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On optimizing the flow shop scheduling with probabilistic processing time for waiting time of jobs

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Abstract: The present paper deals with the Flow Shop Scheduling (FSS) models with special structures. The algorithm to optimize the waiting time of jobs has been proposed. The main goal of the study is to achieve a sequence of jobs that provides the minimum total waiting time of jobs. The numerical example has been solved to present the algorithm structure.

Keywords: Waiting time of jobs, Flow shop, Scheduling, Processing time.

I. INTRODUCTION

Today's global markets and instant communications mean that customers expect high-quality products and services when they need them, where they need them. Organizations, whether public or private, need to provide these products and services as effectively and efficiently as possible. The basic study has been made by Johnson [1954] to find optimal solution using heuristic algorithm for n jobs 2 and 3 machines flow shop problem.

Ignall and Schrage [1965] developed branch and bound algorithms for the permutation flow shop problem with makespan minimization. The work was further developed by Gupta [1969] Lockett and Muhlemann [1972] Crowin and Esogbue [1974], Maggu & Dass [1977], Singh T.P. [1985], [1986] ,Gupta [1982], [1986], Hundal et.al. [1988], Rajendran and Chaudhuri [1992], Singh T.P., Gupta D. et.al. [2004-2005], Narain L. [2006]. Further Singh T.P., Gupta D. [2006] made an attempt to minimize the rental cost of machines including job block through simple heuristic approach.

Singh Vijay [2011] put his efforts to study three machine flow shop scheduling problems with total rental cost. Further Gupta D. [2011] studied minimization of Rental Cost in Two Stage Flow Shop Scheduling Problem, in which Setup Time was separated from Processing Time and each associated with probabilities including Job Block Criteria.

The total waiting time of jobs is defined as the sum of the times of all the jobs which was consumed in waiting for their turn on both of the machines. There are some papers in the literature of scheduling theory which consider the waiting time to be important for scheduling the jobs on the machines.

Though minimization of waiting time may increase some other costs like machine idle cost or penalty cost of the jobs, yet the idea of minimizing the waiting time may be an economical aspect from Factory /Industry manager's view point when he has minimum time contract with a commercial party to complete the jobs. The problem discussed here is wider & practically more applicable and has significant use of theoretical results in process industries or in the situations when the objective is to minimize the total waiting time of jobs.

II. PROBLEM FORMULATION

Let p jobs $1,2,3,4,\dots\dots,p$ be processed through two machines M and N in the order $M\ N$.

Job $i\ (i = 1,2,3,\dots\dots,p)$ has processing time M_i and N_i on each machine respectively assuming their respective probabilities s_i and t_i such that $0 \le s_i \le 1\ ;\ 0 \le t_i \le 1\ \&\ \sum s_i = \sum t_i = 1$ $0 \le s_i \le 1\ ;\ 0 \le t_i \le 1\ \&\ \sum s_i = \sum t_i = 1$

The mathematical model of the problem in matrix form can be stated as:

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	Machine M		Machine N	
Job	M_i	s_i	N_i	t_i
1.	M_1	s_1	N_1	t_1
2.	M_2	s_2	N_2	t_2
3.	M_3	s_3	N_3	t_3
. . .	. . .	. . .	. . .	. . .
p.	M_p	s_p	N_p	t_p

Table 1

Our objective is to find an optimal sequence S of jobs minimizing the total waiting time of all jobs.

III. ASSUMPTIONS

1) p Jobs are processed through two machines M & N in the order M N i.e. no passing is allowed.
2) $\sum s_i = \sum t_i = 1$
3) A job is an entity i.e. even though the job represents a lot of individual part, no job may be processed by more than one machine at a time.

Lemma 1. Let p jobs 1, 2, 3,…,p be processed through two machines M, N in order MN with no passing allowed. Let job i ($i = 1,2,3, \ldots\ldots, p$) has processing times M_i and N_i on each machine respectively assuming their respective probabilities s_i and t_i such that $0 \le s_i \le 1$; $0 \le t_i \le 1$ & $\sum s_i = \sum t_i = 1$. Expected processing times are defined as $M_i' = M_i * s_i$ $N_i' = N_i * t_i$ satisfying expected processing times structural relationship:

Max $M_i' \le$ Min N_i' then for the p job sequence S: $\alpha_1, \alpha_2, \alpha_3, \ldots \ldots \ldots \alpha_p$

$$T_{\alpha_p N} = M'_{\alpha_1} + N'_{\alpha_1} + N'_{\alpha_2} \ldots + N'_{\alpha_p}$$

