675	0.5525	0.5532	0.5543	0.5559
0.1	1	0.6931	0.6215	0.5632	0.5203	0.6215	0.666	0.6195	0.561	0.5632	0.6195	0.627	0.589	0.5203	0.561	0.589	0.5825
	1.5	0.672	0.687	0.6468	0.621	0.5886	0.6547	0.6847	0.6777	0.5456	0.5855	0.6283	0.6558	0.5104	0.5337	0.5651	0.5959
	2	0.6406	0.7003	0.6816	0.6559	0.5737	0.6235	0.6693	0.6911	0.5388	0.5639	0.5978	0.6312	0.5067	0.5206	0.5415	0.5664
	2.5	0.619	0.6902	0.6985	0.6787	0.5663	0.6023	0.6445	0.6771	0.5355	0.5524	0.5771	0.6053	0.5049	0.5141	0.5284	0.5465
	3	0.6052	0.6728	0.7016	0.6938	0.5622	0.5887	0.6239	0.6574	0.5337	0.5457	0.5641	0.5865	0.504	0.5104	0.5207	0.5341
	5	0.5821	0.6191	0.6611	0.6908	0.5559	0.5663	0.5824	0.6024	0.531	0.5355	0.5427	0.5524	0.5026	0.505	0.5088	0.5141
	10	0.5712	0.5821	0.5987	0.6191	0.5532	0.5559	0.5603	0.5663	0.5298	0.531	0.5329	0.5355	0.502	0.5026	0.5036	0.505
0.15	1	0.6814	0.594	0.5168	0.4572	0.594	0.627	0.5639	0.4903	0.5168	0.5639	0.5596	0.5093	0.4572	0.4903	0.5093	0.4937
	1.5	0.6647	0.6834	0.6451	0.6183	0.5638	0.6283	0.662	0.6593	0.5014	0.5376	0.5784	0.6075	0.4489	0.4688	0.4962	0.5245
	2	0.6333	0.6967	0.6846	0.6642	0.5497	0.5978	0.6454	0.6738	0.4952	0.518	0.5495	0.5827	0.4458	0.4576	0.4757	0.4978
	2.5	0.6119	0.6848	0.6992	0.6859	0.5427	0.5771	0.6192	0.6553	0.4923	0.5075	0.5301	0.557	0.4443	0.452	0.4643	0.4801
	3	0.5982	0.6663	0.6995	0.6979	0.5388	0.5641	0.5984	0.6331	0.4907	0.5015	0.5181	0.539	0.4435	0.4489	0.4577	0.4693
	5	0.5754	0.612	0.6543	0.686	0.5329	0.5427	0.558	0.5773	0.4883	0.4923	0.4988	0.5075	0.4423	0.4443	0.4476	0.4521
	10	0.5647	0.5754	0.5918	0.612	0.5303	0.5329	0.537	0.5427	0.4873	0.4883	0.49	0.4923	0.4418	0.4423	0.4432	0.4443
0.2	1	0.666	0.561	0.4676	0.398	0.561	0.5825	0.5065	0.4251	0.4676	0.5065	0.4937	0.4378	0.398	0.4251	0.4378	0.4182
	1.5	0.6547	0.6777	0.6404	0.6102	0.5337	0.5959	0.6322	0.6324	0.4543	0.4866	0.5245	0.5536	0.3912	0.408	0.4316	0.4569
	2	0.6235	0.6911	0.6867	0.6713	0.5206	0.5664	0.6149	0.6485	0.4489	0.4691	0.4978	0.5295	0.3886	0.3985	0.4139	0.4332
	2.5	0.6023	0.6771	0.6988	0.693	0.5141	0.5465	0.5879	0.6264	0.4464	0.4597	0.4801	0.5049	0.3873	0.3938	0.4042	0.4178

	3	0.5887	0.6574	0.6957	0.7015	0.5104	0.5341	0.5672	0.6024	0.4449	0.4544	0.4693	0.4882	0.3866	0.3912	0.3986	0.4084
	5	0.5663	0.6024	0.6451	0.679	0.505	0.5141	0.5285	0.5468	0.4429	0.4464	0.4521	0.4598	0.3857	0.3873	0.3901	0.3938
	10	0.5559	0.5663	0.5825	0.6024	0.5026	0.505	0.5088	0.5141	0.442	0.4429	0.4443	0.4464	0.3852	0.3857	0.3864	0.3873

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