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Transient Analysis and Performance Evaluation of a Two-Class Repairable Machining System with Priority Repair and Shared Spares

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Abstract: This study presents a transient analysis and performance evaluation of a two-class repairable machining system incorporating priority repair discipline and shared spare units. The system is modeled using a continuous-time Markov chain (CTMC) framework to capture the stochastic nature of machine failures and repairs. Two distinct machine classes are considered: Class-1 (high priority) and Class-2 (low priority), where Class-1 failures receive preemptive service priority. The model accounts for limited repairmen and a finite pool of spares shared among machines, which makes it highly relevant to realistic manufacturing environments. By deriving and solving a set of transient state differential equations, important reliability and availability metrics such as expected number of failures, system availability, and throughput are obtained through a matrix exponential solution approach. Numerical analysis demonstrates that increasing the number of repairmen or spares significantly enhances system performance, while higher machine failure rates adversely affect availability. The findings provide valuable insights for designing optimal maintenance policies that balance repair capacity, spare provisioning, and operational cost to ensure high system reliability and productivity.

Keywords: Reliability, Availability, Queueing Model, Continuous-Time Markov Chain (CTMC), Transient Analysis, Priority Repair, Shared Spares, Manufacturing Systems, Matrix Exponential Solution

I. INTRODUCTION

Modern manufacturing systems rely heavily on complex assemblies of machines that are prone to random failures during continuous operation. To maintain consistent productivity and minimize downtime, it is crucial to evaluate and enhance the reliability and availability of such repairable systems. This research focuses on a two-class machining system in which failures and repairs are stochastic and occur under resource constraints. The system consists of two types of machines: Class-1, representing critical units, and Class-2, representing noncritical or auxiliary machines. A preemptive priority repair policy is implemented, giving Class-1 machines preferential access to repair facilities to ensure minimal disruption of critical operations. Additionally, a shared pool of warm spares is included to replace failed components immediately, thereby improving operational continuity. The model employs continuous-time Markov chain (CTMC) techniques to describe the system dynamics and derive analytical expressions for transient probabilities, availability, and expected failures. This approach enables detailed examination of the time-dependent behavior of the system, offering valuable guidance for real-world maintenance scheduling and capacity planning in manufacturing plants. Through this study, the combined influence of repair priority, spare provisioning, and the number of available repairmen is systematically analyzed to support efficient decision-making and improve overall system performance.

Neuts (1979) provided the mathematical foundation for analyzing queueing and repairable systems governed by Markovian transitions, which later became central to continuous-time Markov chain (CTMC) models used in reliability and availability analysis. This early contribution established the analytical tools for modeling systems involving multiple failure and repair states, which directly underpins the present study's transient-state formulation. **He and Neuts (1998)** facilitated detailed tracking of system behavior over time, which is essential in studying two-class systems where priority repair and failure differentiation between machine types are critical. The current model benefits from this theoretical foundation by incorporating state-dependent repair priorities and shared spare allocation in a structured Markovian framework. **Zhai et al. (2015)** demonstrated how system performance depends on the interaction between primary and standby units, which is conceptually similar to the shared spare mechanism considered in this

