IARJSET

ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Impact Factor 8.066 ⪿ Peer-reviewed & Refereed journal ⪿ Vol. 11, Issue 5, May 2024

DOI: 10.17148/IARJSET.2024.11544

The Recurrence Property for the Projective Curvature Tensor in Finsler Space

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Abstract: In this paper, we obtain the necessary and sufficient condition for W^i_{jkh}, N^i_{jkh} and H^i_{jkh} to be recurrent and we get a relationship between them. The projection on indicatrix with respect to Cartan connection has been studied.

Keywords: Recurrence property, Projective curvature tensor, Projection on Indicatrix.

I. INTRODUCTION AND PRELIMINARIES

The recurrent Finsler spaces have been studied by Pande and Tiwari [8], Dikihi [4], Qaseem [10], Saleem and Abdallah [12] and Kim and Parw [6]. Also, P. N. Pandey [9] obtained the relation between the normal projective curvature tensor N^i_{jkh} and Berwald curvature tenser H^i_{jkh}. Let F_n be an $n-$dimensional space equipped with the metric function $F(x,y)$ satisfying the request conditions [1, 11]. The vectors y_i and y^i satisfy

(1.1) a) $y_i y^i = F^2$, b) $\dot{\partial}_i y_j = \dot{\partial}_j y_i = g_{ij}$ and c) $g_{it} y^i = y_t$,
(1.2)
where g_{it} is the metric tensor which homogeneous of degree zero in y^i and symmetric in its lower indices.
Cartan's covariant derivative of the metric function F, vector y^i, unit vector l^i and metric tensor g_{it} vanish identically, i.e.

(1.3) a) $F_{|l} = 0$, b) $y^i_{|l} = 0$, c) $l^i_{|l} = 0$ and d) $g_{jk|l} = 0$,

where e) $l^i = \frac{y^i}{F}$.

Cartan's covariant derivative of an arbitrary tensor T^i_h with respect to x^l is given by [3]

(1.4) $\dot{\partial}_j\left(T^i_{h|l}\right) - \left(\dot{\partial}_j T^i_h\right)_{|l} = T^r_h\left(\dot{\partial}_j\, \Gamma^{*i}_{lr}\right) - T^i_r\left(\dot{\partial}_j\, \Gamma^{*r}_{lj}\right) - (\dot{\partial}_r T^i_h) P^r_{jl}$,

where

(1.4) a) $P^r_{jl} = \left(\dot{\partial}_j\, \Gamma^{*r}_{hl}\right) y^h$ and b) $P^i_{jl} = g^{ih} P_{hjl}$.

The Berwald curvature tensor H^i_{jkh} is positively homogeneous of degree zero in y^i and skew-symmetric in its last two lower indices which defined by [11]

$$H^i_{jkh} = \partial_h G^i_{jk} + G^r_{jk} G^i_{rh} + G^i_{rk} G^r_j - h/k.$$

And satisfy the following relations

(1.5) a) $\dot{\partial}_j H^i_{kh} = H^i_{jkh}$, b) $H^i_{jkh} y^j = H^i_{kh}$, c) $H_{ijkh} = g_{jr} H^r_{ikh}$, d) $H^i_{kh} y^k = H^i_h$,

e) $H^i_{kh} = \dot{\partial}_k H^i_h$, f) $H_{jk} = H^r_{jkr}$, g) $H_k = H^r_{kr}$ and h) $H = \frac{1}{n-1} H^r_r$.

The tensor $H_{jk.h}$ defined by

(1.6) $H_{jk.h} = g_{ik} H^i_{jh}$.

The normal projective curvature tensor N^i_{jkh} and Berwald curvature tenser H^i_{jkh} are connected by [11]

DOI: 10.17148/IARJSET.2024.11544

(1.7) $N^i_{jkh} = H^i_{jkh} - \frac{1}{n+1} y^i \dot{\partial}_j H^r_{rkh}$

where N^i_{jkh} is homogeneous of degree zero in y^i.