Where T_{aB} is the completion time of job a on machine B

Proof. Applying mathematical Induction hypothesis on p:

Let the statement $S(p): T_{\alpha_p N} = M'_{\alpha_1} + N'_{\alpha_1} + N'_{\alpha_2} \ldots + N'_{\alpha_p}$

$$T_{\alpha_1 M} = M'_{\alpha_1}$$
$$T_{\alpha_1 N} = M'_{\alpha_1} + N'_{\alpha_1}$$

Hence for p= 1 the statement $S(1)$ is true.
Let for p= k, the statement $S(k)$ be true, i.e.,

$$T_{\alpha_k N} = M'_{\alpha_1} + N'_{\alpha_1} + N'_{\alpha_2} \ldots + N'_{\alpha_k}$$

Now,
$$T_{\alpha_{k+1} N} = Max\left(T_{\alpha_{k+1} M}, T_{\alpha_k N}\right) + N'_{\alpha_{k+1}}$$

As $Max\ M_i' \le Min\ N_i'$

Hence

$$T_{\alpha_{k+1} N} = M'_{\alpha_1} + N'_{\alpha_1} + N'_{\alpha_2} \ldots + N'_{\alpha_k} + N'_{\alpha_{k+1}}$$

Hence for $p = k+1$ the statement $S(k+1)$ holds true. Since $S(p)$ is true for $p = 1, p = k, p = k+1$, and k being arbitrary. Hence $S(p): T_{\alpha_p N} = M'_{\alpha_1} + N'_{\alpha_1} + N'_{\alpha_2} \ldots + N'_{\alpha_p}$ is true.

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Lemma 2. With the same notations as that of Lemma1, for p- job sequence $S: \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_k, \dots, \alpha_p$

$$W_{\alpha_1} = 0$$

$$W_{\alpha_k} = M'_{\alpha_1} + \sum_{r=1}^{k-1} x_{\alpha_r} - M'_{\alpha_k}$$

Where W_{α_k} is the waiting time of job α_k for the sequence $(\alpha_1, \alpha_2, \alpha_3, \dots \dots \dots \alpha_p)$ and

$$x_{\alpha_r} = N'_{\alpha_r} - M'_{\alpha_r}, \ \ \alpha_r \in (1, 2, 3, \dots., p)$$

Proof. $W_{\alpha_1} = 0$

$$W_{\alpha_k} = Max\left(T_{\alpha_k M}, T_{\alpha_{k-1} N}\right) - T_{\alpha_k M}$$

$$= M'_{\alpha_1} + N'_{\alpha_1} + N'_{\alpha_2} \dots + N'_{\alpha_{k-1}} - M'_{\alpha_1} - M'_{\alpha_2} \dots - M'_{\alpha_k}$$

$$= M'_{\alpha_1} + \sum_{r=1}^{k-1} (x_{\alpha_r}) - M'_{\alpha_k}$$

Theorem 1. Let p jobs 1, 2, 3,…,p be processed through two machines M, N in order MN with no passing allowed. Let job i $(i = 1,2,3, \dots\dots, p)$ has processing times M_i and N_i on each machine respectively assuming their respective probabilities s_i and t_i such that $0 \le s_i \le 1$; $0 \le t_i \le 1$ & $\sum s_i = \sum t_i = 1$. Expected processing times are defined as $M'_i = M_i * s_i$ $N'_i = N_i * t_i$ satisfying expected processing times structural relationship:

Max $M'_i \le$ Min N'_i then for any p job sequence $S: \alpha_1, \alpha_2, \alpha_3, \dots \dots \dots \alpha_p$ the total waiting time T_w (say)

$$T_w = pM'_{\alpha_1} + \sum_{r=1}^{p-1} z_{\alpha_r} - \sum_{i=1}^{p} M'_i$$

$$z_{\alpha_r} = (p - r)x_{\alpha_r} \ ; \alpha_r \epsilon (1, 2, 3, \dots, p)$$

Proof. From Lemma 2 we have

$$W_{\alpha_1} = 0$$

$$k = 2, k - 1 = 1$$

$$W_{\alpha_2} = M'_{\alpha_1} + \sum_{r=1}^{1} x_{\alpha_r} - M'_{\alpha_2}$$

$$= M'_{\alpha_1} + x_{\alpha_1} - M'_{\alpha_2}$$

$$k = 3, k - 1 = 2$$

Continuing in this way

$$k = p, k - 1 = p - 1$$

$$W_{\alpha_p} = M'_{\alpha_1} + \sum_{r=1}^{p-1} x_{\alpha_r} - M'_{\alpha_p}$$

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Hence total waiting time

$$T_w = W_{\alpha_1} + W_{\alpha_2} + W_{\alpha_3} + \cdots + W_{\alpha_p}$$

$$= pM'_{\alpha_1} + \sum_{r=1}^{p-1} (p-r)x_{\alpha_r} - \sum_{i=1}^{p} M'_{\alpha_i}$$