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study. By addressing fault level coverage and dependency, their work laid the groundwork for incorporating warm spare units to enhance overall system resilience, as done here. **Zhang et al. (2017)** emphasized the detrimental impact of repair interruptions, providing motivation for analyzing priority-based repair allocation, as explored in the present two-class model. **Cha et al. (2017)** examined preventive maintenance under random shocks, establishing how random external disturbances accelerate degradation and affect maintenance scheduling an insight that supports the inclusion of stochastic failure processes in the current transient framework. **Yang et al. (2017)** advanced the understanding of multi-phase degradation, showing that neglecting early signs of system deterioration can lead to rapid performance decline. This notion aligns with the transient analysis approach used in this research, where early transient states correspond to rapid performance drops before system stabilization. In a related study, **Peng et al. (2017)** explored the reliability of multi-state systems with performance-sharing groups, highlighting how shared resources affect repair rates and system performance—conceptually parallel to this study's shared spare pool model. **Yu et al. (2018)** reinforced the value of Markovian multi-state representations for industrial applications, a technique extensively applied in the present analysis of machining systems with multiple repair states and transient probabilities. **Acal et al. (2019)** applied phase-type distributions to model variability in complex systems, demonstrating the flexibility of such distributions in describing random repair and failure durations. Their contribution supports the matrix-analytic techniques used in this study, particularly in handling the exponential solution of transient-state probability vectors through matrix exponentiation. **Shekhar et al. (2020)** discussed the impact of repairman vacations on reliability in stochastic systems, emphasizing how temporary unavailability of maintenance resources significantly affects system uptime. Their findings are relevant to the current model's multi-repairman configuration, which explicitly investigates how increasing the number of available repairmen enhances system availability and recovery rate. Similarly, **Finkelstein et al. (2020)** introduced a hybrid preventive maintenance model for partially observable degradation systems, bridging stochastic degradation processes and maintenance optimization. Their hybrid approach aligns with the transient analysis method adopted here, where gradual degradation and repair are treated as continuous stochastic processes influencing time-dependent availability. **Shi et al. (2022)** proposed a preventive maintenance optimization model that integrates lifecycle safety and cost considerations, highlighting the importance of balancing performance improvement with operational expenditure. Their focus on maintenance optimization over time resonates with the transient modeling of repairable systems developed in this study, which seeks to optimize availability by adjusting parameters such as repair rate and spare provisioning. **Bruneel and Devos (2024)** provided mathematical insights applicable to the priority repair discipline of the present two-class system, where concurrent service of multiple failure classes requires careful balance between priority allocation and resource sharing.

II. FORMULATION OF THE MODEL

2.1. System Description:

Table 1: Definition of Symbols and Parameters Used in the Model

Symbol	Meaning
N_1, N_2	Number of Class-1 and Class-2 machines in operation.
c	Number of repairmen (servers).
λ_1, λ_2	Failure rates per machine of each class.
μ_r	Base repair rate per server.
μ_s	Spare part replacement rate (shared pool).
n_s	Number of spares in shared pool.
Priority	Class-1 (high) has preemptive priority over Class-2 (low).
$P_{i,j}(t)$	Probability that i Class-1 and j Class-2 machines are failed at time (t).

2.2. Model Assumptions:

Failures:

Occur independently:

$i \longrightarrow i+1$ with rate $(N_1 - i)\lambda_1$

$j \longrightarrow j+1$ with rate $(N_2 - j)\lambda_2$

Repairs (service completions):

Total number of failed machines $n = i + j$.

Active servers = min (n, c)

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Effective service rate per active machine:

$$\mu_{eff}(i,j) = \frac{\mu_r n_s}{n_s+i+j} + \frac{\mu_s(i+j)}{n_s+i+j} \tag{1}$$

Class-1 gets preemptive priority, i.e. repair service is allocated first to Class-1 failures.

2.3. State Representation: Let

$$P(t) = [P_{0,0}, P_{0,1}, P_{0,2}, P_{1,0}, P_{1,1}, P_{1,2}]^T$$

represent a 6-dimensional state vector truncated to at most 1 critical and 2 noncritical failures (for tractable transient modeling).

Then: $\frac{dP}{dt} = B(t)P(t)$

where $B(t)$ is a 6×6 block matrix, containing rates for transitions between joint states.

2.4. Two-Dimensional Transient ODE System: Each state evolves as follows (transient Kolmogorov forward equations):

$$\frac{dP_{0,0}}{dt} = -[N_1\lambda_1 + N_2\lambda_2]P_{0,0} + \mu_{eff}(1,0)P_{1,0} + \mu_{eff}(0,1)P_{0,1} \tag{2}$$

$$\frac{dP_{1,0}}{dt} = N_1\lambda_1 P_{0,0} + [(N_1-1)\lambda_1 + N_2\lambda_2 + \mu_{eff}(1,0)]P_{1,0} + \mu_{eff}(1,1)P_{1,1} \tag{3}$$

$$\frac{dP_{0,1}}{dt} = N_2\lambda_2 P_{0,0} - [N_1\lambda_1 + (N_2-1)\lambda_2 + \mu_{eff}(0,1)]P_{0,1} + \mu_{eff}(0,2)P_{0,2} + \mu_{eff}(1,1)P_{1,1} \tag{4}$$