Contracting the indices i and j in (1.7) and using the fact that the tensor H^r_{rkh} is positively homogeneous of degree zero in y^i, we get

(1.8) $N^r_{rkh} = H^r_{rkh}$.

Transvecting (1.7) by y^j and using (1.5b), we get

(1.9) $N^i_{jkh} y^j = H^i_{kh}$

The projective curvature tensor W^i_{jkh} and normal projective curvature tensor N^i_{jkh} are connected by [11]

(1.10) a) $W^i_{jkh} = N^i_{jkh} + 2(\delta^i_k M_{hj} - M_{kh}\delta^i_j - k|h)$,

where b) $M_{kh} = -\frac{1}{n^2-1}(nN_{kh} + N_{hk})$ and c) $N_{jk} = N^r_{jkr}$.

The projective curvature tensor W^i_{jkh}satisfies the following [11]

(1.11) a) $W^i_{jkh} y^j = W^i_{kh}$, b) $W^i_{kh} y^k = W^i_h$ and c) $W^i_h y^h = 0$.

Definition 1.1. The projection of any tensor T^i_j on indicatrix is given by [2, 5]

(1.12) $p.T^i_j = T^\alpha_\beta h^i_\alpha h^\beta_j$,

where the angular metric tensor is defined by

(1.13) $h^i_j = \delta^i_j - l^i l_j$.

The projection of the vector y^i and unit vector l^i on indicatrix are given by [5, 7]

(1.14) a) $p.y^i = 0$ and b) $p.l^i = 0$.

II. W –RECURRENT FINSLER SPACE

Definition 2.1. A Finsler space F_n which the projective curvature satisfies the recurrence property i.e. characterized by

(2.1) $W^i_{jkh|l} = \lambda_l W^i_{jkh}$, $W^i_{jkh} \neq 0$,

where λ_l is non-zero covariant vector field. This space will be called a W – *Recurrent Finsler space*. And denote it briefly by $WR - F_n$.

Let us consider $WR - F_n$ characterized by (2.1). Transvecting (2.1) by y^j, using (1.11a) and (1.2b), we get

(2.2) $W^i_{kh|l} = \lambda_l W^i_{kh}$.

Transvecting (2.2) by y^k, using (1.11b) and (1.2b), we get

(2.3) $W^i_{h|l} = \lambda_l W^i_h$.

Thus, we conclude

Theorem 2.1. *In $WR - F_n$, the projective torsion tensor W^i_{jk} and projective deviation tensor W^i_h are recurrent.*

Differentiating (1.10a) covariantly with respect to x^l in sense of Cartan, we get

(2.4) $N^i_{jkh|l} = W^i_{jkh|l} + 2(\delta^i_j M_{kh|l} + \delta^i_h M_{jk|l})$.

IARJSET
ISSN (O) 2393-8021, ISSN (P) 2394-1588
International Advanced Research Journal in Science, Engineering and Technology
Impact Factor 8.066 ⁝⁝ Peer-reviewed & Refereed journal ⁝⁝ Vol. 11, Issue 5, May 2024
DOI: 10.17148/IARJSET.2024.11544

Using (2.1) and (1.10a) in (2.4), we get

$$N^i_{jkh|l} = \lambda_l[N^i_{jkh} - 2(\delta^i_j M_{kh} + \delta^i_h M_{jk})] + 2(\delta^i_j M_{kh|l} + \delta^i_h M_{jk|l}).$$

Contracting i and h in above equation and using (1.10c) and the skew –symmetric property for M_{jk}, we get

$$N_{jk|l} = \lambda_l[N_{jk} - 2(1-n)M_{jk}] + 2(1-n)M_{jk|l}.$$

Using (1.10b) in above equation, we get

$$N_{jk|l} = \lambda_l N_{jk} - \frac{2}{n+1}\lambda_l(nN_{jk} + N_{kj}) + \frac{2}{n+1}(nN_{jk|l} + N_{kj|l}).$$