IV. ALGORITHM

To obtain optimal schedule we proceed as follows:

Step 1: Define expected processing times $M_i^{'}$ and $N_i^{'}$ on machine M & N respectively as follows:

(i) $M_i^{'} = M_i * s_i$
(ii) $N_i^{'} = N_i * t_i$
$Max\ M_i^{'} \leq Min\ N_i^{'}$

Step 2: Fill up the values in the following table

Job	Machine M	Machine N		$z_{ir} = (p-r)x_i$				
I	$M_i^{'}$	$N_i^{'}$	$x_i = N_i^{'} - M_i^{'}$	$r = 1$	$r = 2$	$r = 3$	………………	$r = p-1$
1.	$M_1^{'}$	$N_1^{'}$	x_1	z_{11}	z_{12}	z_{13}	………………	$z_{1\,p-1}$
2.	$M_2^{'}$	$N_2^{'}$	x_2	z_{21}	z_{22}	z_{23}	………………	$z_{2\,p-1}$
3.	$M_3^{'}$	$N_3^{'}$	x_3	z_{31}	z_{32}	z_{33}	………………	$z_{3\,p-1}$
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
p.	$M_p^{'}$	$N_p^{'}$	x_p	z_{p1}	z_{p2}	z_{p3}	……………..	$z_{p\,p-1}$

Table 2

Step 3: Arrange the jobs in increasing order of x_i.

Let the sequence found be $(\alpha_1, \alpha_2, \alpha_3, \dots \dots \dots \alpha_p)$

Step 4: Find min{ $M_i^{'}$ }

Now two cases arise:
If $M_{\alpha_1}^{'} =$ min{ $M_i^{'}$ } then schedule according to step 3 is the required optimal sequence
If $M_{\alpha_1}^{'} \neq$ min{ $M_i^{'}$ } then go to step 5

Step 5: Consider the different sequence of jobs $S_1, S_2, S_3, \dots \dots, S_p$. Where S_1 is the sequence obtained in step 3, Sequence $S_i(i = 2,3, \dots \dots, p)$ can be obtained by placing i^{th} job in the sequence S_1 to the first position and rest of the sequence remaining same.

Step 6: Calculate the total waiting time T_w for all the sequences $S_1, S_2, S_3, \dots \dots, S_p$ using the following formula:

$$T_w = pM_b^{'} + \sum_{r=1}^{p-1} z_{ar} - \sum_{i=1}^{p} M_i^{'}$$

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$M_b' =$ Expected processing time of the first job on machine M in sequence S_i

$z_{ar} = (p - r)x_{ar}$; $a = \alpha_1, \alpha_2, \alpha_3, \dots \dots \dots \alpha_p$

The sequence with minimum total waiting time is the required optimal sequence.

NUMERICAL ILLUSTRATION

Let 5 jobs 1, 2, 3, 4, 5 are processed in a string S on two machines M & N. Let the processing time matrix be seen as given below:

Job	Machine M		Machine N	
I	M_i	s_i	N_i	t_i
1.	4	0.2	9	0.1
2.	3	0.2	4	0.2
3.	2	0.2	3	0.3
4.	2	0.3	5	0.2
5.	6	0.1	4	0.2

Table 3

Our objective is to obtain optimal string, minimizing the total waiting time for the jobs.