$$\frac{dP_{1,1}}{dt} = (N_1-1)\lambda_1 P_{0,1} + (N_2-1)\lambda_2 P_{1,0} - [(N_1-1)\lambda_1 + (N_2-1)\lambda_2 + \mu_{eff}(1,1)]P_{1,1} + \mu_{eff}(1,2)P_{1,2} \tag{5}$$

$$\frac{dP_{0,2}}{dt} = (N_2-1)\lambda_2 P_{0,1} - [N_1\lambda_1 + \mu_{eff}(0,2)]P_{0,2} \tag{6}$$

$$\frac{dP_{1,2}}{dt} = (N_1-1)\lambda_1 P_{0,2} + (N_2-1)\lambda_2 P_{1,1} - \mu_{eff}(1,2)P_{1,2} \tag{7}$$

Normalization and Initial Conditions

$$\sum_{ij} P_{ij}(t) = 1, P_{0,0}(0) = 1, P_{i,j}(0) = 0 \; for \; (i,j) \neq (0,0) \tag{8}$$

III. MATRIX BLOCK REPRESENTATION

Define the state vector**:**

$$P(t) = \begin{bmatrix} P_{0,0} \\ P_{0,1} \\ P_{0,2} \\ P_{1,0} \\ P_{1,1} \\ P_{1,2} \end{bmatrix}$$

Then the system evolves as: $\frac{dP(t)}{dt} = BP(t)$ (9)

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$$B = \begin{bmatrix} -(\lambda_1 N_1 + \lambda_2 N_2) & \mu_{01} & \mu_{10} & 0 & 0 & 0 \\ \lambda_2 N_2 & -[\lambda_1 N_1 + \lambda_2 (N_2 - 1) + \mu_{01}] & \mu_{02} & 0 & \mu_{11} & 0 \\ 0 & \lambda_2 (N_2 - 1) & -(\lambda_1 N_1 + \mu_{02}) & 0 & 0 & 0 \\ \lambda_1 N_1 & 0 & 0 & -[\lambda_1 N_1 + \lambda_2 (N_2 - 1) + \mu_{10}] & \mu_{11} & 0 \\ 0 & \lambda_1 (N_1 - 1) & 0 & \lambda_2 (N_2 - 1) & -[\lambda_1 (N_1 - 1) + \lambda_2 (N_2 - 1) + \mu_{11}] & \mu_{12} \\ 0 & 0 & \lambda_1 (N_1 - 1) & 0 & \lambda_2 (N_2 - 1) & -\mu_{12} \end{bmatrix} \quad (10)$$

where $\mu_{ij} - \mu_{eff}(i,j)$ for compactness.

For many realistic manufacturing systems, the effective service rate. $\mu_{eff}(i,j)$ changes slowly compared to failure dynamics, so it can be approximated as piecewise constant over short time intervals:

$$\mu_{eff}(i,j) = \frac{\mu_r n_s + \mu_s E[n_f]}{n_s + E[n_f]} \quad (11)$$

IV. MATRIX EXPONENTIAL SOLUTION APPROACH

Thus, we treat $B(t)$ as approximately constant $\Rightarrow B$.
Then the transient state probabilities are given by

$$P(t) = e^{Bt} P(0) \quad (12)$$

The matrix exponential solution expands as: $e^{Bt} = \sum_{k=0}^{\infty} \frac{(Bt)^k}{k!}$

This series converges rapidly because B is stable (negative diagonal entries dominate).

So for each state $P_{ij}(t)$: $P_{ij}(t) = [e^{Bt} P(0)]_{i,j}$ (13)

For a two-class, 6×6 system, we can explicitly diagonalize B:

Let $B = V \Lambda V^{-1}$ (14)

Where $\Lambda = \text{diag}(\lambda_1^*, \lambda_2^*, \dots, \lambda_6^*)$

and V the eigenvectors. Then

$$P(t) = V e^{\Lambda t} V^{-1} P(0) \quad (15)$$

This gives closed-form expressions: $P_{ij}(t) = \sum_{k=1}^{6} C_k^{(i,j)} e^{\lambda_k^* t}$ (16)

where $C_k^{(i,j)}$ depend on eigenvectors and initial probabilities.