Using the skew –symmetric property for N_{jk} in above equation, we get

$$N_{jk|l} = \lambda_l N_{jk} - 2\lambda_l N_{jk} + 2N_{jk|l}.$$

which can be written by

(2.5) $$N_{jk|l} = \lambda_l N_{jk}.$$

Thus, we conclude

Theorem 2.2. *In $WR - F_n$, if M_{jk} and N_{jk} satisfy the skew –symmetric property then N_{jk} is recurrent.*

Differentiating (1.10b) covariantly with respect to x^l in the sense of Cartan, using and (2.5), we get

(2.6) $$M_{jk|l} = -\frac{2}{n^2-1}\lambda_l(nN_{jk} + N_{kj}).$$

Using (1.10b) in (2.6), we get

(2.7) $$M_{jk|l} = \lambda_l M_{jk}.$$

Using (2.1) and (2.6) in (2.4), we get

(2.8) $$N^i_{jkh|l} = \lambda_l N^i_{jkh}.$$

From (2.7) and (2.8), we conclude

Theorem 2.3. *In $WR - F_n$, the tensor M_{jk} and normal projective curvature tensor N^i_{jkh} ars recurrent.*

Differentiating (1.5f) partially with respect toy^j, we get

(2.9) $$\dot\partial_j(H^r_{rkh|l}) = (\dot\partial_j\lambda_l)H^r_{rkh} + \lambda_l\dot\partial_j H^r_{rkh}.$$

Differentiating (1.7) covariantly with respect to x^m in the sense of Cartan and using (1.2b), we get

$$N^i_{jkh|l} = H^i_{jkh|l} - \frac{1}{n+1}y^i(\dot\partial_j H^r_{rkh})_{|l}.$$

Using commutation formula exhibited by (1.3) for H^r_{rkh} in above equation, using (2.8) and (1.4a), we get

$$\lambda_l N^i_{jkh} = H^i_{jkh|l} - \frac{1}{n+1}y^i\{(\dot\partial_j\lambda_l)H^r_{rkh} + \lambda_l\dot\partial_j H^r_{rkh} + H^r_{rsh}(\dot\partial_j \Gamma^{*s}_{kl}) + H^r_{rks}(\dot\partial_j \Gamma^{*s}_{hl}) + (\dot\partial_s H^r_{rkh})P^s_{jl}\}.$$

Using (1.7) in above equation, we get

(2.10) $$\lambda_l H^i_{jkh} = H^i_{jkh|l} - \frac{1}{n+1}y^i\{(\dot\partial_j\lambda_l)H^r_{rkh} + H^r_{rsh}(\dot\partial_j \Gamma^{*s}_{kl}) + H^r_{rks}(\dot\partial_j \Gamma^{*s}_{hl}) + (\dot\partial_s H^r_{rkh})P^s_{jl}\}.$$

This shows that

$$H^i_{jkh|l} = \lambda_l H^i_{jkh}$$

if and only if

IARJSET

ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Impact Factor 8.066 ⋇ Peer-reviewed & Refereed journal ⋇ Vol. 11, Issue 5, May 2024

DOI: 10.17148/IARJSET.2024.11544

(2.11) $(\dot{\partial}_j \lambda_l) H^r_{rkh} + H^r_{rsh}(\dot{\partial}_j \Gamma^{*s}_{kl}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{hl}) + (\dot{\partial}_s H^r_{rkh}) P^s_{jl} = 0.$

Contracting the indices i and h in (2.10) and using (1.5f), we get

(2.12) $\lambda_l H_{jk} = H_{jk|l} - \frac{1}{n+1} y^t \{(\dot{\partial}_j \lambda_l) H^r_{rkt} + H^r_{rst}(\dot{\partial}_j \Gamma^{*s}_{kl}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{tl}) + (\dot{\partial}_s H^r_{rkt}) P^s_{jl}\}.$