SOLUTION

As per step 1- define new expected processing time M_i' & N_i' on machine M & N respectively as shown in the following table

JOB	M_i'	N_i'
1.	$4 * 0.2 = 0.8$	$9 * 0.1 = 0.9$
2.	$3 * 0.2 = 0.6$	$4 * 0.2 = 0.8$
3.	$2 * 0.2 = 0.4$	$3 * 0.3 = 0.9$
4.	$2 * 0.3 = 0.6$	$5 * 0.2 = 1.0$
5.	$6 * 0.1 = 0.6$	$4 * 0.2 = 0.8$

Table 4

Max $M_i' = 0.8 \leq$ Min $N_i' = 0.8$

As per step 2- Fill up the values in the following table

Job	Machine M	Machine N					
I	M_i'	N_i'	$x_i = N_i' - M_i'$	$r = 1$	$r = 2$	$r = 3$	$r = 4$
1.	0.8	0.9	0.1	0.4	0.3	0.2	0.1
2.	0.6	0.8	0.2	0.8	0.6	0.4	0.2
3.	0.4	0.9	0.5	2.0	1.5	1.0	0.5
4.	0.6	1.0	0.4	1.6	1.2	0.8	0.4
5.	0.6	0.8	0.2	0.8	0.6	0.4	0.2

Table 5

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As per step 3- Arrange the jobs in increasing order of x_i i.e. the sequence found be 1, 2, 5, 4, 3

As per step 4- $Min\{M_i'\} = 0.4 \neq M_1'$

As per step 5- Consider the following different sequences of jobs

S_1: 1, 2, 5, 4, 3
S_2: 2, 1, 5, 4, 3
S_3: 5, 1, 2, 4, 3
S_4: 4, 1, 2 ,5, 3
S_5: 3,1, 2, 5, 4

As per step 6- Calculate the total waiting time for the sequences S_1, S_2, S_3, S_4, S_5

For the sequence S_1: 1, 2, 5, 4, 3
Hence total waiting time $T_w = 2.8$
For the sequence S_2: 2, 1, 5, 4, 3
Total waiting time $T_w = 1.9$
For the sequence S_3: 5, 1, 2, 4, 3
Total waiting time $T_w = 1.9$
For the sequence S_4: 4, 1, 2 ,5, 3
Total waiting time $T_w = 1.6$
For the sequence S_5: 3,1, 2, 5, 4
Total waiting time $T_w = 2.1$

Hence schedule S_4: 4, 1, 2 ,5, 3 is the required schedule with minimum total waiting time.

REFERENCES

[1]. **Johnson [1954],** *optimal two and three stage production schedule with set up times included.* Nay Res Log Quart Vol 1 pp 61-68.
[2]. **Ignall, E. and Schrage, L.E., [1965]**, *Application of branch and bound techniques to some flow shop problems*. Operation Research 13, 400-412.
[3]. **Lockett, A.G. and Muhlemann, A.P., [1972],** *Technical notes: a scheduling Problem involving sequence dependent changeover times*. Operation Research 20, 895- 902.
[4]. **Maggu P.L. and Dass G. [1977]**, *equivalent jobs for job block in job sequencing,* Operations Research, Vol 14 No. 4, pp 277-281.
[5]. **Singh T.P. [1985],** *on n×2 shop problem involving job block. Transportation times and Break-down Machine times,* PAMS Vol.XXI No.1-2.
[6]. **Gupta, J.N.D.,[1986].** *Flow shop schedules with sequence dependent setup times.* Journal of the Operations Research Society of Japan 29, 206-219
[7]. **Rajendran, C. and Chaudhuri, D. [1992],** *an efficient heuristic approach to the scheduling of jobs in a flow-shop.* European Journal of Operational Research 61, 318. 325.
[8]. **Singh T.P., Gupta D. and Kumar R. [2006],** *Optimal two stage production schedule with Group job-restrictions having set up times separated from processing time associated with probabilities.* Reflections des ERA, (JMS) Vol. I pp 53-70.
[9]. **Singh T.**P., **Gupta D. & Kumar R. [2006],** *Bi-criteria in scheduling under specified rental policy, processing time associated with probabalities including job block concept,* presented at National Conference on information technology at NCCEI, March 18-20.
[10]. **Narrain L.**, **Gupta D. & Kumar R.[2006],** *Minimization rental cost under specified rental policy in two stage flow shop the processing times associated with probabilities including job block criteria,* Reflections des ERA, (JMS) Vol. 2 pp 107-120.
[11]. **Singh Vijay [2011],***Three machines flow shop scheduling problems with total rental cost,* International referred journal, Jan 2011, Vol-II pp79-80
[12]. **Gupta D. [2011],** *Minimizing rental cost under specified rental policyin two stage flowshop, the processing time associated with probabilities including break down interval and job block criteria,* European Journal of Business and Management (USA), Vol.3 No.2 pp 85-103.