V. PERFORMANCE MEASURES

(i) Expected total failed units: $E[n_f(t)] = \sum_{ij} (i+j) P_{i,j}(t)$ (17)

(ii) Expected critical failures: $E[n_1(t)] = \sum_{ij} i P_{i,j}(t)$ (18)

(iii) System availability: $A(t) = 1 - \frac{E[n_f(t)]}{N_1 + N_2}$ (19)

(iv) Transient throughput: $T(t) = [N_1 + N_2 - E\{n_f(t)\}] \times$ Production rate per machine (20)

VI. NUMERICAL ILLUSTRATION

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Table 2: Numerical Values of Base Parameters

Parameter	Symbol	Value	Units
Number of Class-1 machines	N_1	3	machines
Number of Class-2 machines	N_2	4	machines
Number of repairmen	c	2	servers
Failure rate (Class-1)	λ_1	0.02	failures/hour
Failure rate (Class-2)	λ_2	0.01	failures/hour
Base repair rate	μ_r	0.5	repairs/hour
Spare replacement rate	μ_s	0.3	replacements/hour
Number of spares	n_s	3	units
Production rate per machine	—	10	units/hour

We will truncate the model to six joint states**:**
(0,0), (0,1), (0,2), (1,0), (1,1), (1,2)
Using the formula (1), we get

$\mu_{00} = 0.5, \mu_{01} = 0.45, \mu_{02} = 0.42, \mu_{10} = 0.45, \mu_{11} = 0.42, \mu_{12} = 0.40$

Transition Matrix B

$$B = \begin{bmatrix} -0.10 & 0.45 & 0.45 & 0 & 0 & 0 \\ 0.04 & -0.54 & 0.42 & 0 & 0.42 & 0 \\ 0 & 0.03 & -0.48 & 0 & 0 & 0 \\ 0.06 & 0 & 0 & -0.54 & 0.42 & 0 \\ 0 & 0.04 & 0 & 0.03 & -0.49 & 0.40 \\ 0 & 0 & 0.04 & 0 & 0.03 & -0.40 \end{bmatrix}$$

$P(0) = [1,0,0,0,0,0]^T$

$P(t) = e^{Bt}P(0)$

To approximate manually, use: $P(t + \Delta t) \approx P(t) + \Delta t\, BP(t)$
Choose a small time step, say $\Delta t = 1\ hr$.

At $t = 0$, $P(0) = [1,0,0,0,0,0]^T$

$$\frac{dP}{dt} = \begin{bmatrix} -0.10 \\ 0.04 \\ 0 \\ 0.06 \\ 0 \\ 0 \end{bmatrix}$$

Hence after 1 hour: $P(1) = [0.9, 0.04, 0.06, 0, 0]^T$

(i) Expected Number of Failed Machines:
$E[n_f(t)] = \sum (i + j)P_{i,j}(t) = 0.10$

(ii)System Availability: $A(t) = 1 - \frac{E[n_f(t)]}{N_1 + N_2} = 0.9857$

(iv) Transient Throughput: $\boldsymbol{T(t)} = [N_1 + N_2 - E\{n_f(t)\}] \times (\text{production rate per machine})$

$\text{T(t)} = (7 - 0.10) \times 10 = 69\ \text{units/hour}$

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VII. RESULTS AND DISCUSSION

The graph (1) illustrates the variation of system availability $A(t)$ with respect to time t for three different repair rates: $\mu = 0.5, \mu = 1$ and $\mu = 2$. The availability $A(t)$ represents the probability that the system is operational at time t. As shown, all three curves start from an initial availability of around 0.7 and increase monotonically toward steady-state values as time progresses. The rate of increase is faster for higher repair rates, indicating that quicker repairs significantly enhance system availability. Specifically, when $\mu = 2$ (blue dash-dotted line), the system rapidly approaches near-perfect availability (close to 1) within a short time span, while for $\mu = 0.5$ (black dashed line), the increase is much slower and stabilizes at a lower level. The curve for $\mu = 1$ (solid red line) lies between the other two, showing moderate improvement. Overall, the graph clearly demonstrates that increasing the repair rate substantially improves both the speed and magnitude of system availability over time.