This shows that

$$H_{jk|l} = \lambda_l H_{jk}.$$

if and only if

(2.13) $y^t \{(\dot{\partial}_j \lambda_l) H^r_{rkt} + H^r_{rst}(\dot{\partial}_j \Gamma^{*s}_{kl}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{tl}) + (\dot{\partial}_s H^r_{rkt}) P^s_{jl}\} = 0.$

Thus, we conclude

Theorem 2.4. *In $WR - F_n$, Berwald curvature tensor H^i_{jkh} and Ricci tensor H_{jk} are recurrent if and only if (2.11) and (2.13) hold.*

Transvecting (2.10) by g_{ti}, using (1.5c), (1.1c) and (1.2d), we get

$$\lambda_l H_{jtkh} = H_{jtkh|m} - \frac{1}{n+1} y_t \{(\dot{\partial}_j \lambda_l) H^r_{rkh} + H^r_{rsh}(\dot{\partial}_j \Gamma^{*s}_{kl}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{hl}) + (\dot{\partial}_s H^r_{rkh}) P^s_{jl}\}.$$

This shows that

$$H_{jtkh|m} = \lambda_m H_{jtkh}$$

if and only if

(2.14) $y_t \{(\dot{\partial}_j \lambda_l) H^r_{rkh} + H^r_{rsh}(\dot{\partial}_j \Gamma^{*s}_{kl}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{hl}) + (\dot{\partial}_s H^r_{rkh}) P^s_{jl}\} = 0.$

Thus, we conclude

Theorem 2.5. *In $WR - F_n$, the associate tensor H_{jtkh} of the curvature tensor H^i_{jkh} behaves as recurrent if and only if (2.14) holds.*

III. PROJECTION ON INDICATRIX WITH RESPECT TO CARTAN'S CONNECTION

Since W^i_{jkh} is recurrent in sense of Cartan, i.e. characterized by (2.1). Now, in view of (1.12), the projection of W^i_{jkh} on indicatrix is given by

(3.1) $p.W^i_{jkh} = W^a_{bcd} h^i_a h^b_j h^c_k h^d_h$.

Taking covariant derivative of (3.1) with respect to x^l in sense of Cartan and using the fact that $h^i_{j|l} = 0$, then using (2.1) in the resulting equaion, we get

$$(p\,.W^i_{jkh})_{|l} = \lambda_l W^a_{bcd} h^i_a h^b_j h^c_k h^d_h.$$

In view of (1.12), above equation can be written as

$$(p\,.W^i_{jkh})_{|l} = \lambda_l (p\,.W^i_{jkh}).$$

This shows that $p\,.W^i_{jkh}$ is recurrent. Thus, we conclude

Theorem 3.1. *$WR - F_n$, the projection of the projective curvature tensor W^i_{jkh} on indicatrix is recurrents in sense of Cartan.*

Since W^i_{jk} is recurrent in sense of Cartan, i.e. characterized by (2.2). In view of (1.12), the projection of W^i_{jk} on indicatrix is given by

(3.2) $p\,.W^i_{jk} = W^a_{bc} h^i_a h^b_j h^c_k.$

Taking covariant derivative of (3.2) with respect to x^l in sense of Cartan and using the fact that $h^i_{j|l} = 0$, then using (2.2) in the resulting equaion, we get

DOI: 10.17148/IARJSET.2024.11544

$$\left(p.W^i_{jk}\right)_{|l} = \lambda_l W^a_{bc} h^i_a h^b_j h^c_k.$$

In view of (1.12), above equation can be written as

$$\left(p.W^i_{jk}\right)_{|l} = \lambda_l \left(p.W^i_{jk}\right).$$

This shows that $p.W^i_{jk}$ is recurrent.. Thus, we conclude

Theorem 3.2. *$WR - F_n$, the projection of the torsion tensor W^i_{jk} on indicatrix is recurrent in sense of Cartan.*