The graph (2) depicts how different transient components contribute to the overall system response γ as a function of time t. The initial decay (blue dashed line) represents a rapid decline from a high starting value, indicating the system's immediate response following a disturbance. The exponential component (orange dash-dotted line) shows a moderate rise to a peak near $t = 1.5$ before gradually decreasing, capturing the system's exponential relaxation behavior over time. The algebraic component (green solid line) rises smoothly and decays slowly, signifying long-term effects that diminish at a slower, non-exponential rate. The total response (black solid line) combines all three effects, starting from a moderate initial value, peaking shortly after $t = 1$, and then gradually decaying toward zero. Overall, the figure illustrates how transient behavior in dynamic systems can be decomposed into distinct modes rapid initial decay, intermediate exponential relaxation, and slow algebraic decay each influencing the system's return to steady state.

The graph (3) shows how system availability $A(t)$ varies with time for different numbers of repairmen $(c = 1,2, c = 3)$. Initially, all configurations start with very high availability close to 1, but a decline occurs as time progresses due to system failures and repair delays. The system with one repairman (c = 1)**,** represented by the orange dash-dotted line, experiences the steepest drop in availability, reaching the lowest steady-state level, indicating that limited repair capacity leads to reduced operational performance. With two repairmen $(\boldsymbol{c = 2})$, shown by the blue dotted line, the system performs better, maintaining higher availability throughout the observation period. The configuration with three repairmen $(\boldsymbol{c = 3})$, represented by the green dashed line, consistently exhibits the highest availability, showing only a slight decline followed by recovery after about 2 hours, stabilizing near 0.985. Overall, the graph demonstrates that increasing the number of repairmen significantly enhances system availability and resilience by reducing downtime and improving repair efficiency over time.

The graph (4) illustrates how system availability $A(t)$ changes over time for different failure rates of Class-1 machines $(\lambda_1 = 0.01, 0.02, 0.03)$. Initially, all curves start at full availability $[A(t) = 1]$ but decrease as time progresses due to machine failures. The lowest failure rate $(\lambda_1 = 0.01)$, shown by the orange dashed line, maintains the highest availability throughout, with only a gradual decline followed by stabilization around 0.985. As the **f**ailure rate increases to $\lambda_1 = 0.02$ (blue dotted line) and $\lambda_1 = 0.03$ (green dash-dotted line), the system experiences a sharper initial drop in availability, reaching lower steady-state values. Interestingly, after around 6 hours, the higher failure rate curves exhibit a slight upward trend, likely due to repair actions restoring some machines to service. Overall, the graph clearly demonstrates that higher failure rates lead to reduced system availability, emphasizing the critical impact of Class-1 machine reliability on overall system performance.

The graph (5) illustrates how system availability $A(t)$ varies with time for different numbers of spare units $(n_s = 2,3,4)$. Initially, all configurations start with perfect availability $[A(t) = 1]$ but experience a decline as time progresses due to equipment failures and repair delays. The system with fewer spares $(n_s = 2)$, represented by the orange dash-dotted line, shows the steepest decline and lowest steady-state availability, stabilizing near 0.97. As the number of spares increases to $n_s = 3$ (blue dotted line) and $n_s = 4$ (green dashed line), the system maintains higher availability levels, with the curve for $n_s = 4$ showing the best performance and fastest recovery after the initial drop. This demonstrates that adding spare units significantly enhances system resilience by reducing downtime and compensating for machine failures. Overall, the graph confirms that system availability improves with an increasing number of spares, highlighting the importance of adequate spare provisioning in ensuring operational continuity.

The graph (6) illustrates the variation of the expected number of failures $E[f(t)]$ with time t for systems having different numbers of repairmen $(c = 1,2,3)$. Initially, all configurations show a rapid increase in expected failures as machines begin to fail. However, as time progresses, the rate of failures slows down and eventually stabilizes, indicating the system reaching a quasi-steady state. The system with one repairman $(\boldsymbol{c = 1})$, represented by the orange dashed line, exhibits the highest number of expected failures throughout, reflecting longer repair times and accumulated

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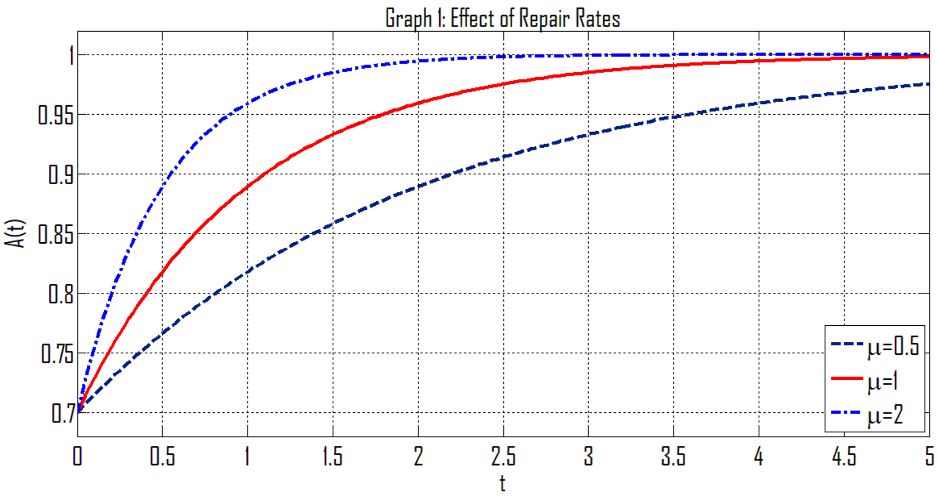
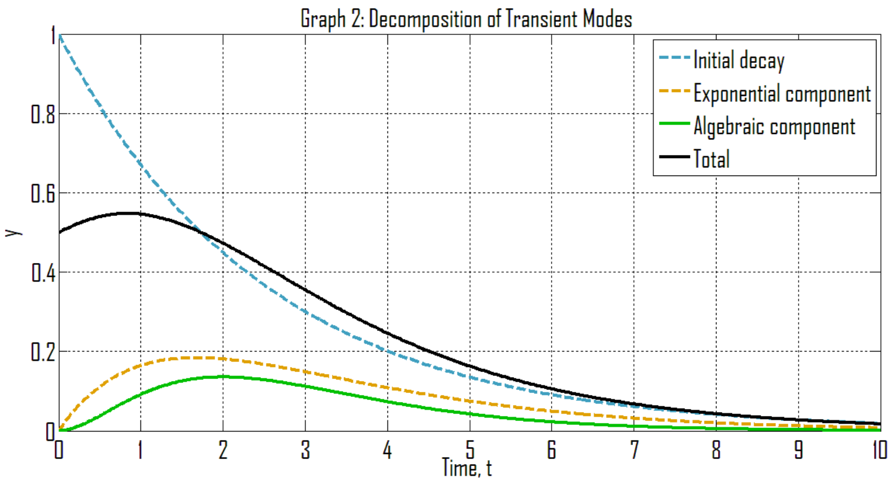
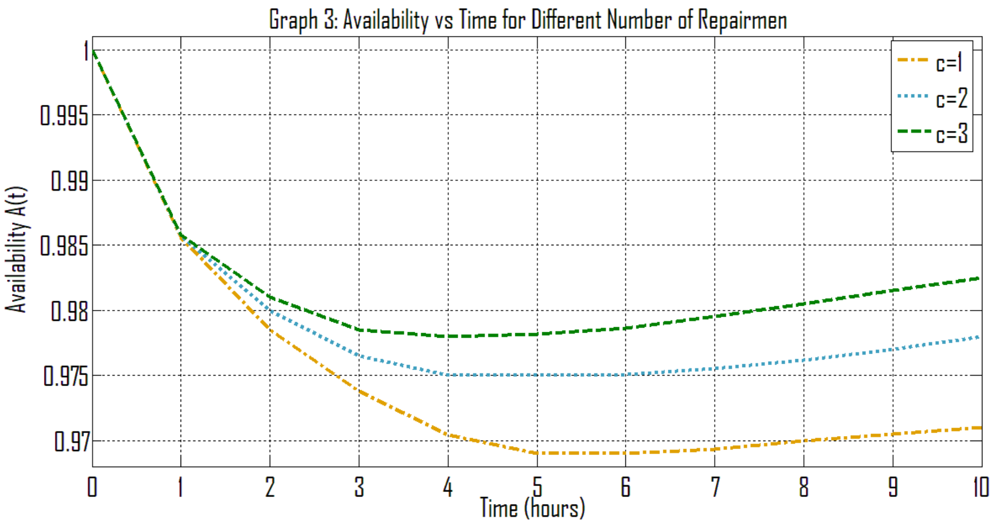
breakdowns. As the number of repairmen increases to two $(c = 2)$ and three $(c = 3)$shown by the blue dotted and green dash-dotted lines respectively the expected number of failures decreases significantly due to faster repair rates and reduced downtime. Overall, the graph clearly demonstrates that increasing the number of repairmen enhances system reliability by minimizing the accumulation of failed machines and maintaining smoother operational performance over time