Since W^i_j is recurrent in sense of Cartan, i.e. characterized by (2.3). In view of (1.12), the projection of W^i_j on indicatrix is given by

(3.3) $$p\,.W^i_j = W^a_b h^i_a h^b_j.$$

Taking covariant derivative of (3.3) with respect to x^l in sense of Cartan and using the fact that $h^i_{j|l} = 0$, then using (2.3) in the resulting equaion, we get

$$\left(p.W^i_j\right)_{|l} = \lambda_l W^a_b h^i_a h^b_j.$$

In view of (1.12), above equation can be written as

$$\left(p.W^i_j\right)_{|l} = \lambda_l \left(p.W^i_j\right).$$

This shows that $p.W^i_j$ is recurrent. Thus, we conclude

Theorem 3.3. *$WR - F_n$, the projection of the deviation tensor W^i_j on indicatrix is recurrent in the sense of Cartan.*

Let us consider a Finsler space F_n which the projection of W^i_{jkh} on indicatrix is recurrent with respect to Cartan's connection. i.e characterized by (2.1). Using (1.12) in (2.1), we get

$$\left(W^a_{bcd} h^i_a h^b_j h^c_k h^d_h\right)_{|l} = \lambda_l W^a_{bcd} h^i_a h^b_j h^c_k h^d_h.$$

Using (1.13) in above equation, we get

$$\{W^a_{bcd}(\delta^i_a - \ell^i\ell_a)(\delta^b_j - \ell^b\ell_j)(\delta^c_k - \ell^c\ell_k)(\delta^d_h - \ell^d\ell_h)\}_{|l}$$
$$= \lambda_l\{W^a_{bcd}(\delta^i_a - \ell^i\ell_a)(\delta^b_j - \ell^b\ell_j)(\delta^c_k - \ell^c\ell_k)(\delta^d_h - \ell^d\ell_h)\}$$

which can be written as

$$(W^i_{jkh} - W^i_{jkd}\ell^d\ell_h - W^i_{jch}\ell^c\ell_k + W^i_{jcd}\ell^c\ell_k\ell^d\ell_h - W^a_{jkh}\ell^i\ell_a$$
$$+W^a_{jkd}\ell^i\ell_a\ell^d\ell_h + W^a_{jch}\ell^i\ell_a\ell^c\ell_k - W^a_{jcd}\ell^i\ell_a\ell^c\ell_k\ell^d\ell_h)_{|l}$$
$$= \lambda_l(W^i_{jkh} - W^i_{jkd}\ell^d\ell_h - W^i_{jch}\ell^c\ell_k + W^i_{jcd}\ell^c\ell_k\ell^d\ell_h - W^a_{jkh}\ell^i\ell_a$$
$$+W^a_{jkd}\ell^i\ell_a\ell^d\ell_h + W^a_{jch}\ell^i\ell_a\ell^c\ell_k - W^a_{jcd}\ell^i\ell_a\ell^c\ell_k\ell^d\ell_h).$$

Using (1.11a), (1.11b), (1.2a) and (1.2c) in above equation, we get

$$(W^i_{jkh} - \tfrac{1}{F}W^i_{jk}\ell_h - \tfrac{1}{F}W^i_{jh}\ell_k - W^a_{jkh}\ell^i\ell_a + W^a_{jk}\ell^i\ell_a\ell_h + \tfrac{1}{F}W^a_{jh}\ell^i\ell_a\ell_k)_{|l}$$
$$= \lambda_l(W^i_{jkh} - \tfrac{1}{F}W^i_{jk}\ell_h - \tfrac{1}{F}W^i_{jh}\ell_k - W^a_{jkh}\ell^i\ell_a + W^a_{jk}\ell^i\ell_a\ell_h + \tfrac{1}{F}W^a_{jh}\ell^i\ell_a\ell_k).$$

Now, since the torsion tensor W^i_{jk} is recurrent, i.e characterized by (2.2), then in view of (2.2), (1.2a) and (1.2c), above equation can be written as