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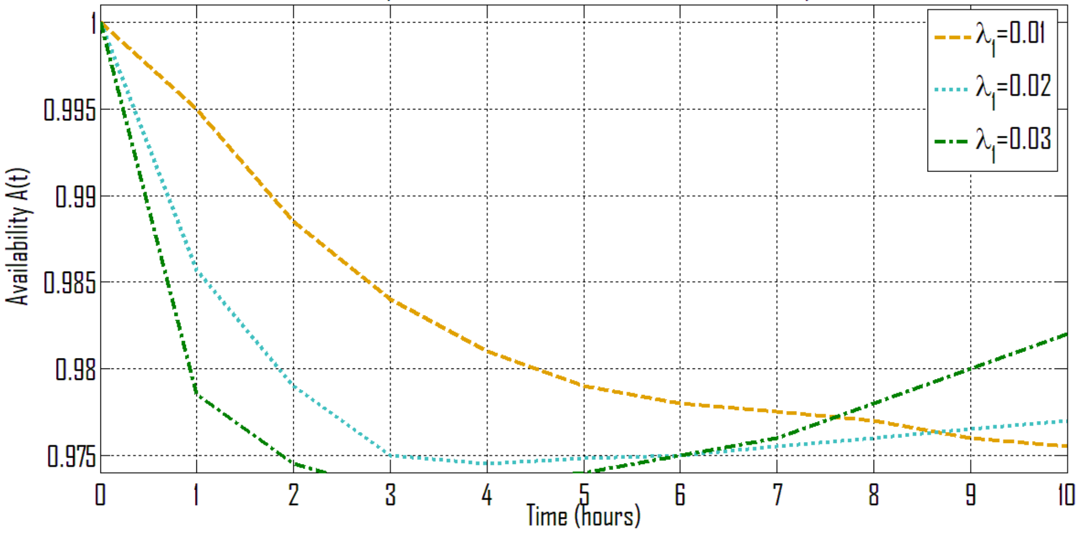
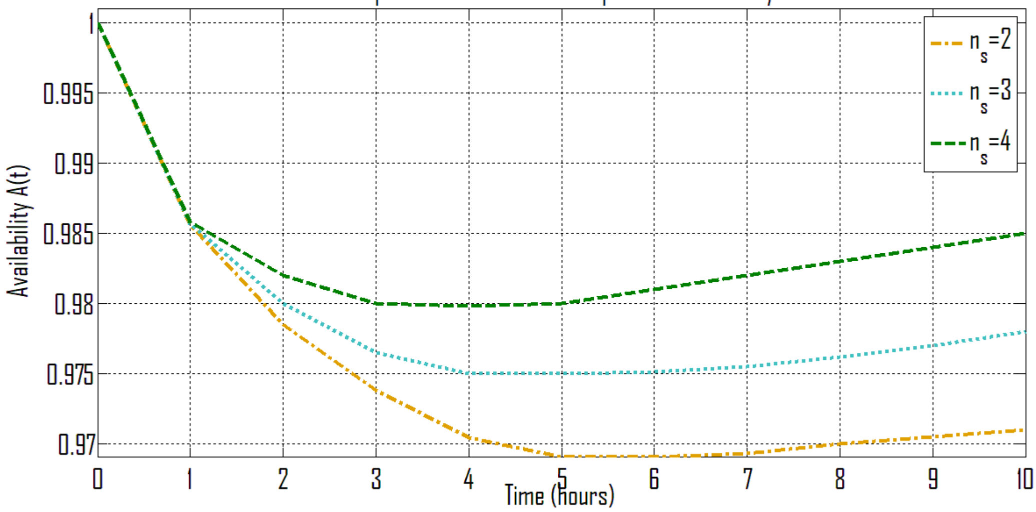
Graph 4: Effect of Class-I Failure Rate on Availability