(3.4) $$(W^i_{jkh} - W^a_{jkh}\ell^i\ell_a)_{|l} = \lambda_l(W^i_{jkh} - W^a_{jkh}\ell^i\ell_a).$$

IARJSET ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Impact Factor 8.066 ⪤ Peer-reviewed & Refereed journal ⪤ Vol. 11, Issue 5, May 2024

DOI: 10.17148/IARJSET.2024.11544

Thus, we conclude

Theorem 3.4. *If the projection of* $(W^i_{jkh} - W^a_{jkh}\ell^i\ell_a)$ *on indicatrix is recurrent, then the space is* $WR - F_n$*, provided* W^i_{jk} *is recurrent in sense of Cartan.*

From (3.4), we get

Corallary 3.1. *In* $WR - F_n$*, the projection of* W^i_{jkh} *on indicatrix is recurrent, if and only if* $W^a_{jkh}\ell_a$ *is recurrent.*

Let us consider a Finsler space F_n which the projection of W^i_{jk} on indicatrix is recurrent with respect to Cartan's connection characterized by (2.2). Using (1.12) in (2.2), we get

$$\left(W^a_{bc}h^i_a h^b_j h^c_k\right)_{|l} = \lambda_l W^a_{bc}h^i_a h^b_j h^c_k.$$

Using (1.13) in above equation, we get

$$\{W^a_{bc}(\delta^i_a - \ell^i\ell_a)(\delta^b_j - \ell^b\ell_j)(\delta^c_k - \ell^c\ell_k)\}_{|l} = \lambda_l\{W^a_{bc}(\delta^i_a - \ell^i\ell_a)(\delta^b_j - \ell^b\ell_j)(\delta^c_k - \ell^c\ell_k)\}$$

which can be written as

$$\begin{aligned}(W^i_{jk} &- \tfrac{1}{F}W^i_k\ell_h - \tfrac{1}{F}W^i_h\ell_k + \tfrac{1}{F^2}W^i_c y^c\ell_h - W^a_{kh}\ell^i\ell_a - \tfrac{1}{F}W^a_k\ell^i\ell_a\ell_h \\ &+ \tfrac{1}{F}W^a_h\ell^i\ell_a\ell_k - W^a_{bc}\ell^i\ell_a\ell^c\ell_h\ell^b\ell_k)_{|l} = \lambda_l(W^i_{jk} - \tfrac{1}{F}W^i_k\ell_h - \tfrac{1}{F}W^i_h\ell_k \\ &+ \tfrac{1}{F^2}W^i_c y^c\ell_h - W^a_{kh}\ell^i\ell_a - \tfrac{1}{F}W^a_k\ell^i\ell_a\ell_h + \tfrac{1}{F}W^a_h\ell^i\ell_a\ell_k - W^a_{bc}\ell^i\ell_a\ell^c\ell_h\ell^b\ell_k).\end{aligned}$$

Now, since the deviation tensor W^i_j is recurrent, i.e characterized by (2.3), then in view of (2.3), (1.11b), (1.11c), (1.2a) and (1.2c), above equation can be written as

(3.5) $$(W^i_{jk} - W^a_{jk}\ell^i\ell_a)_{|l} = \lambda_l\left(W^i_{jk} - W^a_{jk}\ell^i\ell_a\right).$$

Thus, we conclude

Theorem 3.5. *If the projection of* $(W^i_{jk} - W^a_{jk}\ell^i\ell_a)$ *on indicatrix is recurrent, then the space is* $WR - F_n$.

From (3.5), we get

Corollary 3.2. *In* $WR - F_n$*, the projection of* W^i_{jk} *on indicatrix is recurrent, if and only if* $W^a_{jk}\ell_a$ *is recurrent.*

IV. CONCLUSION

We introduced a Finsler space which W^i_{jkh} satisfies the recurrence property in sense of Cartan. Also, we proved that some tensors behave as recurrent.

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