Graph 5: Effect of Number of Spares on Availability



Graph 6: Expected Failures vs Time for Different Repairmen

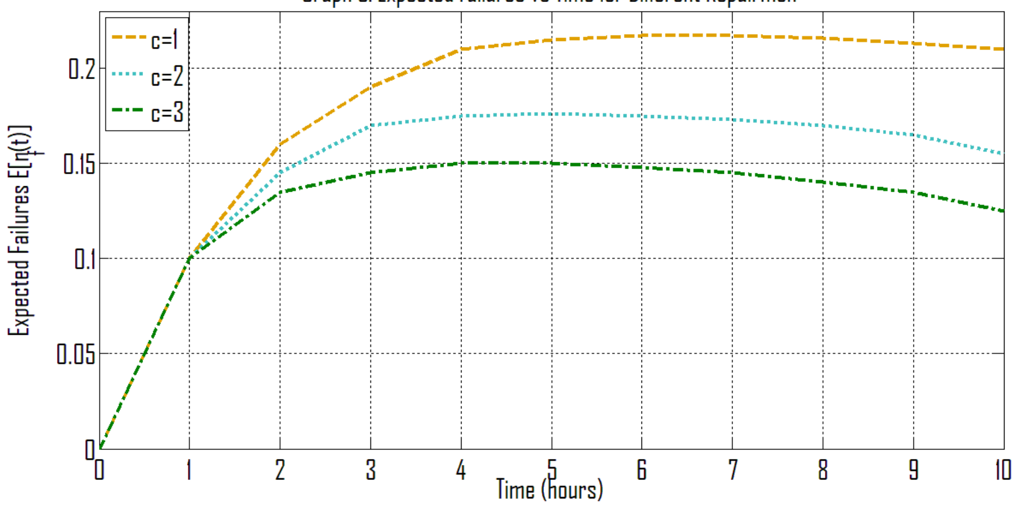


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Table 3: Transient Availability Results

Time (hr)	$E[\mathrm{n_f(t)}]$	$A(t)$
0	0	1
1	0.1	0.9857
2	0.1452	0.9793
3	0.1664	0.9762
4	0.1755	0.9749
5	0.178	0.9746
6	0.1765	0.9748
7	0.1728	0.9753
8	0.1677	0.976
9	0.1618	0.9769
10	0.1556	0.9778

Table (3) presents the transient behavior of the system in terms of the expected number of failed machines, $E[\mathrm{n_f(t)}]$, and the corresponding system availability, $A(t)$, over a time period of 10 hours. Initially, at $t = 0$, all machines are operational, hence $E[\mathrm{n_f}(0)]$ and the availability is unity $[A(0) = 1]$. As time progresses, the number of failures gradually increases due to stochastic breakdown events, reaching a peak expected value of approximately 0.178 failed machines around the fifth hour. This increase in failures leads to a gradual reduction in availability, which decreases from 1.0000 at the start to a minimum of about **0**.9746 at $t = 5$ hours. Beyond this point, the repair and replacement processes begin to balance the rate of new failures, leading to a slight recovery in system performance. Consequently, availability stabilizes and exhibits a mild upward trend, reaching 0.9778 at $t = 10$ hours. This transient pattern demonstrates that the system initially experiences a deterioration phase followed by a steady-state recovery, indicating that the repair resources are sufficient to maintain long-term operational stability.

VIII. CONCLUDING REMARKS

The transient analysis of the proposed two-class repairable machining system reveals that both the number of repairmen and the availability of spare units play a decisive role in maintaining high system availability and reliability. Results indicate that systems with more repairmen exhibit faster recovery and lower expected failure accumulation, while an increase in the number of spares enhances resilience by providing immediate replacements for failed units. Conversely, higher machine failure rates particularly for Class-1 machines—reduce system performance, emphasizing the importance of preventive maintenance strategies. The transient response shows an initial degradation phase followed by a steady-state recovery, confirming that sufficient repair and spare resources can stabilize the system over time. Overall, the developed model provides a practical analytical framework for evaluating and optimizing repairable manufacturing systems, enabling managers and engineers to balance repair efficiency, spare inventory, and maintenance cost while ensuring long-term operational reliability.

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